Entrainment Threshold of Loose Boundary Streams

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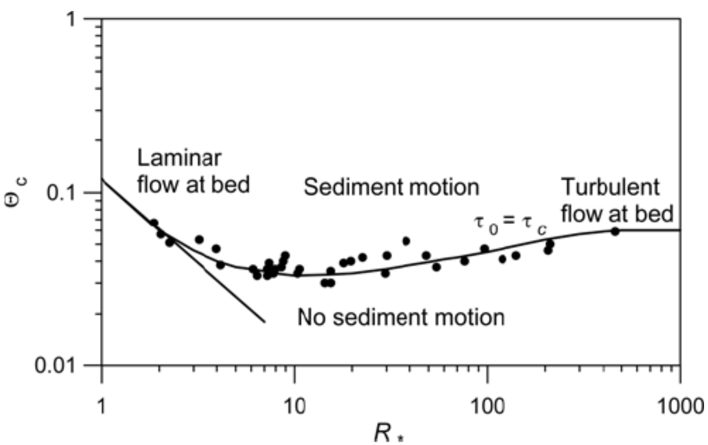
An Overview of Studies on Entrainment Threshold

A.F. Shields

A.F. Shields' doctoral research on sediment transport in the Technischen Hochshcule Berlin becomes a legend

Shields AF (1936) Application of similarity principles and turbulence research to bed-load movement. *Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau*, Berlin, Germany



Shields parameter θ_c versus particle Reynolds number R_*

Shields diagram is well-know for the criterion for the *threshold of sediment entrainment*

Since his pioneering work, numerous attempts have so far been made

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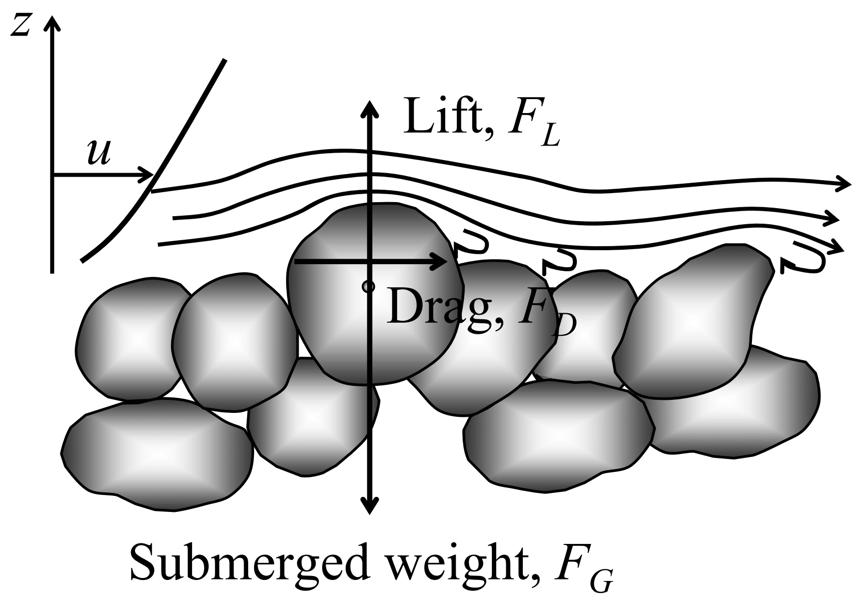
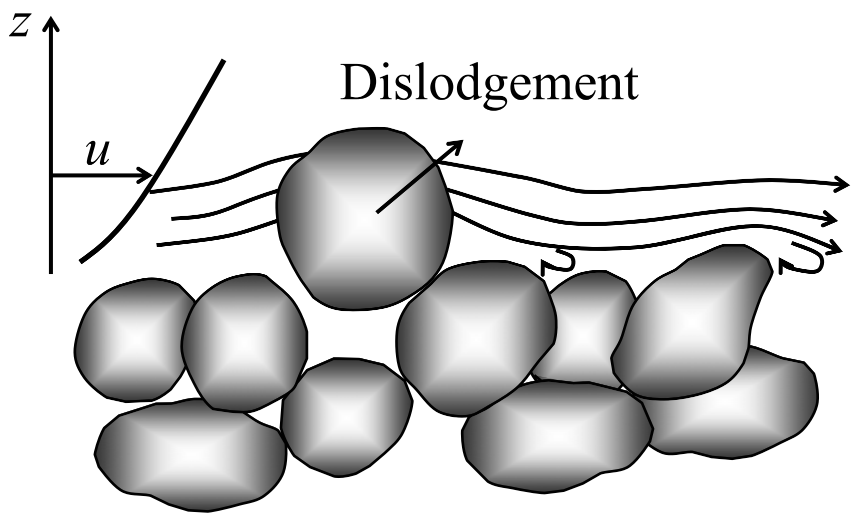
What is an Entrainment Threshold?

- When a stream flows over a loose sedimentary bed, hydrodynamic forces are exerted on the sediment particles at the bed surface
- An increase in flow velocity causes an increase in the magnitude of hydrodynamic forces



- Sediment particles start to move if a situation is eventually reached when the hydrodynamic forces induced by the flow exceed a certain limiting value

- Initial movement of sediment particles is frequently called entrainment threshold of sediment



Definitions of Sediment Threshold

First type of definition based on sediment flux concept

- **Shields** (1936): A concept that the bed shear stress has a value for which the extrapolated sediment flux becomes zero
- **USWES** (1936): A concept that the tractive force brings about general motion of bed particles. For sediment less than 0.6 mm size, this concept was inadequate. The general motion was redefined that sediment in motion should be represented by all sizes of bed particles and that sediment flux should exceed 4.1×10^{-4} kg /sm

Second type of definition based on bed particle motion concept

- **Kramer** (1935) indicated four different bed shear conditions
  - No particles in motion, termed no transport
  - A few of the smallest particles in motion at isolated zones, termed weak transport
  - Many particles of mean size in motion, termed medium transport
  - Particles of all sizes in motion at all points and at all times, termed *general transport*

- **Kramer** (1935) pointed out the difficulty of setting up clear limits between these regimes but defined threshold bed shear stress to be that stress initiating general transport

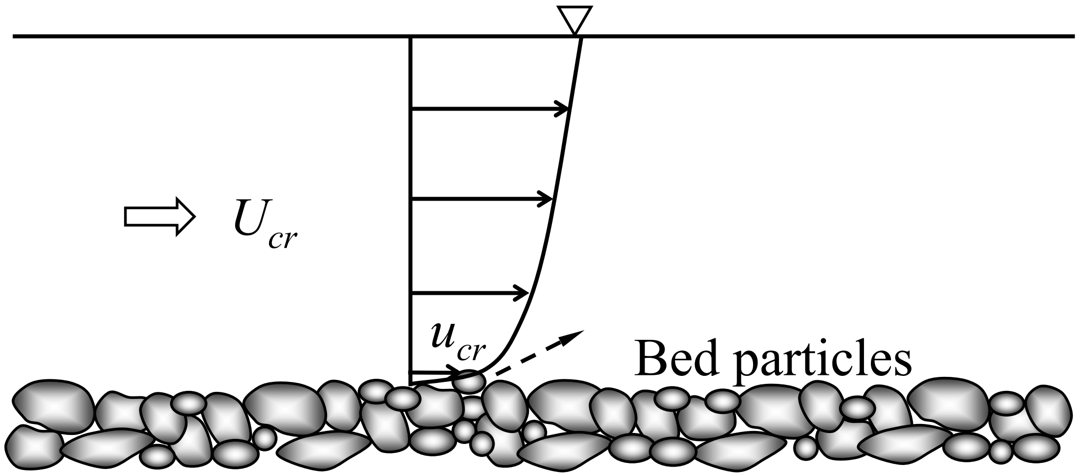
- **Vanoni** (1964) proposed that the sediment threshold is the condition of particle motion in every 2 s at any bed position

Concepts of Entrainment Threshold

- Competent velocity
- Lift force
- Threshold bed shear stress
- Probabilistic
- Latest development ~ turbulent bursting

Competent Velocity Concept

- Competent velocity is a velocity at particle level u_{cr} or mean velocity U_{cr}, which is just enough to move the particles



- **Goncharov** (1964) defined threshold velocity as detachment velocity U_n, which was defined as the lowest average velocity at which individual particles detach

$$U_n = \log(8.8h/d)\sqrt{0.57\Delta g d} \qquad (1)$$

where h = flow depth; d = median particle diameter; g = acceleration due to gravity; $\Delta = s - 1$; s = relative density of sediment particles, that is ρ_s/ρ; ρ_s = mass density of sediment; and ρ = mass density of fluid

- **Carstens** (1966) reported an equation of threshold velocity u_{cr} at the particle level

$$u_{cr}^2 / \Delta g d \approx 3.61(\tan\varphi \cos\theta - \sin\theta) \qquad (2)$$

where φ = angle of repose; and θ = angle made by the streamwise sloping bed with the horizontal

- **Neill** (1968) presented a conservative design curve for the movement of coarse uniform gravel in terms of average threshold velocity U_{cr} and represented it in an equation

$$U_{cr}^2 / \Delta g d = 2(h / d)^{1/3} \qquad (3)$$

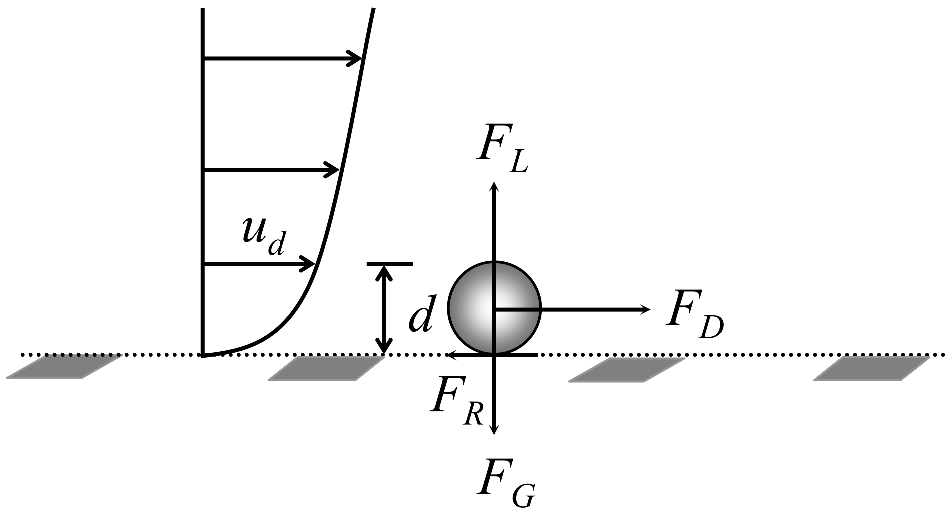
- **Zanke** (1977) proposed the following equation

$$U_{cr} = 2.8\sqrt{\Delta g d} + 14.7 c_1 \upsilon / d \tag{4}$$

where c_1 = cohesiveness coefficient varying from 1 for non-cohesive to 0.1 for cohesive sediments; and υ = kinematic viscosity

- There remains confusion regarding the competent velocity at particle level u_{cr} and average threshold velocity U_{cr}

- **Yang** (1973) developed a promising model for the estimation of average velocity for entrainment threshold



Forces acting on a spherical sediment particle

Yang (1973) proposed a model for competent velocity

- The drag force F_D is expressed as

$$F_D = C_D \frac{\pi}{8} d^2 \rho u_d^2 \qquad (5)$$

where C_D = drag coefficient; and u_d = velocity at a distance d above the bed

- The terminal fall velocity w_{ss} of a spherical particle is reached when the fall drag force equals submerged weight F_G

$$C_{D1}\frac{\pi}{8}d^2\rho w_{ss}^2 = \frac{\pi}{6}d^3(\rho_s - \rho)g \quad (= F_G) \tag{6}$$

where C_{D1} = drag coefficient at w_{ss}, assumed as $\psi_1 C_D$

$$F_D = \frac{\pi}{6\psi_1 w_{ss}^2}d^3(\rho_s - \rho)g u_d^2 \tag{7}$$

- Considering log-law, velocity at particle level u_d and depth-averaged velocity U are

$$u_d = B_r u_* \tag{8a}$$

$$U = u_* \left[5.75 \left(\log \frac{h}{d} - 1 \right) + B_r \right] \tag{8b}$$

where B_r = roughness function; and u_* = shear velocity

Using Eqs. (8a) and (8b) into Eq. (7), yields

$$F_D = \frac{\pi}{6\psi_1} d^3 (\rho_s - \rho) g \left(\frac{U}{w_{ss}} \right)^2 \frac{B_r^2}{\left[5.75 \left(\log \frac{h}{d} - 1 \right) + B_r \right]^2} \tag{9}$$

- The lift force F_L acting on the particle is given by

$$F_L = C_L \frac{\pi}{8} d^2 \rho u_d^2 \qquad (10)$$

where C_L = lift coefficient, assumed as C_D/ψ_2

Using Eqs. (8a) and (8b) into Eq. (10), yields

$$F_L = \frac{\pi}{6\psi_1\psi_2} d^3 (\rho_s - \rho) g \left(\frac{U}{w_{ss}}\right)^2 \frac{B_r^2}{\left[5.75\left(\log\frac{h}{d} - 1\right) + B_r\right]^2} \qquad (11)$$

- Force balance equation

$$F_D = F_R = \psi_3 (F_G - F_L) \quad (12)$$

where ψ_3 = friction coefficient

Inserting Eqs. (6), (9) and (11) in Eq. (12), the equation of average threshold velocity U_c is

$$\frac{U_c}{w_{ss}} = \sqrt{\frac{\psi_1 \psi_2 \psi_3}{\psi_2 + \psi_3}} \left[\frac{5.75}{B_r} \left(\log \frac{h}{d} - 1 \right) + 1 \right] \quad (13)$$

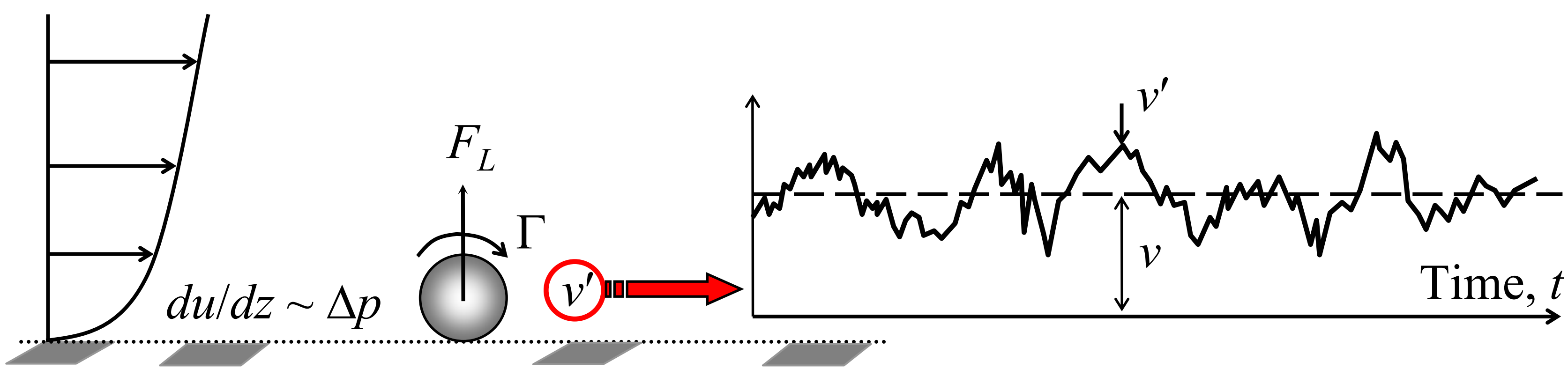
- **Yang** (1973) gave equations empirically for different R_* ranges

$$\frac{U_c}{w_{ss}} = \frac{2.5}{\log R_* - 0.06} + 0.66 \qquad \text{for } 0 < R_* < 70 \qquad (14)$$

$$\frac{U_c}{w_{ss}} = 2.05 \qquad \text{for } R_* \geq 70 \qquad (15)$$

Lift Force Concept

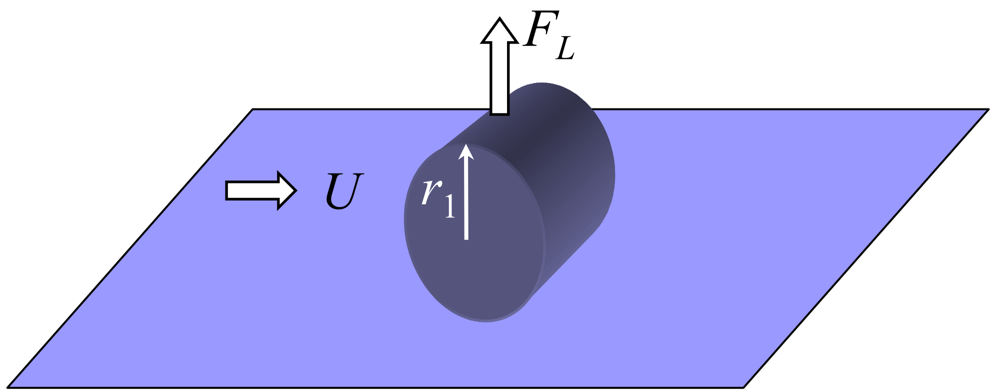
- Lift force arises for following reasons:
  - Pressure difference due to a steep velocity gradient on the bed
  - Instantaneous vertical velocity fluctuations adjacent to the bed as a result of turbulence
  - Spinning motion of particles resulting in Magnus lift



- **Jeffreys** (1929) assumed a potential flow over a circular cylinder and found lifting of cylinder takes place if

$$(3+\pi^2)U^2 > 9\Delta g r_1 \quad (16)$$

where r_1 = radius of the cylinder



- Shortcoming of Jeffreys model was that the drag force was discarded

- **Reitz** (1936) discussed a similar idea and suggested to express the beginning of sediment motion with a lift model

- **Lane and Kalinske** (1939) stressed on turbulence for determination of lift and assumed
  - Particles having a settling velocity smaller than the instantaneous turbulent fluctuations at bed experience lift
  - Velocity fluctuations vary according to the normal error law
  - Turbulent fluctuations and shear velocities are correlated

- **Einstein and El-Samni** (1949) measured the lift force directly as a pressure difference

$$f_L = 0.5 C_L \rho u_{0.35d}^2 \qquad (17)$$

where f_L = lift force per unit area of the particle; C_L = lift coefficient assumed as 0.178; and $u_{0.35d}$ = velocity of flow at a distance of 0.35 diameter (equivalent) from the theoretical wall

- The results of the study of **Einstein and El-Samni** (1949) were used by **ASCE Task Committee** (1966), who calculated $f_L/\tau_c \approx 2.5$; where τ_c = threshold bed shear stress

- **Chepil** (1961) pointed out that, once the particle is displaced, lift force tend to diminish and drag force to increase. He measured $F_L/F_D = 0.85$ for $47 < UD/\upsilon < 5\times10^3$
- **Apperley** (1968) measured $F_L/F_D = 0.5$ for $R_* = 70$
- **Aksoy** (1973) measured $F_L/F_D = 0.1$ for $R_* = 300$
- **Bagnold** (1974) measured $F_L/F_D = 0.5$ for $R_* = 800$
- **Brayshaw et al.** (1983) measured $F_L/F_D = 1.8$ for $R_* = 5.2\times10^4$

- **Watter and Rao** (1971) observed negative lift for $20 < R_* < 100$
- **Davis and Samad** (1978) observed negative lift for $R_* < 5$

- While the lift forces obviously contribute to the sediment entrainment, the occurrence of lift on a sediment particle is still unclear
- A critical lift criterion has so far not been obtained which could have been a ready reference for the determination of sediment entrainment
- The occurrence of negative lift at low R_* has been well established, but its cause and magnitude remain uncertain
- For higher R_*, the correlation between lift and drag is another uncertain issue, although the lift is definitely positive

Threshold Shear Stress Concept

Empirical Equations of Threshold Shear Stress

- **Kramer** (1935) carried out experiments using quartz particles

$$\tau_c = 29\sqrt{(\rho_s - \rho) g d / M} \qquad (18)$$

where τ_c = threshold bed shear stress; and M = uniformity coefficient

- **USWES** (1936) recommended the formula

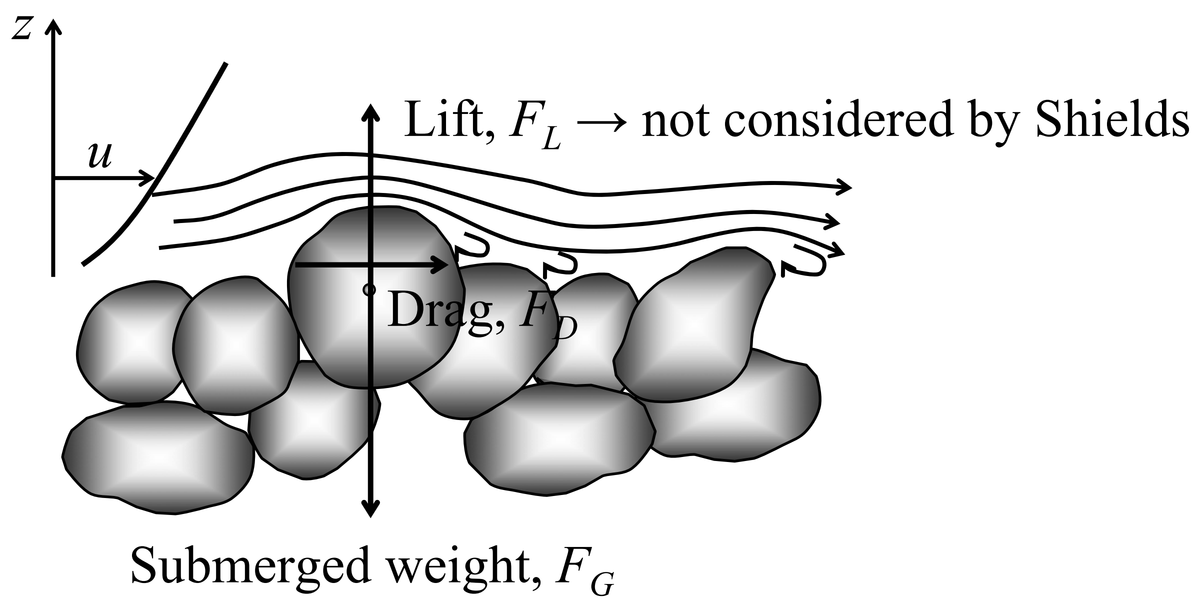
$$\tau_c = 0.285\sqrt{\Delta d / M} \qquad (19)$$

- **Leliavsky** (1955) gave a simple relationship as

$$\tau_c = 166 d \qquad (20)$$

Semi-Theoretical Analyses

Shields (1936) was pioneer to present a semi-theoretical theory



Forces acting on a sediment particle resting on bed

- The drag force F_D exerted on the sediment particle

$$F_D = C_D \frac{1}{2} \rho u^2 A = f_1\left(a_1, \frac{ud}{\upsilon}\right)\rho d^2 u^2 \qquad (21)$$

where u = velocity at elevation $z = a_2 d$; A = frontal area of the particle; and a_1 = particle shape factor

- Velocity distributions for rough and smooth flows

$$\frac{u}{u_*} = 5.75 \log \frac{z}{k_s} + \frac{z u_*}{\upsilon} = 5.75 \log a_2 + f_2\left(\frac{u_* d}{\upsilon}\right) \qquad (22)$$

where k_s = roughness height being proportional to d

- Drag force is $F_D = \tau_0 d^2 f_3(a_1, a_2, R_*)$ (23)

- The resistance F_R to motion was assumed to be dependent only upon k_s and F_G

$$F_R = a_3 \Delta \rho g d^3 \tag{24}$$

where a_3 = roughness factor

- At the threshold condition, when the sediment particle is about to move, $u_* \rightarrow u_{*c}$ (that is threshold shear velocity), then drag force is balanced by the resistance

$$F_D = F_R \tag{25}$$

Rearranging the terms

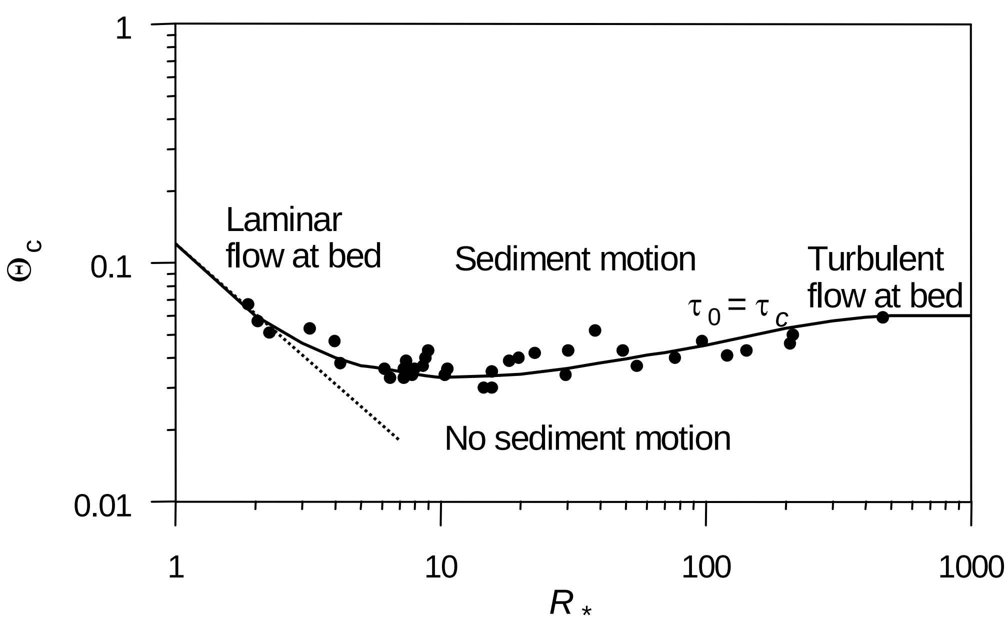
$$\frac{u_{*c}^2}{\Delta g d} = \frac{\tau_c}{\Delta \rho g d} = f(R_*) \tag{26}$$

- The Shields parameter Θ is defined as

$$\Theta = \frac{u_*^2}{\Delta g d} \tag{27}$$

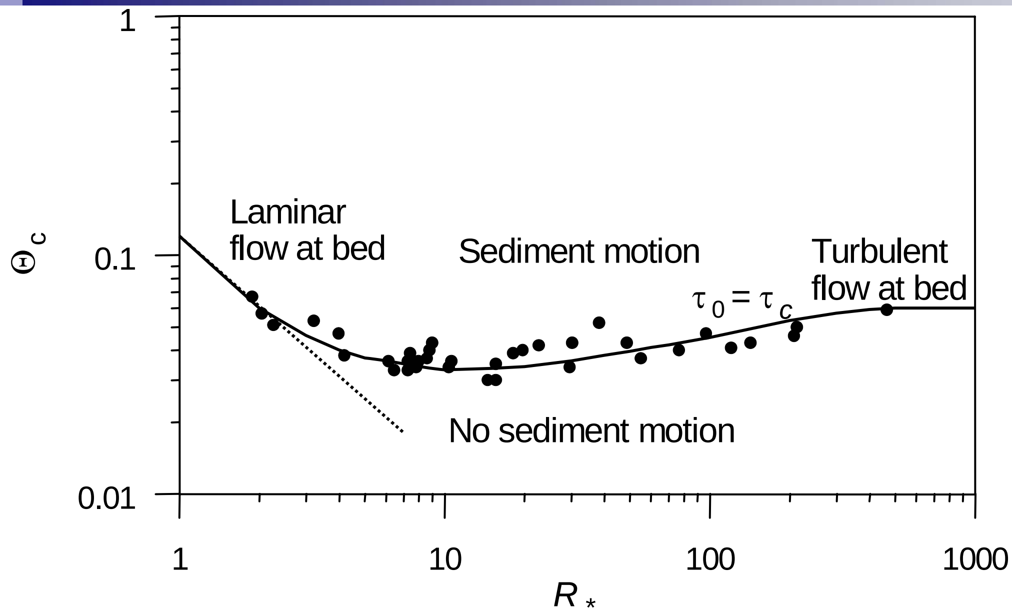
- The critical Shields parameter Θ_c is defined as

$$\Theta_c = f(R_*) \tag{28}$$



Shields Diagram:

Shields parameter Θ_c as a function of particle Reynolds number R_*



The Shields diagram has three distinct zones

- Smooth flow ($R_* \leq 2$): d is much smaller than the viscous sub-layer thickness; and $\Theta_c = 0.1/R_*$
- Rough flow ($R_* \geq 500$): Θ_c is independent of the fluid viscosity and has a constant value of 0.056
- Transitional flow ($2 \leq R_* \leq 500$): d is of the order of viscous sub-layer thickness. Θ_c is minimum as 0.032 for $R_* = 10$

- Drawbacks of the Shields Diagram are as follows:

- The viscous sub-layer does not have any effect on the velocity distribution when $R_* \geq 70$, but the diagram shows that Θ_c still varies with R_* when the latter is greater than seventy

- Shields used threshold shear velocity u_{*c} in the diagram as dependent and independent variables, which is not appropriate as Θ_c and R_* are interchangeable

- Threshold bed shear stress to be determined through trial and error method

Explicit equations of Shields diagram

- Brownlie (1981):

$$\Theta_c = 0.22 R_b^{-0.6} + 0.06 \exp(-17.77 R_b^{-0.6}) \qquad (29)$$

where $R_d = d(\Delta g d)^{0.5}/\upsilon$

- van Rijn (1984):

$$\begin{aligned}
&\Theta_c (D_* \le 4) = 0.24 / D_* \\
&\Theta_c (4 < D_* \le 10) = 0.14 / D_*^{0.64} \\
&\Theta_c (10 < D_* \le 20) = 0.04 / D_*^{0.1} \\
&\Theta_c (20 < D_* \le 150) = 0.013 D_*^{0.29} \\
&\Theta_c (D_* > 150) = 0.055
\end{aligned} \qquad (30)$$

where $D_* = d(\Delta g/\upsilon^2)^{1/3}$

- Soulsby and Whitehouse (1997):

$$\Theta_c = \frac{0.24}{D_*} + 0.055\ [1 - \exp(-0.02 D_*)] \qquad (31)$$

- Paphitis (2001):

$$\Theta_c (10^{-2} < R_* < 10^4) = \frac{0.273}{1 + 1.2 D_*} + 0.046[1 - 0.57 \exp(-0.02 D_*)] \qquad (32)$$

Other Studies

- **White** (1940) classified high-speed and low speed flows

- High-Speed Case ($R_* \geq 3.5$): It is required to move larger sediment particles. If p_f is the packing coefficient defined by Nd^2, where N = number of particles per unit area, the shear drag per particle (that is τ_0/N) is given by $\tau_0 d^2/p_f$

- At threshold, the shear drag is balanced by the product of submerged weight of particle and frictional coefficient tanφ

$$\Theta_c = \frac{\pi}{6} p_f \tan\varphi \qquad (33)$$

- Introducing *turbulence factor* T_f, which is the ratio of the instantaneous bed shear stress to the mean bed shear stress

$$\Theta_c = \frac{\pi}{6} p_f T_f \tan\varphi \tag{34}$$

- Low-Speed Case ($R_* < 3.5$): Upper portion of particle is exposed to shear drag that acts above center of gravity of particle
- Effect is taken into account introducing a coefficient α_f

$$\Theta_c = \frac{\pi}{6} p_f \alpha_f \tan\varphi \tag{35}$$

- White experimentally obtained $p_f \alpha_f = 0.34$ as an average value

- **Kurihara** (1948) extended work of **White** (1940) obtaining expression of T_f in terms of R_*, turbulence intensity and probability of bed shear stress increment. The resulting equations are quite complex.
- He proposed the following simpler empirical equations

$$\Theta_c(X_2 \leq 0.1) = (0.047 \log X_2 - 0.023)/\beta_2$$

$$\Theta_c(0.1 < X_2 \leq 0.25) = (0.01 \log X_2 + 0.034)/\beta_2 \qquad (36)$$

$$\Theta_c(X_2 > 0.25) = (0.0517 \log X_2 + 0.057)/\beta_2$$

where $X_2 \approx 4.67\times10^{-3}[\Delta g/(\upsilon^2\beta_2)]^{1/3}d$; $\beta_2 = (M + 2)/(1 + 2M)$; and M = uniformity coefficient of **Kramer** (1935) varying from 0.265 to 1

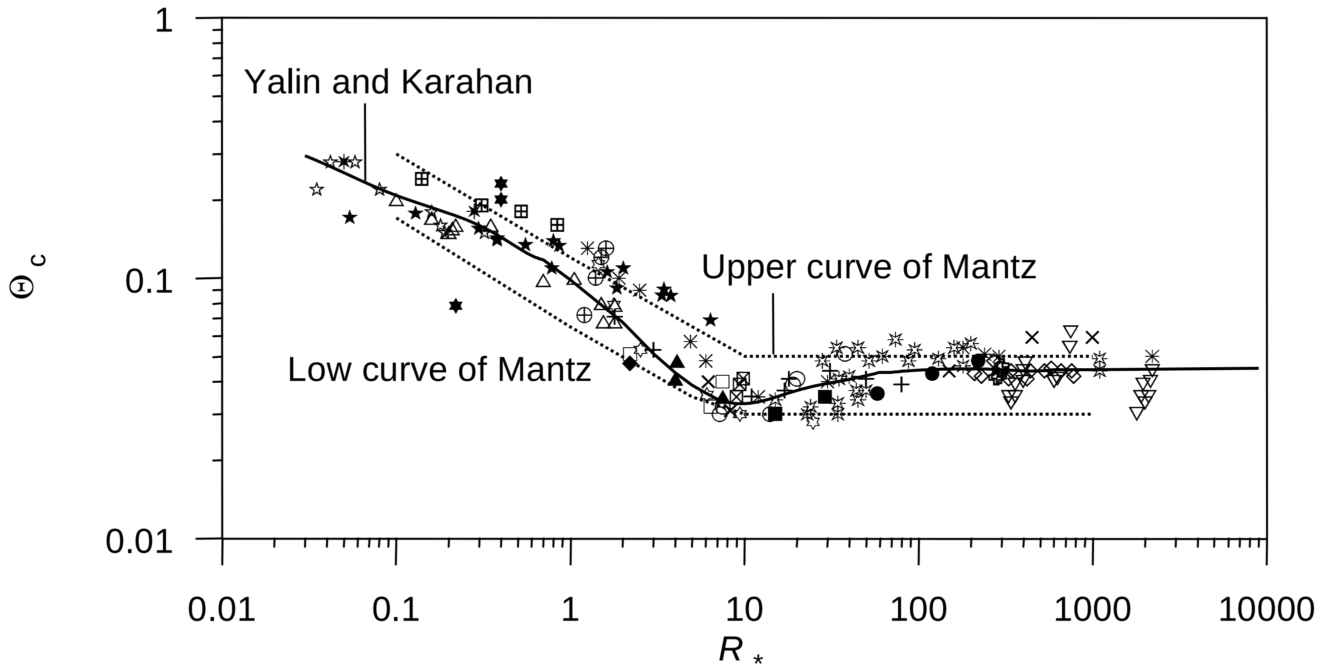
- **Egiazaroff** (1965) assumed that at threshold condition, the velocity at an elevation of 0.63d (above the bottom of particle) equals the fall velocity w_{ss} of particle. His equation is

$$\Theta_c = \frac{1.33}{C_D[a_r + 5.75\log(0.63)]} \tag{37}$$

where a_r = 8.5; and C_D = drag coefficient = 0.4 for large R_*, and both a_r and C_D increase for low R_*

- **Mantz** (1977) proposed the extended Shields diagram

- **Yalin and Karahan** (1979) presented a diagram for Θ_c versus R_*. It is regarded as a superior curve to the Shields Diagram



Curves (Θ_c versus R_*) of **Mantz** and **Yalin and Karahan**

- **Cao et al.** (2006) derived the explicit equation for the curve of **Yalin and Karahan** (1979). It is

$$\Theta_c(R_d \le 6.61) = 0.1414 / R_d^{0.23}$$

$$\Theta_c(6.61 < R_d \le 282.84) = \frac{[1 + (0.0223 R_b)^{2.84}]^{0.35}}{3.09 R_d^{0.68}} \qquad (38)$$

$$\Theta_c(R_d \ge 282.84) = 0.045$$

- **Iwagaki** (1956) considered the equilibrium of a single spherical particle, placed on a rough surface

$$\Theta_c = \frac{\cot\varphi}{\varepsilon_s \Psi_s R_*} \qquad (39)$$

where ε_s = empirical coefficient to take care of the sheltering effect; and Ψ_s = function of R_*

- The analysis of **Ikeda** (1982) was based on works of **Iwagaki** (1956) and **Coleman** (1967). He considered forces on a solitary particle placed on a sediment bed obtaining an equation that could approximately derive the Shields diagram

$$\Theta_c = \frac{4}{3} \cdot \frac{\tan\varphi}{(C_D + \tan\varphi C_L)} \cdot \left\{ 10.08 R_*^{-10/3} + \left[\kappa^{-1} \ln\left(1 + \frac{4.5 R_*}{1 + 0.3 R_*} \right) \right]^{-10/3} \right\}^{0.6} \quad (40)$$

where κ = von Kármán constant

- **Wiberg and Smith** (1987) gave the force balance as

$$(F_G - F_L)\tan\varphi = F_D \tag{41}$$

$$F_G = \Delta\rho g V \tag{42}$$

$$F_D = C_D \frac{1}{2}\rho u^2 A_x = C_D \frac{1}{2}\tau_0[f^2(z/z_0)]A_x \tag{43}$$

$$F_L = C_L \frac{1}{2}\rho(u_T^2 - u_B^2)A_x = C_L \frac{1}{2}\tau_0[f^2(z_T/z_0) - f^2(z_B/z_0)]A_x \tag{44}$$

where V = particle volume; A_x = frontal area of particle; u = velocity at z above bed; z_0 = zero-velocity level; u_T = velocity at top of particle; u_B = velocity at bottom of particle; z_T = height of top point of particle; and z_B = height of bottom point of particle

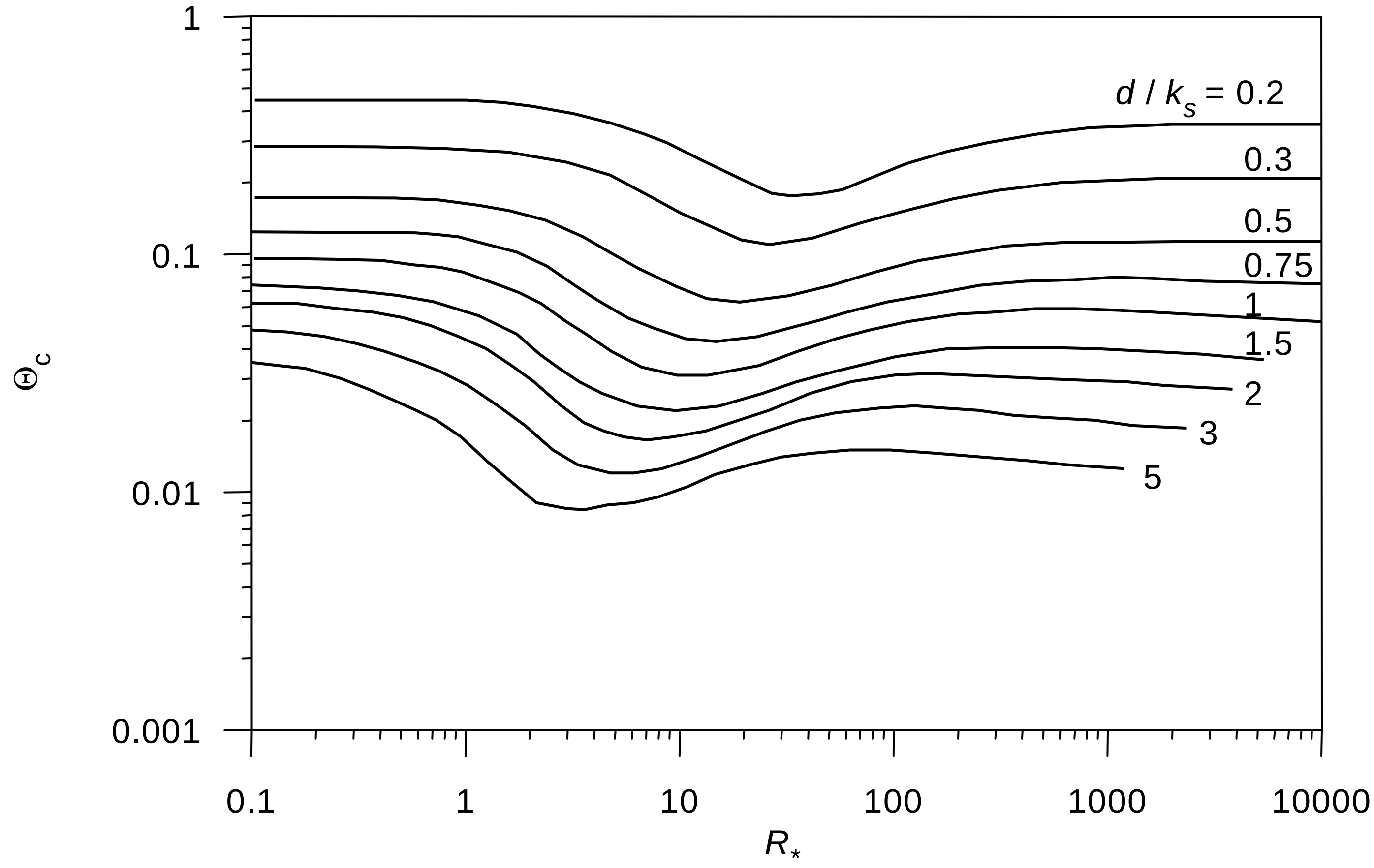
- Assuming the bed level is passing through the mid points (those are the contact points) of the bed particles

Using Eqs. (41) – (44), the expression for Θ_c is obtained as

$$\Theta_c = \frac{2}{C_D \alpha_0} \cdot \frac{1}{f^2(z/z_0)} \cdot \frac{\tan\varphi}{1 + (F_L/F_D)_c \tan\varphi} \qquad (45)$$

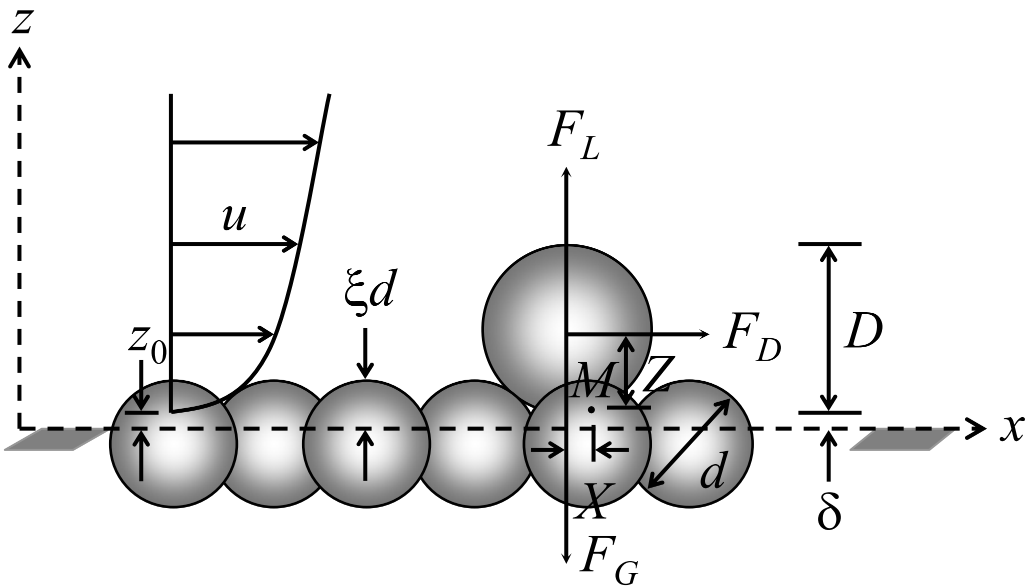
where $\alpha_0 = A_x d/V$

- C_D is a function of particle Reynolds number, $C_L = 0.2$ and $\cos\varphi = [(d/k_s) + z_*]/[(d/k_s) + 1]$
- For natural sands, $z_* = -0.02$



Θ_c as a function of R_* for different d/k_s

- **Dey** (1999) put forward a sediment threshold model



Forces acting on a spherical solitary particle

- Depending on the orientation, the solitary particle has a tendency either to roll over the valley formed by the two particles or to roll over the summit of a single particle or in between

- The equation of moment about the point of contact *M*

$$(F_L - F_G)X + F_D Z = 0 \qquad (46)$$

- Horizontal and vertical lever arms *X* and *Z*

$$X = \frac{\sqrt{3}}{4} \cdot \frac{Dd}{D+d} \qquad (47)$$

$$Z = \frac{1}{2\sqrt{3}} \cdot \frac{D}{D+d} (3D^2 + 6Dd - d^2)^{0.5} \qquad (48)$$

- Submerged weight

$$F_G = \frac{\pi}{6} D^3 (\rho_s - \rho) g \qquad (49)$$

- Drag force

$$F_D = C_D \frac{\pi}{8} D^2 \rho u_m^2 \qquad (50)$$

where u_m = mean flow velocity on the frontal area of particle

- Empirical equation for C_D by **Morsi and Alexander** (1972)

$$C_D = a + bR^{-1} + cR^{-2} \tag{51}$$

where $R = u_m D/\upsilon$; and a, b and c = coefficients dependent on R

- The lift due to shear effect

$$F_{Ls} = C_L \, \rho \, D^2 \, u_m \left(\upsilon \frac{\partial u}{\partial z} \right)^{0.5} \tag{52}$$

- For low particle Reynolds number R_*, Eq. (52) is applicable
- For large Reynolds number ($R_* > 3$), the solitary particle spins into the groove

- The lift due to Magnus effect

$$F_{Lm} = C_L \rho D^3 u_m \omega \qquad (53)$$

where ω = angular velocity of spinning particle

- According to **Saffman** (1965), maximum $\omega = 0.5\partial u/\partial z$

$$F_{Lm} = 0.5 C_L \rho D^3 u_m \frac{\partial u}{\partial z} \qquad (54)$$

- The total lift force F_L, a combination of F_{Ls}- and F_{Lm}

$$F_L = C_L \rho D^2 u_m \left(\frac{\partial u}{\partial z}\right)^{0.5} \left[\upsilon^{0.5} + 0.5 f(R_*) D \left(\frac{\partial u}{\partial z}\right)^{0.5}\right] \qquad (55)$$

where $f(R_*) = 1$ for $R_* \geq 3$; $f(R_*) = 0$ for $R_* < 3$

- Using Eqs. (47) - (55) into Eq. (46), the equation for the threshold of sediment motion is obtained as

$$\Theta_c = \frac{2\pi \hat{d} / (1 + p\sqrt{\alpha - 1}\cos\psi)^2}{\pi C_D \hat{u}_m^2 (3 + 6\hat{d} - \hat{d}^2)^{0.5} + 6C_L \hat{d}\,\hat{u}_m (\partial\hat{u}/\partial\hat{z})\{2[(R_*/\hat{d})\partial\hat{u}/\partial\hat{z}]^{-0.5} + f(R_*)\}}$$

(56)

where $\hat{u}_m = u_m/u_{*c}$; $\hat{d} = d/D$; $\hat{u} = u/u_{*c}$; and $\hat{z} = z/D$;

p = probability of occurring sweep event; ψ = sweep angle; $\alpha = \tau_t/\tau_c$; and τ_t = instantaneous shear stress

- The expression given by **Ippen and Eagleson** (1955) for the spherical sediments

$$\hat{d} = \frac{2\tan\varphi\ \ [6\tan\varphi + (48\tan^2\varphi + 27)^{0.5}]}{4\tan^2\varphi + 9} \tag{57}$$

where φ = angle of repose

- The particle parameter $\tilde{d}$ is given by $(d/\upsilon)[gd(\rho_s\text{-}\rho)/\rho]^{0.5}$. The following equation is used to compute $\tilde{d}$

$$\tilde{d} = R_*(\hat{d}/\Theta_c)^{0.5} \tag{58}$$

Case 1 ($R_* < 3$):

- The velocity distribution of the flow is linear for $R_* < 3$

$$\hat{u} = \frac{z u_*}{\upsilon} \tag{59}$$

Case 2 ($3 \le R_* \le 70$):

- The velocity distribution proposed by **Reichardt** (1951) is

$$\hat{u} = \frac{1}{\kappa}\left\{ \ln\left(1 + \frac{\kappa \hat{z} R_*}{\hat{d}}\right) - \left[1 - \exp\left(-\frac{\hat{z} R_*}{11.6\hat{d}}\right) - \frac{\hat{z} R_*}{11.6\hat{d}} \exp\left(-\frac{\hat{z} R_*}{3\hat{d}}\right)\right] \ln\left(\frac{\kappa \hat{z}_0 R_*}{\hat{d}}\right)\right\} \tag{60}$$

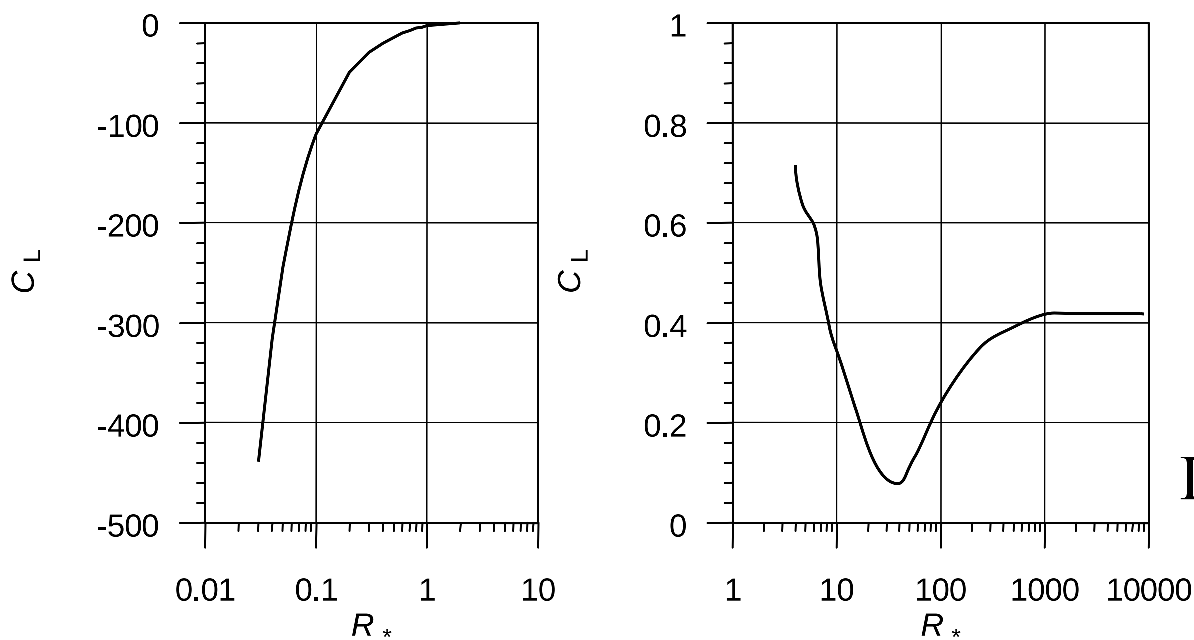
where z_0 = zero-velocity level (= $0.033k_s$)

Case 3 ($R_* > 70$):

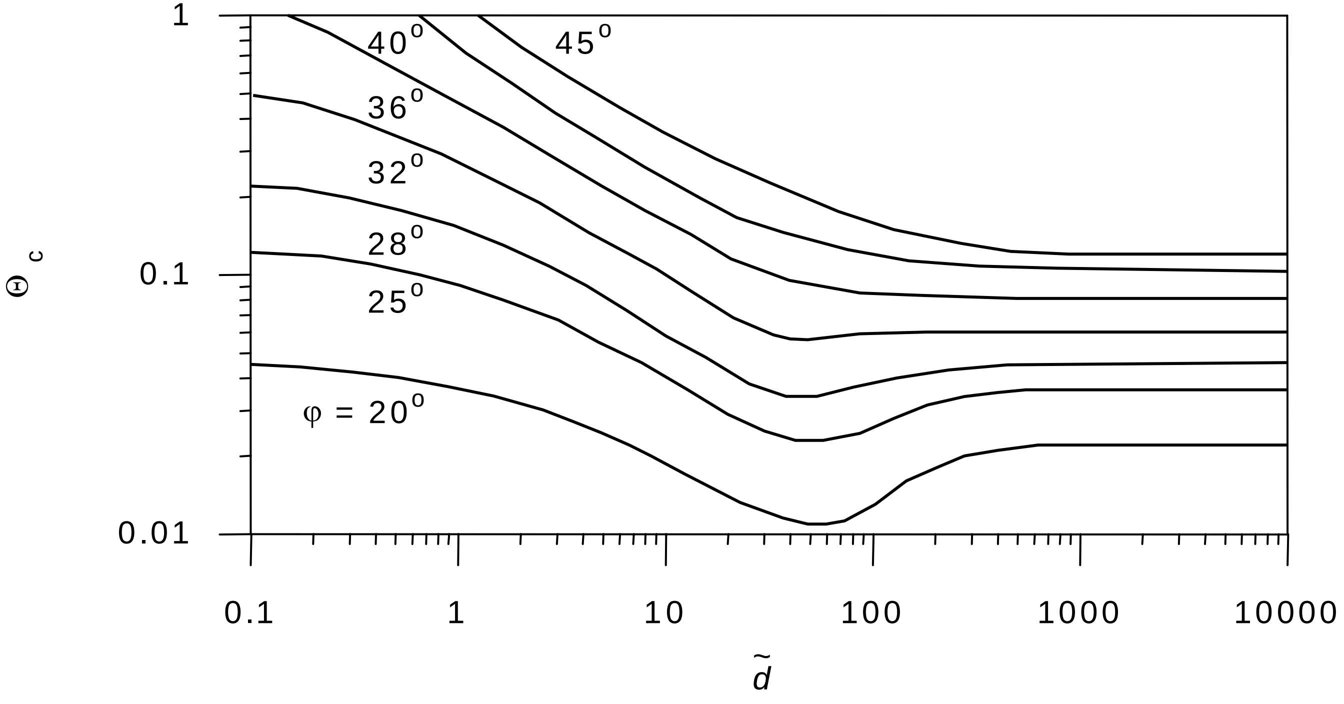
- The velocity distribution in rough regime is

$$\hat{u} = \frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right) \qquad (61)$$

- Eq. (56) is calibrated extensively making C_L a free parameter



Dependency of C_L on R_*



Dependency of Θ_c on particle parameter $\tilde{d}$ for different φ

- It enables direct estimation of u_{*c}

Probabilistic Concept

- Threshold of sediment motion is probabilistic in nature

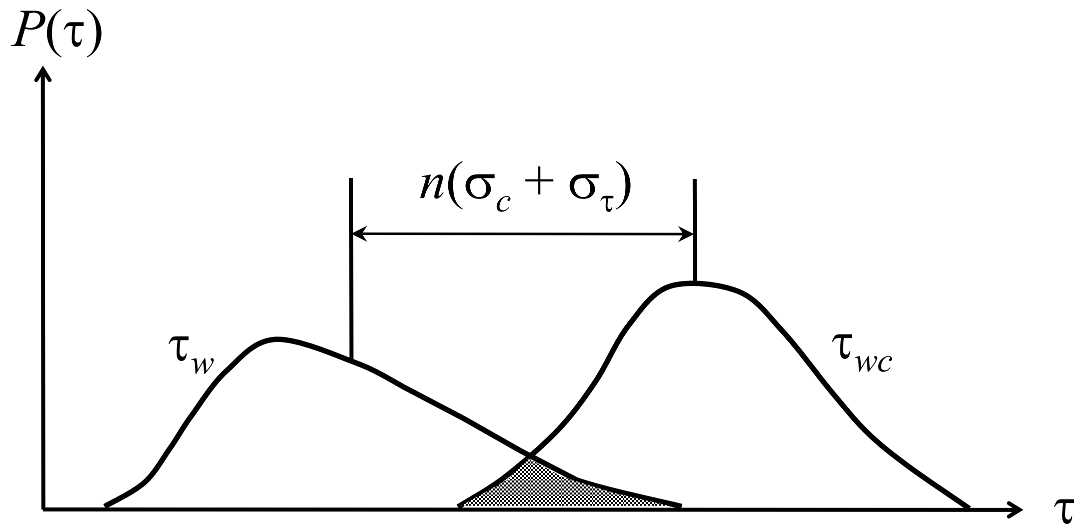
- **Gessler** (1970) measured the probability of particles of a specific size to stay

- It depends strongly on Θ and weakly on R_*

- **Grass** (1970) identified two probability distributions:
- One for the boundary shear stress τ_w induced by the fluid and other for the boundary shear stress τ_{wc} required to put the particle in motion
- When these two distributions start overlapping, the particles that have the lowest threshold boundary shear stress start to move
- The representative magnitudes of the probability distributions are their standard deviations used to describe the distance of the two mean boundary shear stresses as

$$\bar{\tau}_{wc} - \bar{\tau}_w = n(\sigma_c - \sigma_\tau)$$

- Experimentally, $\sigma_\tau = 0.4\bar{\tau}_{wc}$ and $\sigma_c = 0.3\bar{\tau}_w$
- For $n = 0.625$, the result collapses on that of Shields



Probabilities of boundary shear stress τ_w due to flow and threshold boundary shear stress τ_{wc} corresponding to the motion of individual particles

- **Papanicolaou et al**. (2002) developed a stochastic sediment threshold model considering the role of near-bed turbulent structures and bed micro-topography upon the sediment threshold

- It was based on the hypothesis that the probability of occurrence of exceeding the minimum momentum required to initiate rolling motion equals the probability of first displacement of a particle

- The theoretical derivation was complemented by the experimental measurements of the probability and near-bed turbulence for different bed particle packing regimes

- They found that the probability of the occurrence of intermittent turbulent events equals the sediment entrainment probability

- **Dancey et al.** (2002) proposed a criterion, that might be interpreted as the probability of individual particle motion, considering the statistical nature of sediment motion in turbulent flow and the time-scale of flow

- The sediment threshold was specified by a constant value of the probability

- The mechanism is strongly dependent upon the sediment packing density

Role of Turbulence on Sediment Entrainment

- **Cao** (1997) proposed a model for the sediment entrainment based on the bursting structures arguing that the sediment entrainment is strongly dependent on u_*

- **Zanke** (2003) recognized two important effects as (a) the effective-τ_0 acting on a particle increases above the time-averaged-τ_0 owing to turbulent stress peaks and (b) the particles exposed to the flow become effectively lighter due to lift force. Both the turbulence induced effects are randomly distributed

- **Nikora and Goring** (2000) and **Dey and Raikar** (2007) observed a reduction in κ-value from its traditional value (0.41) over an entrained bed

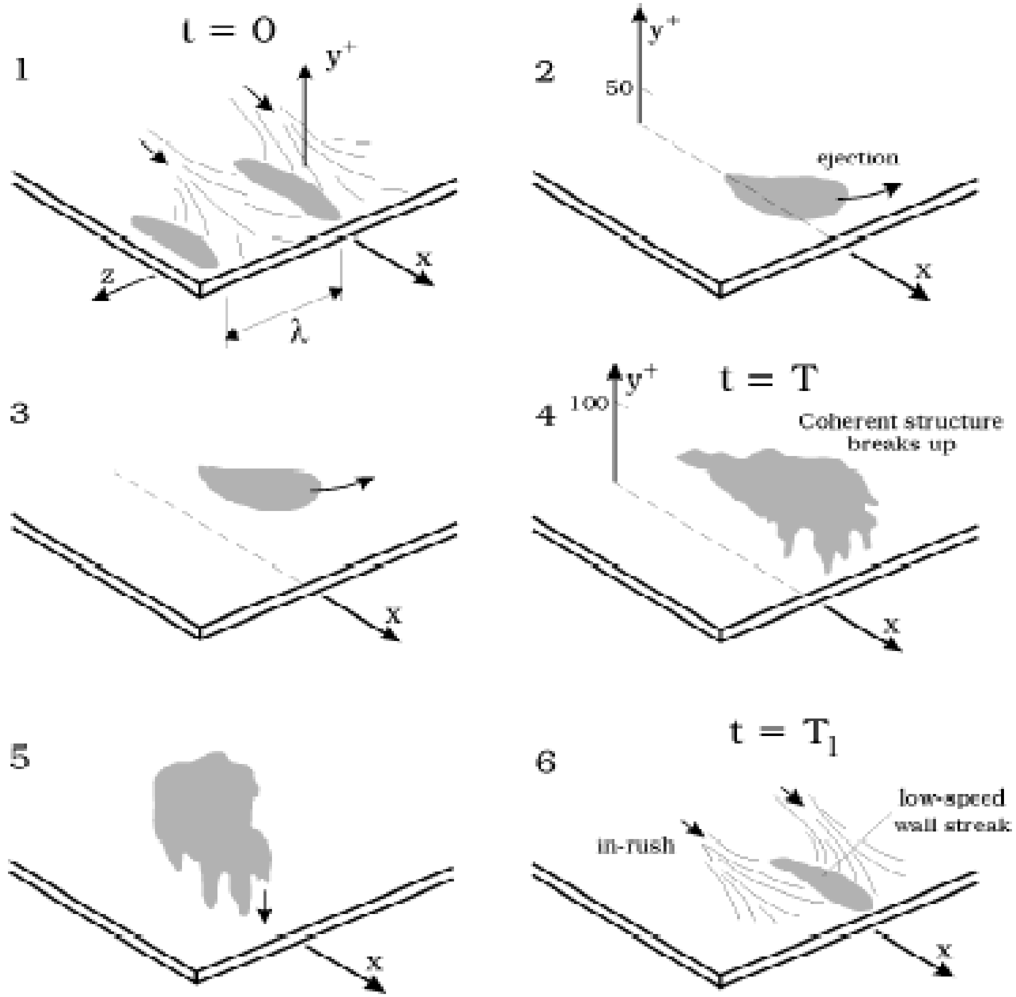
Turbulent Burst

- The sequence turbulence bursting is described by ejections and sweeps which play an important role on sediment entrainment

- During the ejections, the upward flow enlarges the shear layer and the associated small-scale flow structures to a wide region

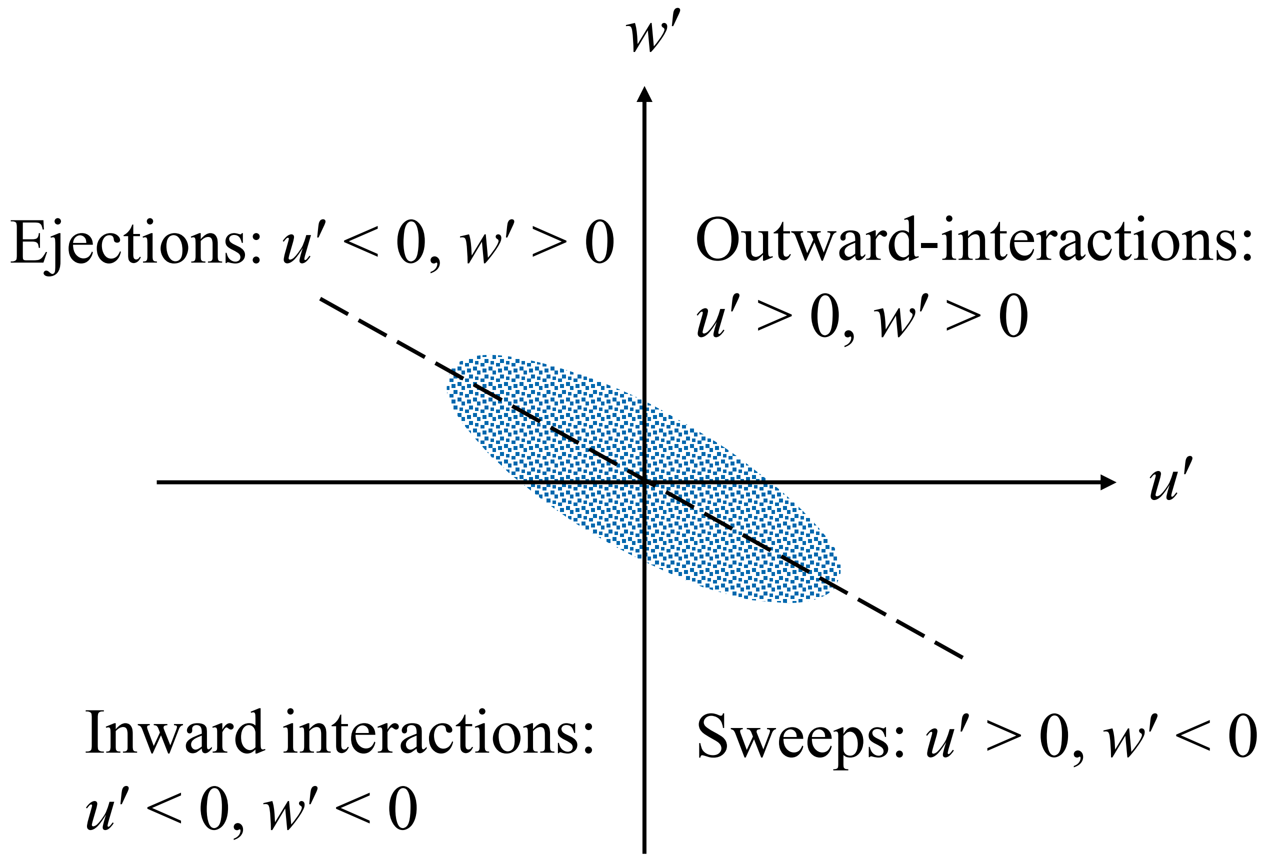
- The ejected fluid streaks which remain as a result of retardation are brushed away by high-speed fluid approaching to the bed in a process called the sweeps

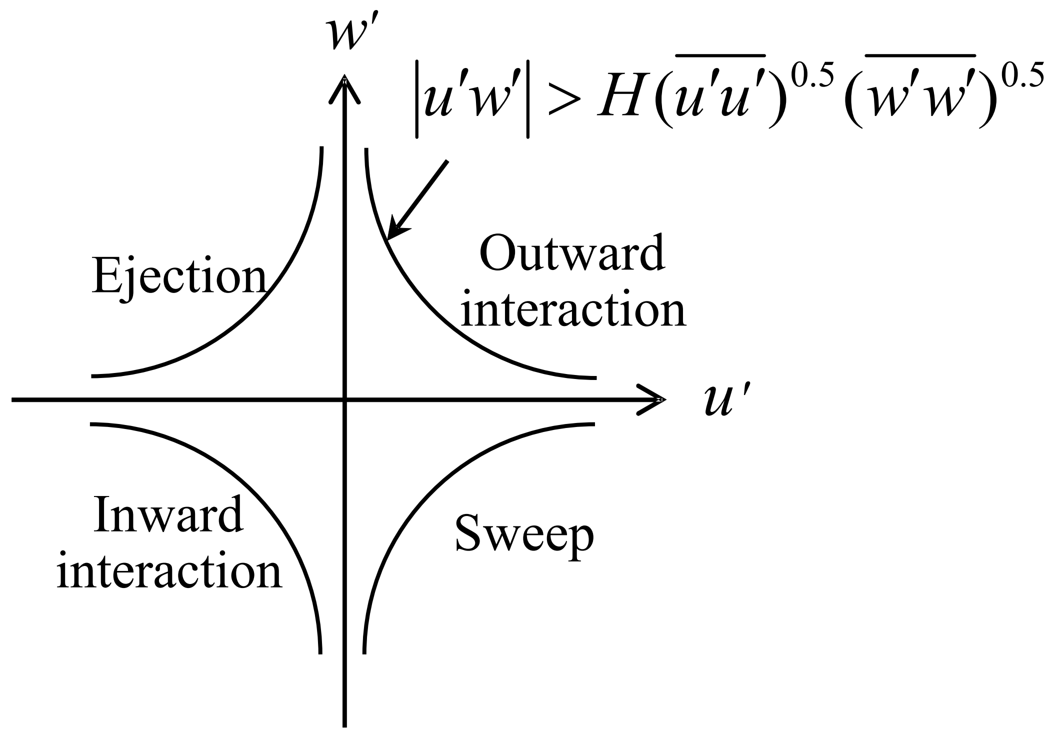
- The turbulent bursting process can be described by a quadrant analysis



Sequence of bursting process

Quadrant Analysis





- The hyperbolic zone bounded by the curve $|u'v'|$ = constant is called a *hole*. Introducing a parameter *H* called *hole-size* that represents threshold level

- The conditional stochastic analysis can be performed introducing a detection function $\lambda_{i,H}(t)$ defined as

$$\lambda_{i,H}(z,t) = \begin{cases} 1, \text{ if } (u', w') \text{ is in quadrant } i \text{ and if } |u'w'| > H\left(\overline{u'u'}\right)^{0.5}\left(\overline{w'w'}\right)^{0.5} \\ 0, \text{ otherwise} \end{cases} \tag{62}$$

- At any point, contributions to the total Reynolds shear stress from the quadrant i outside the hole region of size H is

$$\langle u'w' \rangle_{i,H} = \lim_{T\to\infty} \frac{1}{T} \int_0^T u'(t) w'(t) \lambda_{i,H}(z,t) dt \tag{63}$$

Earlier Developments

- **Sutherland** (1967) observed that the sediment threshold is associated with a near-bed eddy impact onto the bed particles to produce a streamwise drag force to roll the particles

- **Heathershaw and Thorne** (1985) argued that the entrainment is not correlated with the instantaneous Reynolds shear stress but correlated with the near-wall instantaneous streamwise velocity

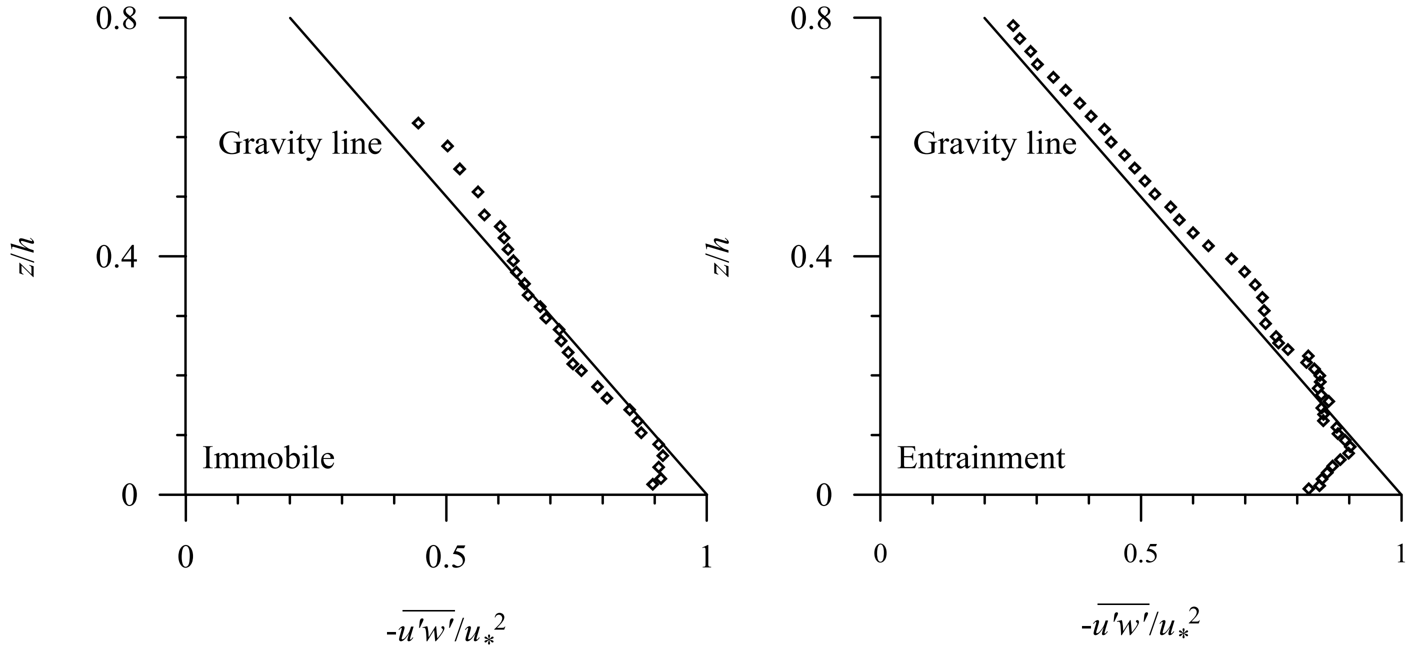
- **Drake et al.** (1988) reported that the majority of the gravel entrainment is associated with the sweep events

- **Thorne et al.** (1989) observed that the sweeps and outward interactions govern sediment entrainment. It is the instantaneous increase in streamwise velocity fluctuations that generate excess bed shear stresses, governing entrainment processes

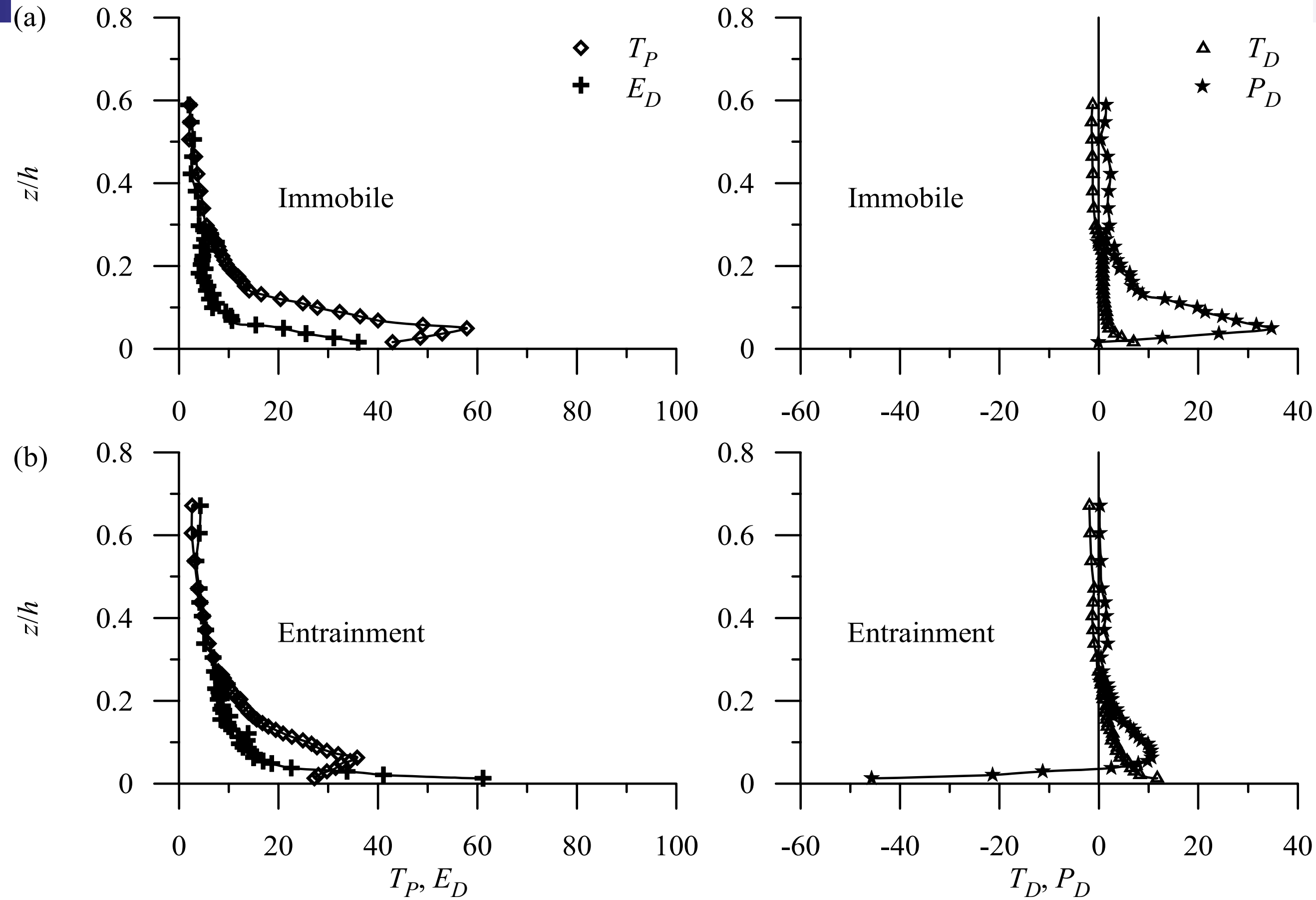
- **Nelson et al.** (1995) reported that when the outward interactions increases relative to the other events, then the sediment flux increases albeit the bed shear stress decreases

Recent Developments

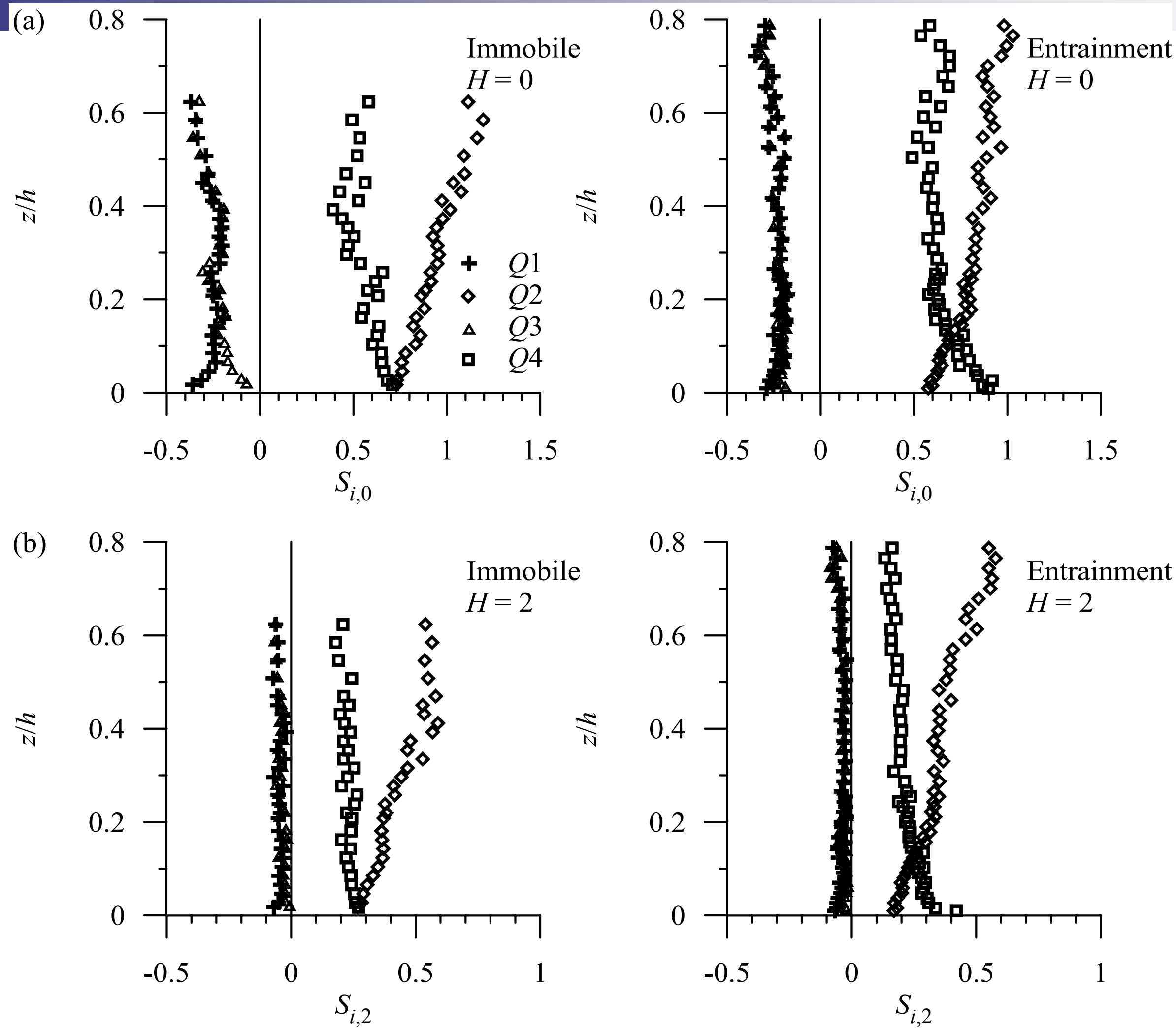
- Sarkar (2010) studied the turbulence characteristics over immobile and entrainment threshold sediment beds



- Larger near-bed damping in Reynolds stress for sediment entrainment is observed

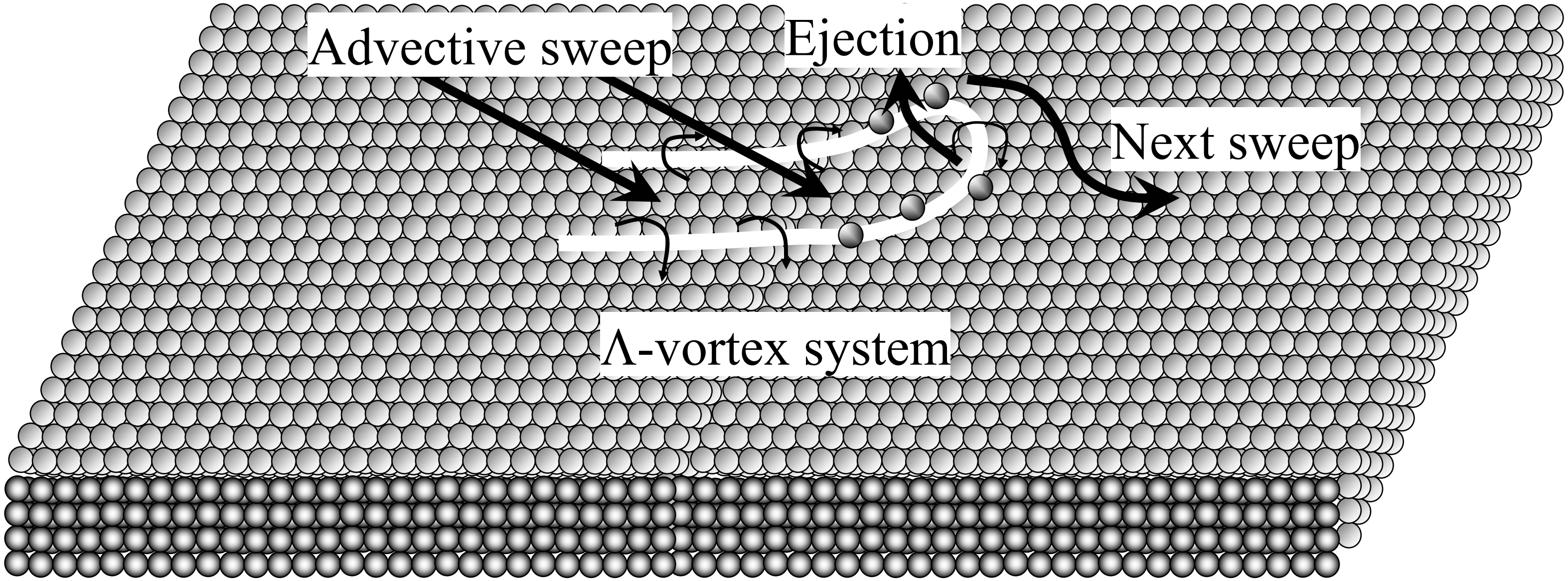


- TKE budget: In near-bed flow over entrainment threshold beds, the dissipation exceeds the production and the pressure energy diffusion becomes considerably negative



- Sweeps are the dominant mechanism towards the sediment entrainment

Physics of Sediment Entrainment



Schematic of coherent structure during sediment entrainment

Physics of Sediment Entrainment: A Conceptual Framework

At an entrainment-threshold regime:

- The entrainment does not initiate dislodging single particles from the isolated regions, but as a common temporal motion dislodging many particles from the isolated regions, whose locations change for a given area
- The near-bed shearing flow is highly retarded due to interaction with the bed developing front vortex (Λ-vortex) having an intense vorticity core under pressure, rising bed particles through its low-pressure core

- The most provoking turbulence characteristic for the sediment entrainment is a sweep producing a threshold low-pressure field, as confirmed by the drastic change in pressure energy diffusion to a negative

- It induces a lift force transporting the bed particles collectively from the isolated regions

- This concept is the basis of the sediment entrainment by the turbulent flows, where this aspect has not so far been given much attention in modeling the sediment entrainment processes

Closure

- The exact interaction between the particles and the fluid, in the level of particle micromechanics in association with the probabilistic feature of turbulence eddies, has not been completely revealed

- Not many researchers have tried to explore the threshold of sediment entrainment for water worked beds

- Sediment threshold under the sheet flows or shallow flow depths seems to remain unattended

Thank You