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Flow Characteristics through Emerged Rigid Vegetation over a Perturbed Bed

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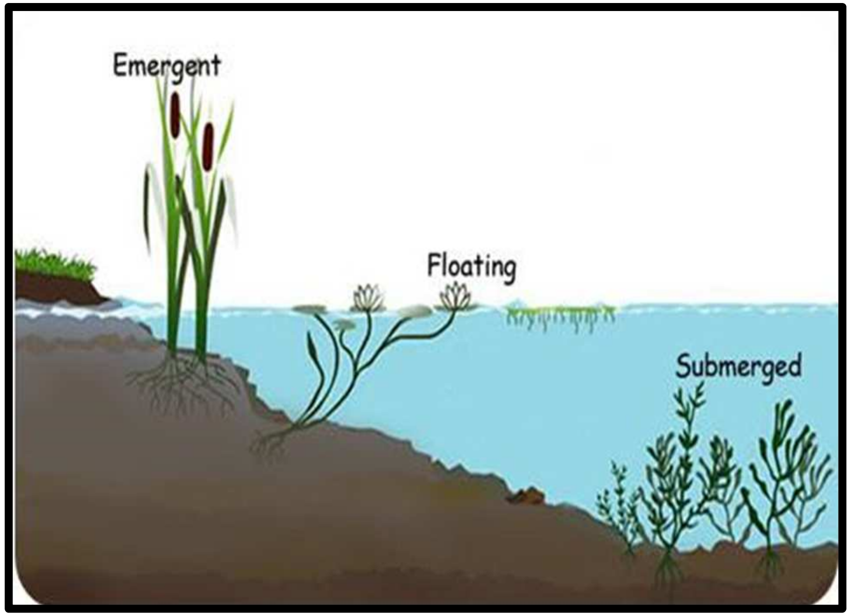
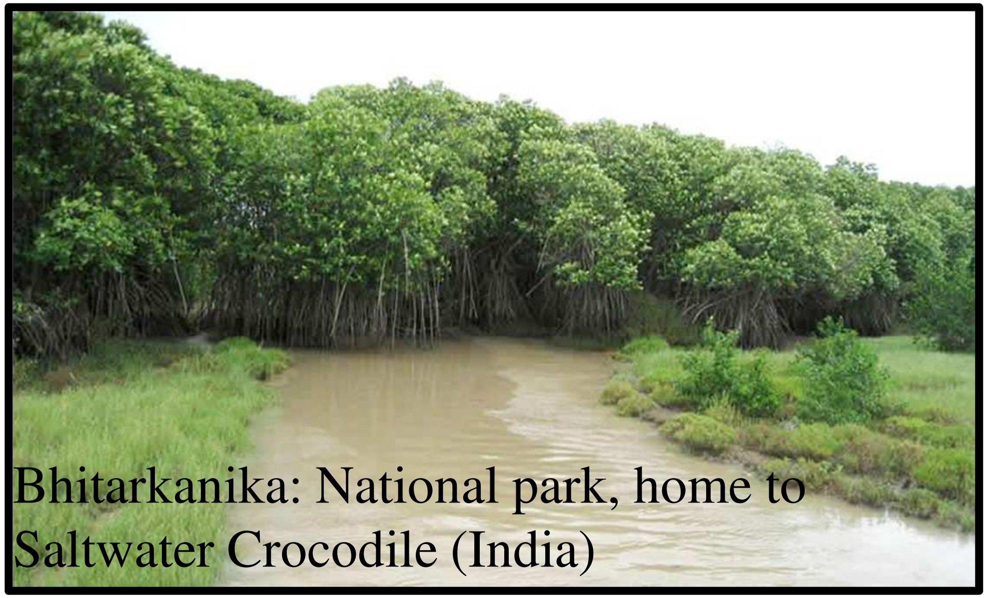
Wetlands

- a land area that is saturated with water
- applications in storm water management, habitat restoration etc.
- reduces river bed erosion, increase river bank stability.
- beautiful landscape



Sunderbans: Largest single block of tidal mangrove forest in the world (India)



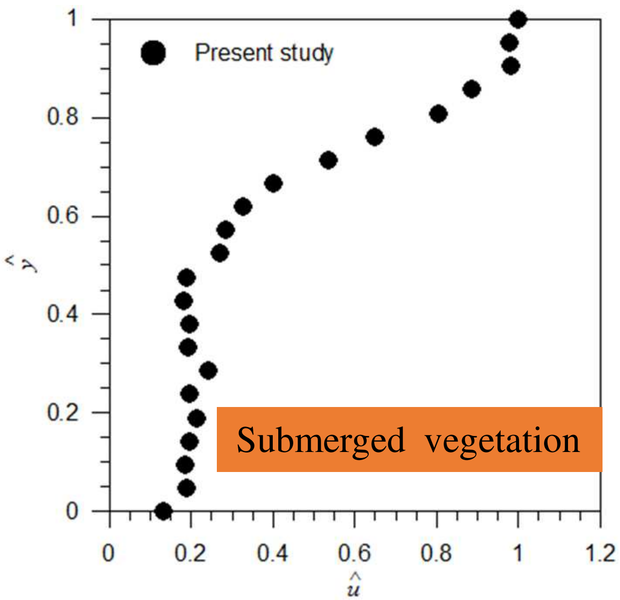
Bhitarkanika: National park, home to Saltwater Crocodile (India)

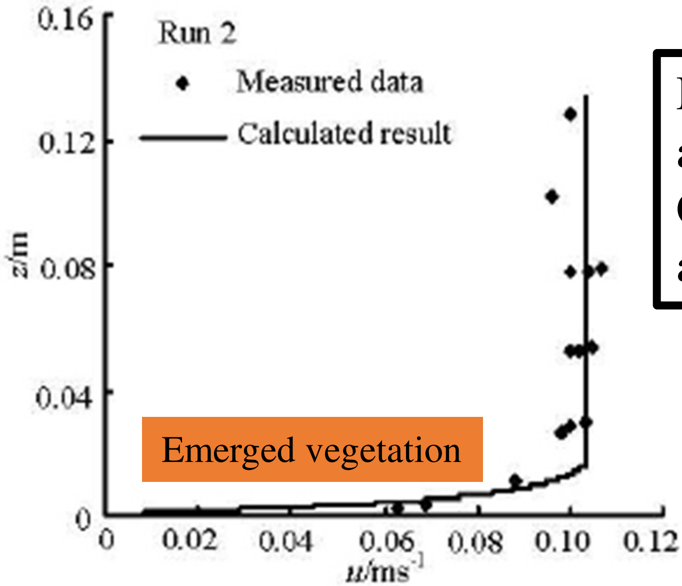
Problem Definition

In the present study, an analytical approach for flow through emerged vegetation over a sinusoidal bed for low flow velocities is proposed focusing on the influences on the flow process of following:

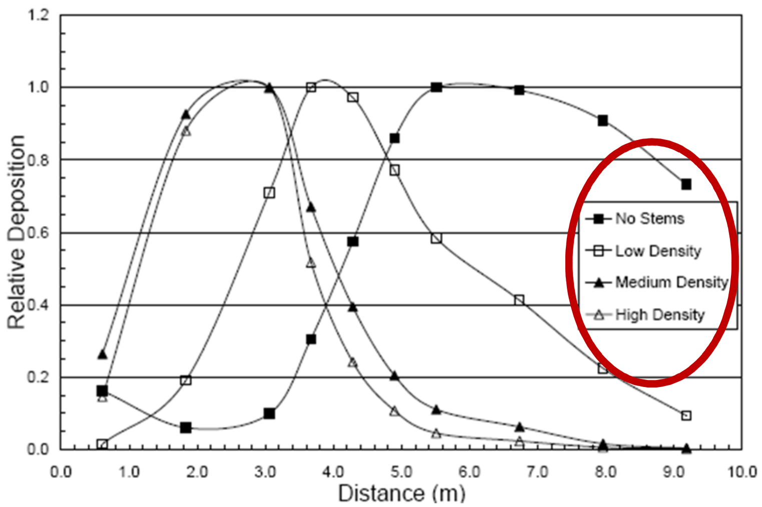
- vegetation density
- bed form

Some known facts



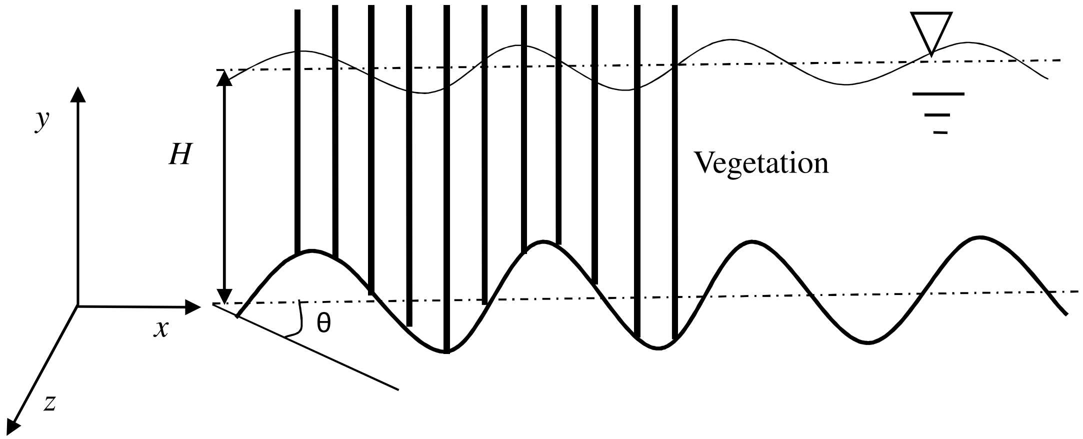


Measured data : Rowinski and Kubrak (2002)
Calculated results : Huai et al. (2009)



Sharpe and James (2006)

Problem Formulation



Schematic for flow through emerged vegetation over sinusoidal bed

Assumptions

- Vegetation distribution pattern is not considered
- Steady Open channel Flow
- The bed slope and velocity to be small

Governing Equations

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

X-momentum Equation:

Inertia term, Pressure term, Viscous term, Gravity term, Porosity term

$$\eta\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \eta\left[-\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \rho g \sin\theta\right] - T_1$$

Y-momentum Equation:

$$\eta\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \eta\left[-\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \rho g \cos\theta\right] - T_2$$

$$T_1 = \frac{\mu\eta^2 u}{kp} \qquad T_2 = \frac{\mu\eta^2 v}{kp}$$

Non-Dimensional Variables

$$\hat{x} = \frac{x}{H}, \hat{y} = \frac{y}{H}, \hat{u} = \frac{u}{u_0}, \hat{v} = \frac{v}{u_0}, \hat{P} = \frac{P}{P_0}, \hat{\tau} = \frac{\tau}{\tau_0}$$

where,

The effect of inertia is considered to be negligible, and the flow is mainly gravity driven rendering the order of the gravity force and viscous force to be same. Equating these forces the scales for u and P are obtained as

$$u_0 = \frac{\rho g H^2 \sin\theta}{\mu}, \quad P_0 = \rho g H \sin\theta, \quad \tau_0 = \frac{\mu u_0}{H}$$

Non-dimensional Governing Equations

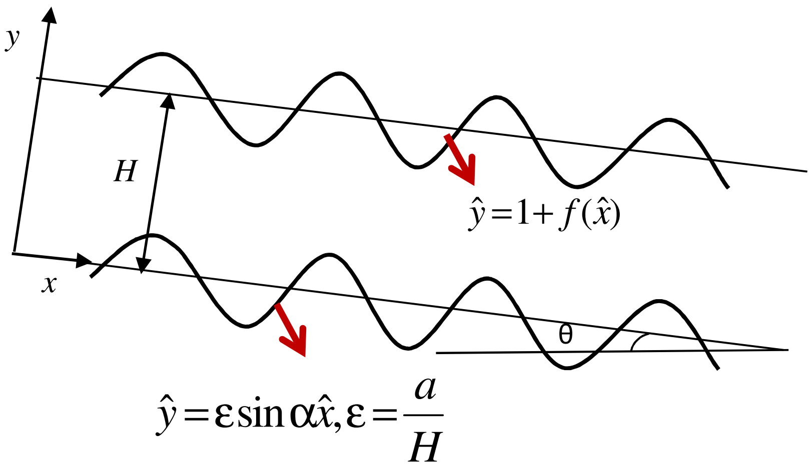
$$\frac{\partial \hat{u}}{\partial \hat{x}}+\frac{\partial \hat{v}}{\partial \hat{y}}=0$$

$$R_e(\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{u}}{\partial \hat{y}})=-\frac{\partial \hat{P}}{\partial \hat{x}}+(\frac{\partial^2 \hat{u}}{\partial \hat{x}^2}+\frac{\partial^2 \hat{u}}{\partial \hat{y}^2})+\frac{\rho g H^2 \sin\theta}{\mu u_0}-T_3$$

$$R_e(\hat{u}\frac{\partial \hat{v}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{v}}{\partial \hat{y}})=-\frac{\partial \hat{P}}{\partial \hat{y}}+(\frac{\partial^2 \hat{v}}{\partial \hat{x}^2}+\frac{\partial^2 \hat{v}}{\partial \hat{y}^2})-\frac{\rho g H^2 \cos\theta}{\mu u_0}-T_4$$

$$where, R_e=\frac{u_0 H\rho}{\mu}, T_3=\frac{H^2\eta\hat{u}}{kp}, T_4=\frac{H^2\eta\hat{v}}{kp}$$

Introducing bed Perturbation



Bed variation(Kang and Chen 1995)

The profile of the bed suggests that a series solution may be assumed for all the flow variables. That is all the variables can be expanded with respect to the small perturbation parameter (ε).

$$\hat{u}=\hat{u}_0+\varepsilon\hat{u}_1+\varepsilon^2\hat{u}_2+\ldots,\ \hat{v}=\hat{v}_0+\varepsilon\hat{v}_1+\varepsilon^2\hat{v}_2+\ldots,\ \hat{P}=\hat{P}_0+\varepsilon\hat{P}_1+\varepsilon^2\hat{P}_2+\ldots,$$

$$f(\hat{x})=\varepsilon f_1(\hat{x})+\varepsilon^2 f_2(\hat{x})\ldots,\ \hat{\tau}=\hat{\tau}_0+\varepsilon\hat{\tau}_1+\varepsilon^2\hat{\tau}_2+\ldots$$

where, $\alpha=\frac{2\Pi H}{\lambda}, \varepsilon=\frac{a}{H}$ where, a is the amplitude and λ is the wavelength

0^{th} order Equations

$$\frac{\partial \hat{u}_0}{\partial \hat{x}} = 0$$

$$\frac{d^2 \hat{u}_0}{d\hat{y}^2} = -\frac{\rho g H^2 \sin\theta}{\mu u_0} + T_3$$

$$\frac{d\hat{P}_0}{d\hat{y}} = -\frac{\rho g H^2 \cos\theta}{\mu u_0}$$

1^{st} order Equations

$$\frac{\partial \hat{u}_1}{\partial \hat{x}} + \frac{\partial \hat{v}_1}{\partial \hat{y}} = 0$$

$$\frac{\partial^2 \hat{u}_1}{\partial \hat{x}^2} + \frac{\partial^2 \hat{u}_1}{\partial \hat{y}^2} - \frac{\partial \hat{P}_1}{\partial \hat{x}} - T_3 = 0$$

$$\frac{\partial^2 \hat{v}_1}{\partial \hat{x}^2} + \frac{\partial^2 \hat{v}_1}{\partial \hat{y}^2} - \frac{\partial \hat{P}_1}{\partial \hat{y}} - T_4 = 0$$

Boundary conditions

(0^{th} order)

No slip condition at the bed gives at, $\hat{y} = 0, \hat{u}_0 = 0$

At the free surface, $\hat{y} = 1$

zero shear stress at the free surface is, $\left.\frac{d\hat{u}_{10}}{d\hat{y}}\right|_{(\hat{y}=1)} = 0$

Boundary conditions (Contd.)

(1st order)

No slip condition at the bed gives at, $\hat{y} = 0$

No slip condition at the bed and no infiltration through the bed gives,

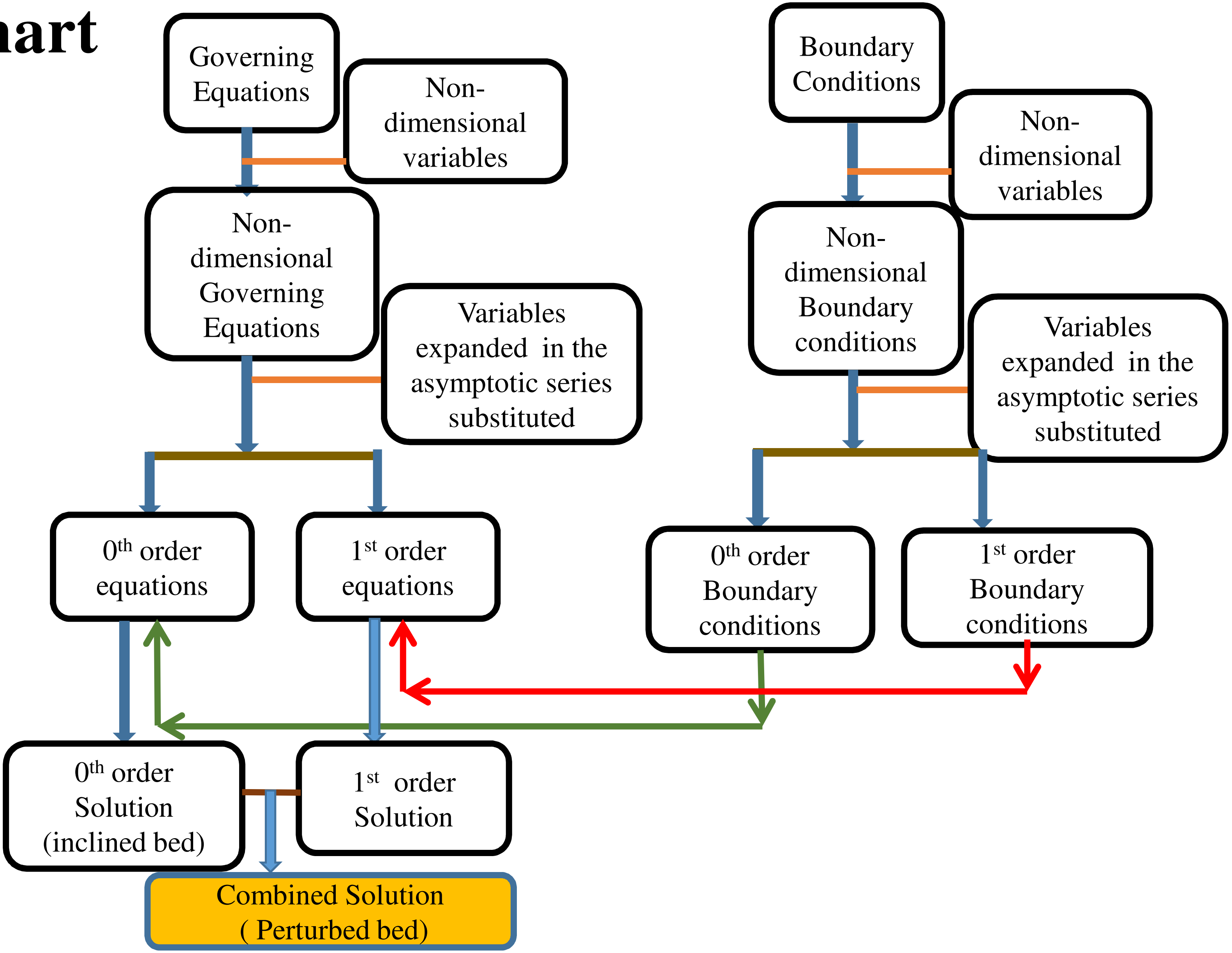
$$\hat{u}_1 + \frac{d\hat{u}_0}{d\hat{y}} \sin\alpha\hat{x} = 0, \hat{v}_1 = 0$$

At the free surface, $\hat{y} = 1$

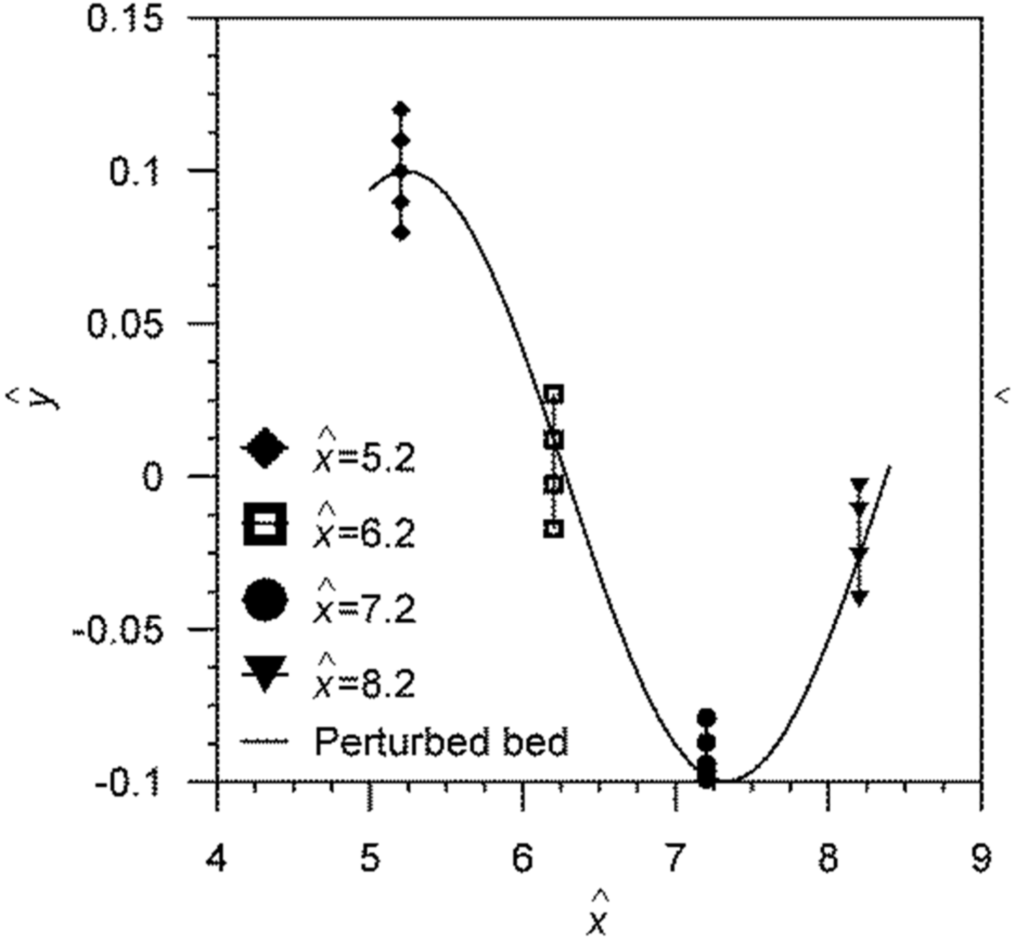
Kinematic boundary condition at the free surface is $\hat{u}_0 \dfrac{\partial f_1}{\partial \hat{x}} = \hat{v}_1$

Zero shear and continuity of normal stress at the free surface

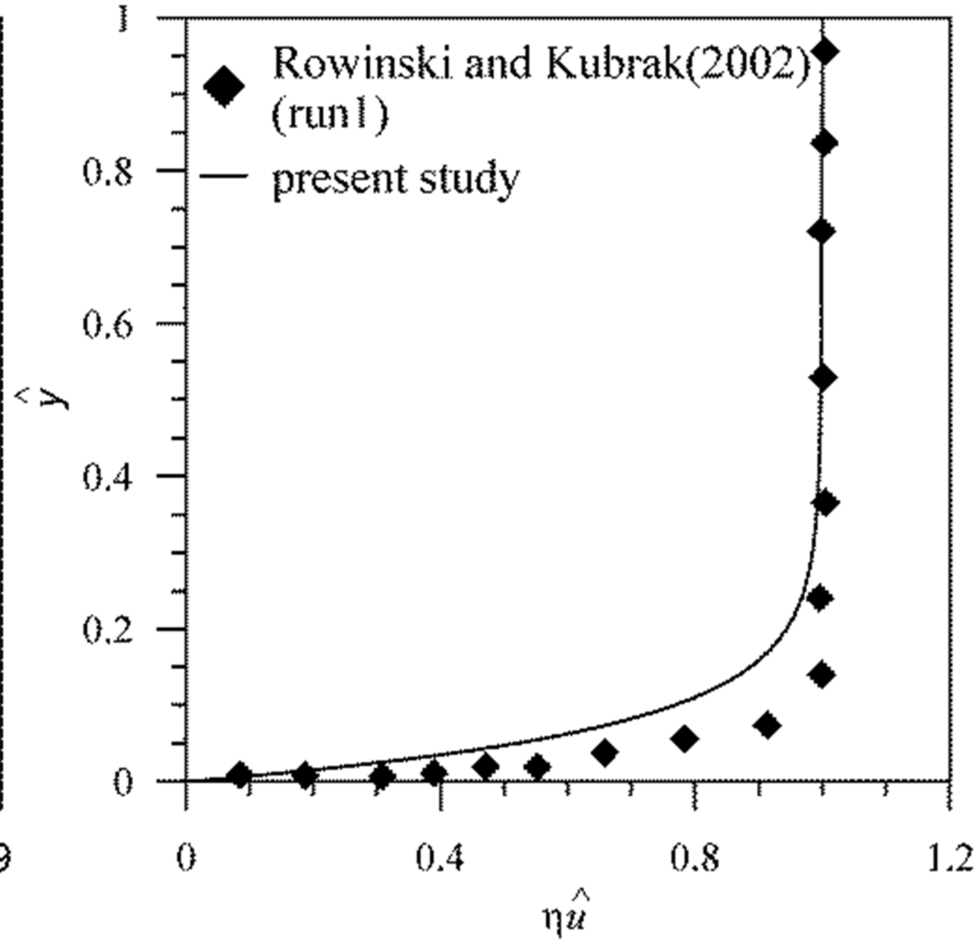
Flow Chart


Governing Equations
Non-dimensional variables
Non-dimensional Governing Equations
Variables expanded in the asymptotic series substituted
Boundary Conditions
Non-dimensional variables
Non-dimensional Boundary conditions
Variables expanded in the asymptotic series substituted
0th order equations
1st order equations
0th order Boundary conditions
1st order Boundary conditions
0th order Solution (inclined bed)
1st order Solution
Combined Solution (Perturbed bed)

Validation

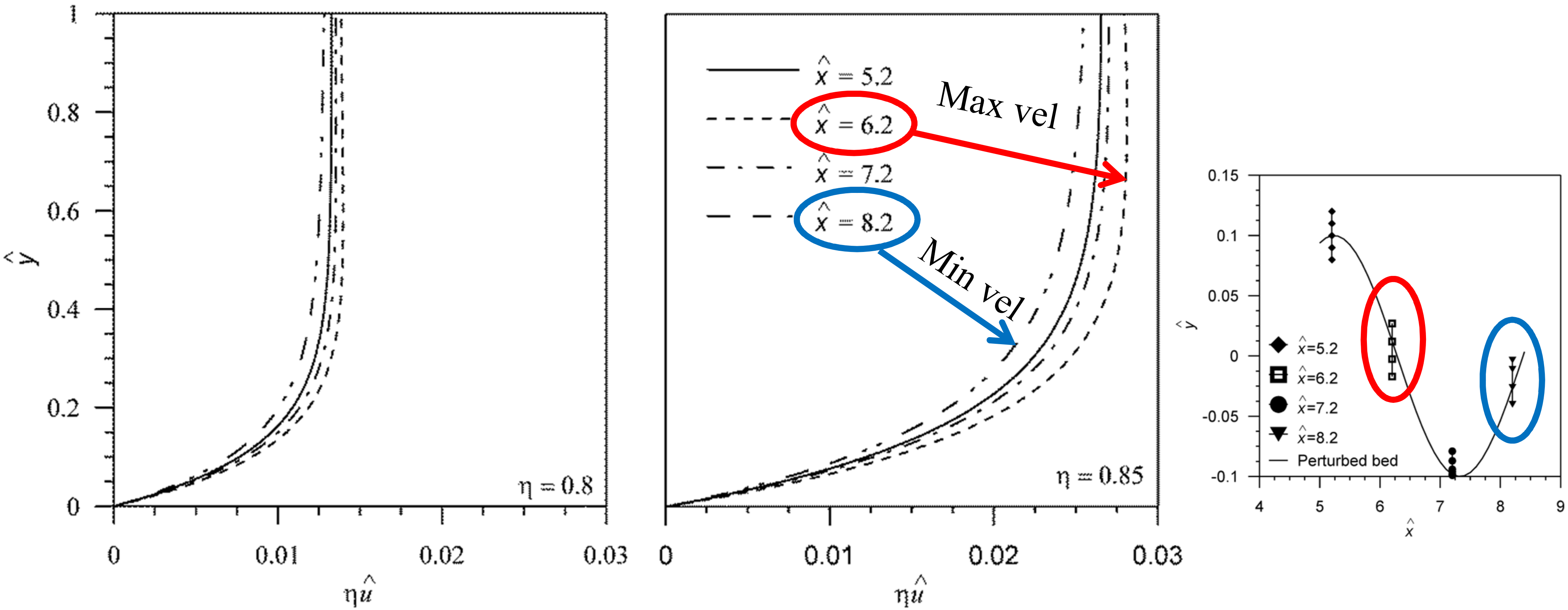


Different sections over the sinusoidal bed



Validation for a plane inclined bed

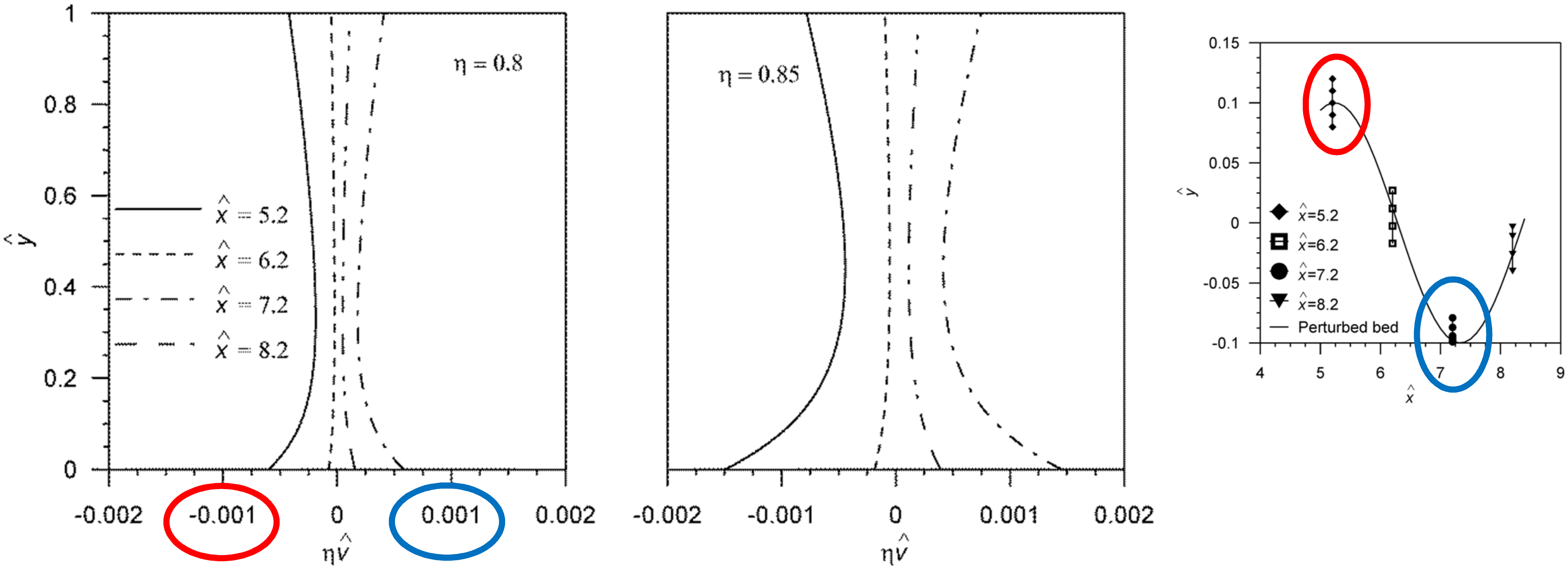
Longitudinal velocity profile



Vertical distributions of longitudinal velocity component at different sections for various η

- for higher values of η, the variations of the velocities at different sections are quite significant.

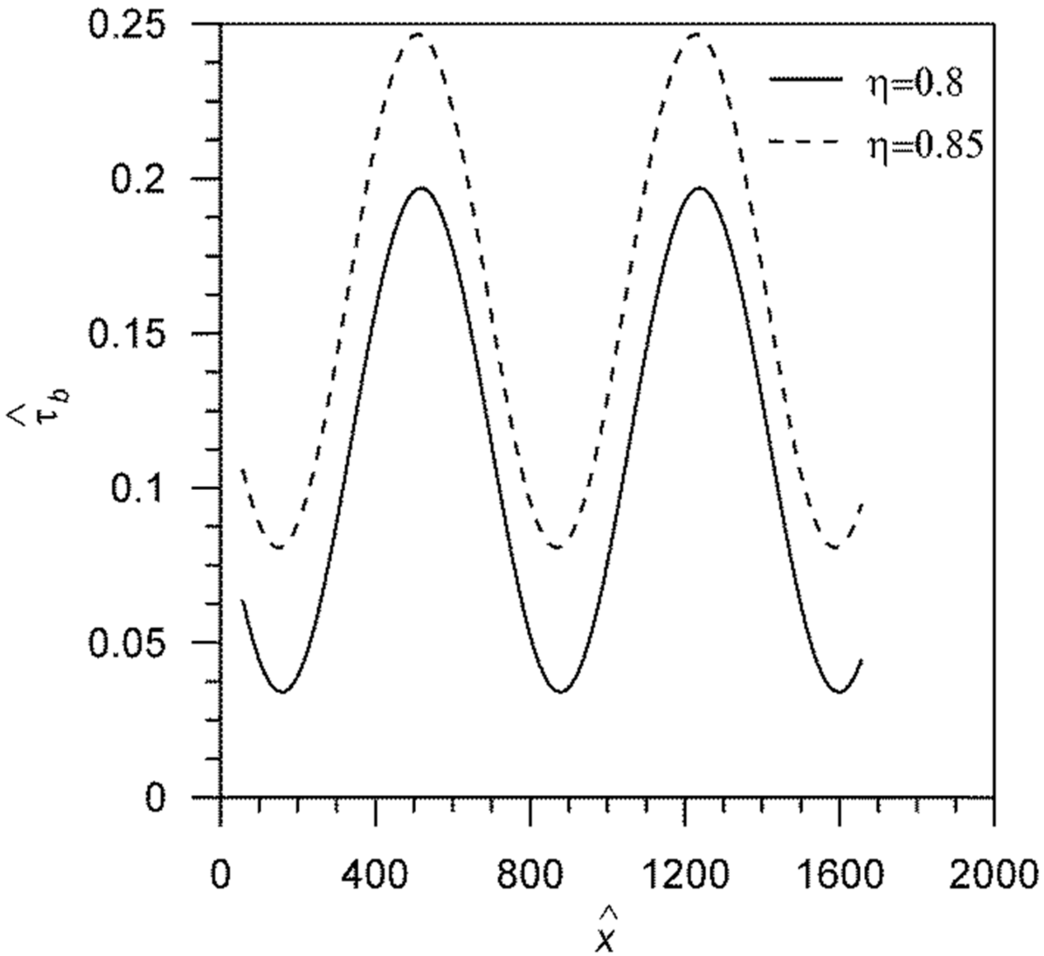
Vertical velocity profile



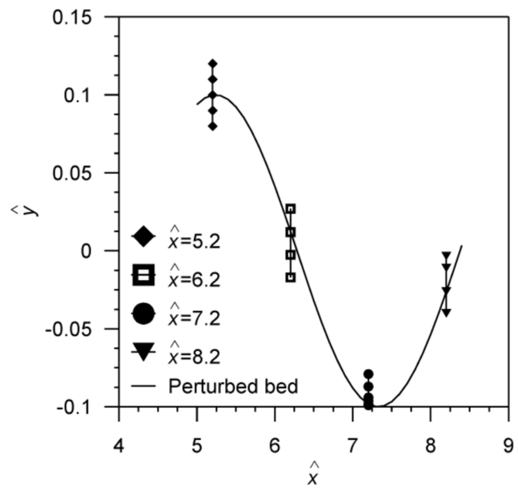
Vertical distribution of vertical velocity at different sections for different η

- at the crest and trough, vertical components of velocity are showing totally opposite trend
- At the crest: velocity is in upward direction
- At the trough: velocity is in downward direction

Bed Shear



Variation of the bed shear along the bed for different η



- the bed shear increases with the decrease in the vegetation density
- 7% decrease in vegetation density increases the bed shear stress by nearly 20%

Conclusions

- Bed shear increases with decrease of vegetation density
- At the crest: velocity is in upward direction
- At the trough: velocity is in downward direction
- Longitudinal velocities are maximum at the downstream slope

Thanks

Experiments

Flume

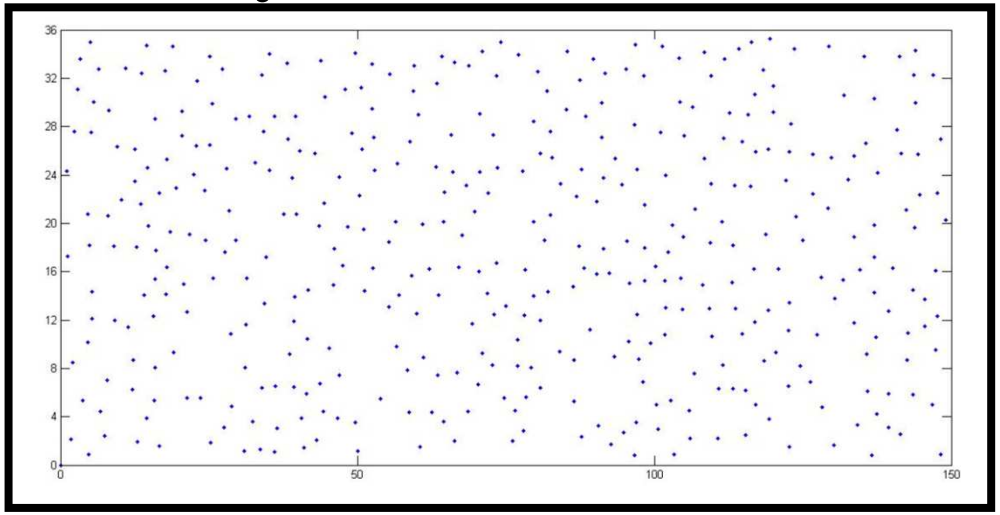
Experiment with submerged rigid vegetation along with sediment transport

Pictorial view of ADV

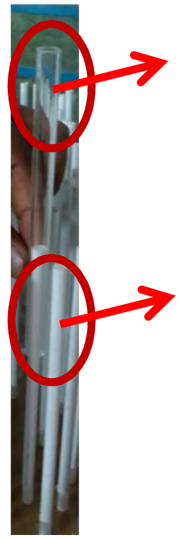
Pictorial view of ADP

Generation of Random Array

- minimum distance between the centres of adjacent cylinders = 2 cm
- minimum of a 0.75 cm distance between the cylinder centres and the flume wall
- number density of the cylinder arrays, *Ng* is 900 cylinders/m^2



Generation of Random distribution of array by Matlab



Glass tube of dia 9 mm

PVC pipes of dia 6 mm

Random distribution of rigid vegetation array

Solutions

Solutions of the 0th order and 1st order problem.

(0th Order) Solving the continuity and momentum Eqs. (8)-(10),

$$\hat{u}_0 = c_1 e^{m_2 \hat{y}} + c_2 e^{-m_2 \hat{y}} + \frac{1}{m_2^2} \qquad where, m_2^2 = \frac{H^2 \eta}{kp} = C_s T_0^2 S_s^2 H^2 (\frac{1}{\eta} - 1)^2$$

where, *Cs* is the pore shape factor, *To* is pore tortuosity, *Ss* is the pore specific surface area.

c_1 and c_2 are the constants of integration. the solution of the 0th order problem are obtained for various values of m_2^2
and these values are used for the solution of 1st order problem.

(1st Order) The boundary conditions suggest the solution of the 1st order to be assumed as follows:

$$\hat{u}_1 = \hat{U}_1(\hat{y}) e^{j\alpha\hat{x}}, \hat{v}_1 = \hat{V}_1(\hat{y}) e^{j\alpha\hat{x}}, g_1 = A e^{j\alpha\hat{x}}, \hat{P}_{11} = \hat{P}_1(\hat{y}) e^{j\alpha\hat{x}}$$

where, $j = \sqrt{-1}$ and A and B are complex constants and $\hat{U}_1, \hat{V}, \hat{P}_1$ are the non dimensional function of

Here only the real parts of the variables are to be considered. Substituting the above Eq.(20) in the continuity and momentum Eqs. (11)-(13) and simplifying, following ordinary differential equations and their respective solutions are written below:

$$\hat{U}_1^{IV} - (2\alpha^2 + m_2^2)\hat{U}_1^{II} + \left(\alpha^4 + \alpha^2 m_2^2\right)\hat{U}_1 = 0$$

$$\hat{U}_1 = A_1^{/} e^{(\sqrt{\alpha^2 + m_2^2})\hat{y}} + B_1^{/} e^{-(\sqrt{\alpha^2 + m_2^2})\hat{y}} + C_1^{/} e^{\alpha\hat{y}} + D_1^{/} e^{-\alpha\hat{y}}$$

$$\hat{V}_1^{IV} - (2\alpha^2 + m_2^2)\hat{V}_1^{II} + \left(\alpha^4 + \alpha^2 m_2^2\right)\hat{V}_1 = 0$$

$$\hat{V}_1 = A_2^{/} e^{(\sqrt{\alpha^2 + m_2^2})\hat{y}} + B_2^{/} e^{-(\sqrt{\alpha^2 + m_2^2})\hat{y}} + C_2^{/} e^{\alpha\hat{y}} + D_2^{/} e^{-\alpha\hat{y}}$$

where, $\hat{U}_1^{IV}, \hat{U}_1^{II}, \hat{V}_1^{IV}, \hat{V}_1^{II}$ are fourth order and second order derivatives of $\hat{U} and \hat{V}$ respectively

Substituting the values of $\hat{U} and \hat{V}$

in the 1st order boundary conditions (16)-(18), 10 linear algebraic equations with 10 complex constants as unknowns $A, B, A_1^/, A_2^/, B_1^/, B_2^/, C_1^/, C_2^/, D_1^/ D_2^/$

are formed and solved. Then substituting these unknowns, the combined solution is obtained.

Shear stress profile

Non-dimensional bed shear stress at the perturbed bed may be written as

$$\hat{\tau}_{\mathrm{b}} = \hat{\tau} + \frac{\partial \hat{\tau}}{\partial \hat{y}} \varepsilon \sin \alpha \hat{x}$$

Substituting the shear stress,

$$Bedshear = \hat{\tau}_b = \hat{\tau}_{20} + \varepsilon\hat{\tau}_{21} + [\frac{\partial}{\partial \hat{y}}(\hat{\tau}_{20} + \varepsilon\hat{\tau}_{21})]\varepsilon \sin \alpha\hat{x}$$

$$= \left.\frac{d\hat{u}_{20}}{d\hat{y}}\right|_{\hat{y}=0} + \varepsilon[\left.(\frac{\partial \hat{u}_{21}}{\partial \hat{y}} + \frac{\partial \hat{v}_{21}}{\partial \hat{x}})\right|_{\hat{y}=0} + \left.\frac{d^2\hat{u}_{20}}{d\hat{y}^2}\right|_{\hat{y}=0} \sin \alpha\hat{x}]$$

Zero shear and continuity of normal stress at the free surface

$$\hat{f}_1 \frac{d^2\hat{u}_0}{d\hat{y}^2} + \frac{\partial \hat{u}_1}{\partial \hat{y}} + \frac{\partial \hat{v}_1}{\partial \hat{x}} = 0, -\frac{d\hat{P}_0}{d\hat{y}}\hat{f}_1 - \hat{P}_1 - 2\frac{\partial f_1}{\partial \hat{x}}\left(\frac{d\hat{u}_0}{d\hat{y}}\right) + 2\frac{\partial \hat{v}_1}{\partial \hat{y}} = 0$$