# Some properties of Lagrangian modeling of saltating grains over movable bed

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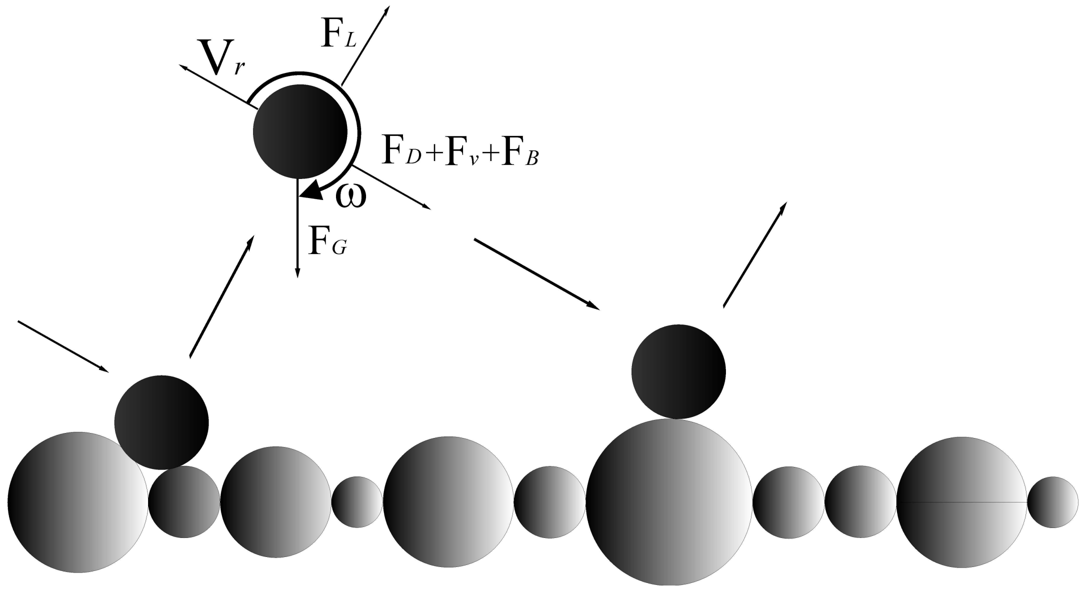
# Talk Outline

# XXVI International School of Hydraulics (2006)

International Scientific Committee of ISH

- Prof. Ian Guymer, University of Warwick, UK;
- Prof. Andrea Marion, University of Padova, Italy;
- Prof. Vladimir Nikora, University of Aberdeen, UK;
- Dr. Steve Wallis, Heriot-Watt University, Edinburgh, UK;

# XXVI International School of Hydraulics (2006)




$$m_s \frac{d\mathbf{u_s}}{dt} = \frac{\rho C_D A_D}{2} u_r \mathbf{u_r} + m_f C_m \frac{\partial \mathbf{u_r}}{\partial t} + \\ + \frac{1}{2}\rho C_L A_D \left( |\mathbf{u_r}|_T^2 - |\mathbf{u_r}|_B^2 \right) + \mathbf{g}(m_f - m_s)$$

where: $\mathbf{u_r} = \mathbf{u_f} - \mathbf{u_s}$ the relative water and particle velocity vector, $d$ the particle diameter, $C_D$ the drag coefficient, $C_L$ the lift coefficient, $C_m = 0.5$ the virtual mass coefficient, $A_D$ the cross-section area, $m_p$ and $m_f$ the particle and water masses, $\rho$ the water density, $t$ the time of saltation, subscript $B$ and $T$ denote the particle bottom and particle top.

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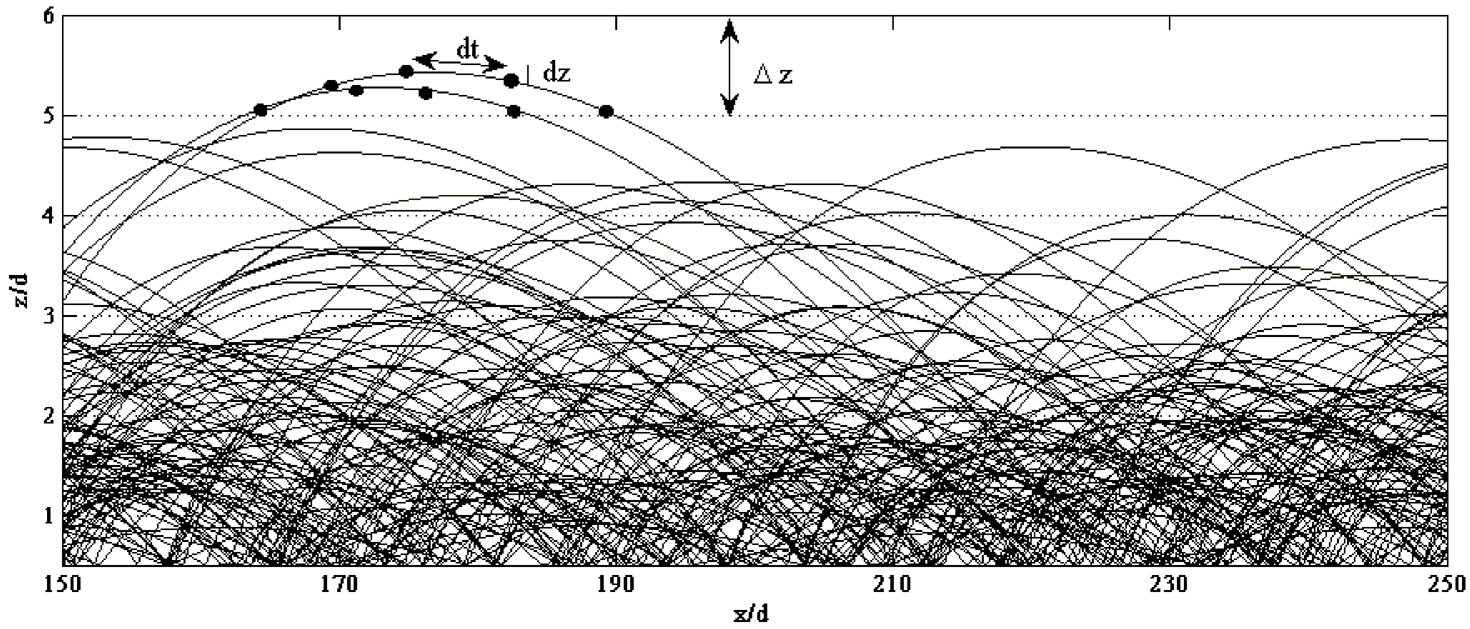



Figure: Typical particle trajectories obtained using presented Lagrangian model

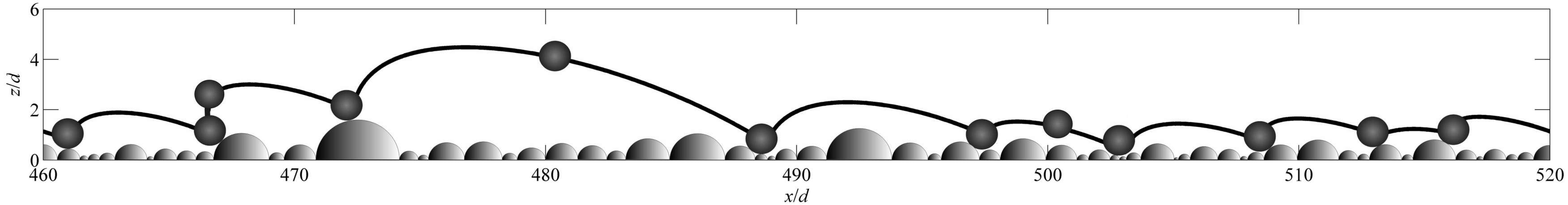
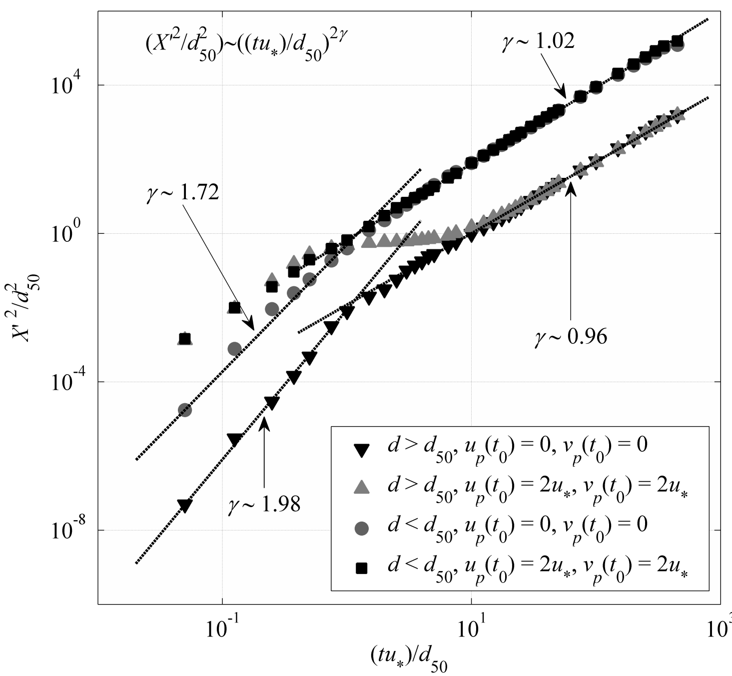
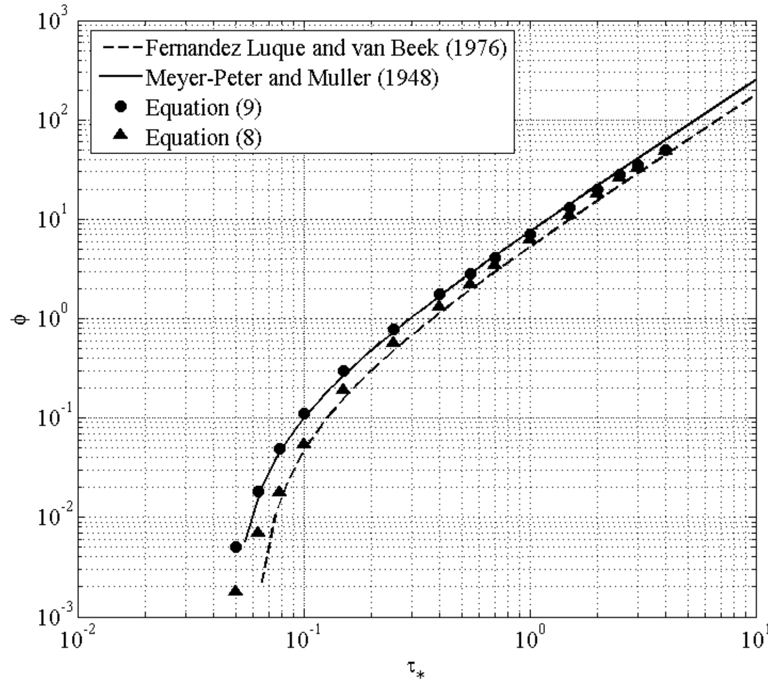



Figure: Modeling of saltation of non-uniform grains




(a) Diffusion of saltating particles




(b) Bed-load transport

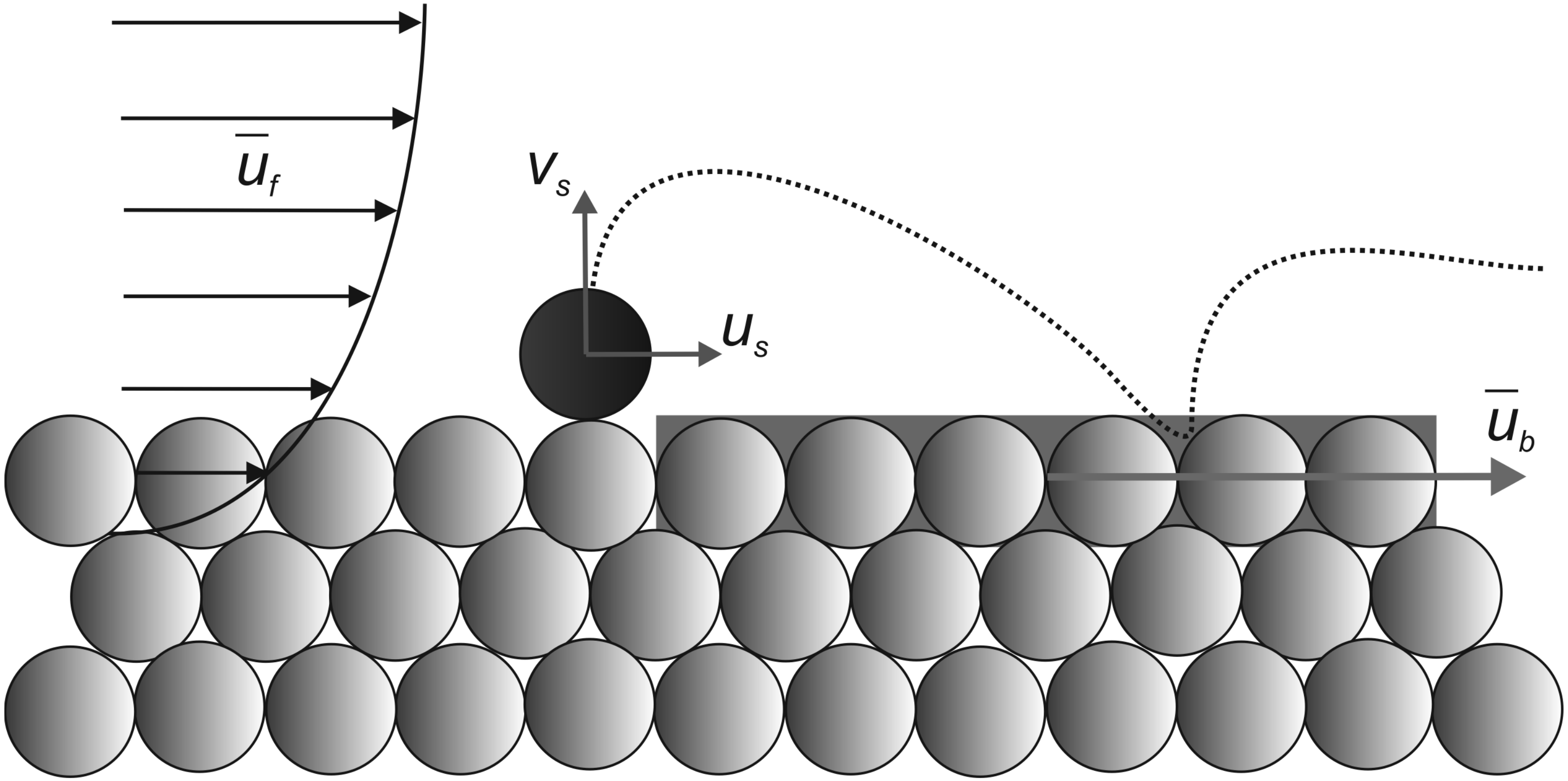
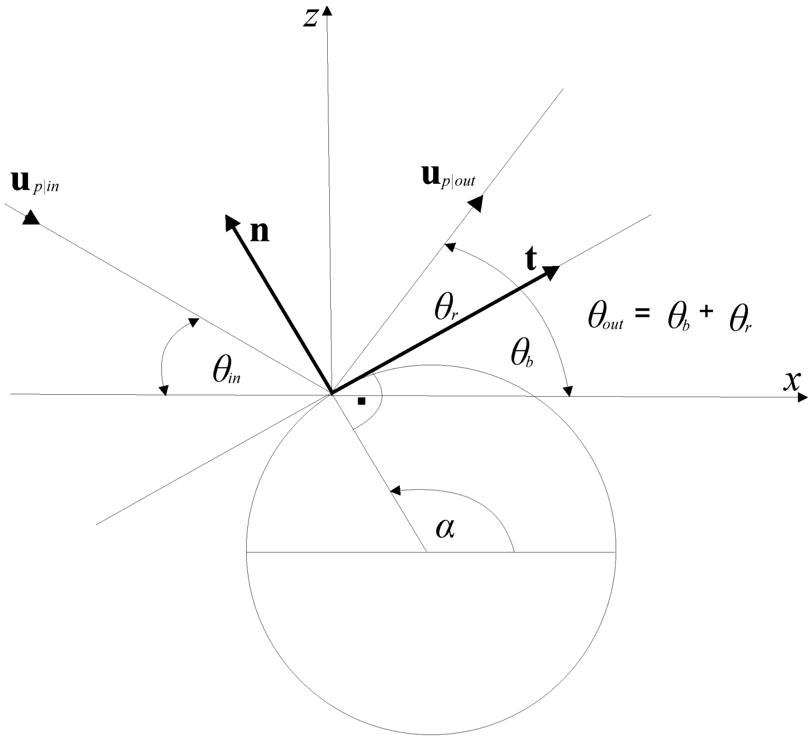



Figure: Scheme of the particles movement over movable bed, $v_s$ and $u_s$ are the longitudinal and vertical velocities of particles, respectively, $u_f$ is the flow velocity and $u_b$ is the mean velocity of the channel bed

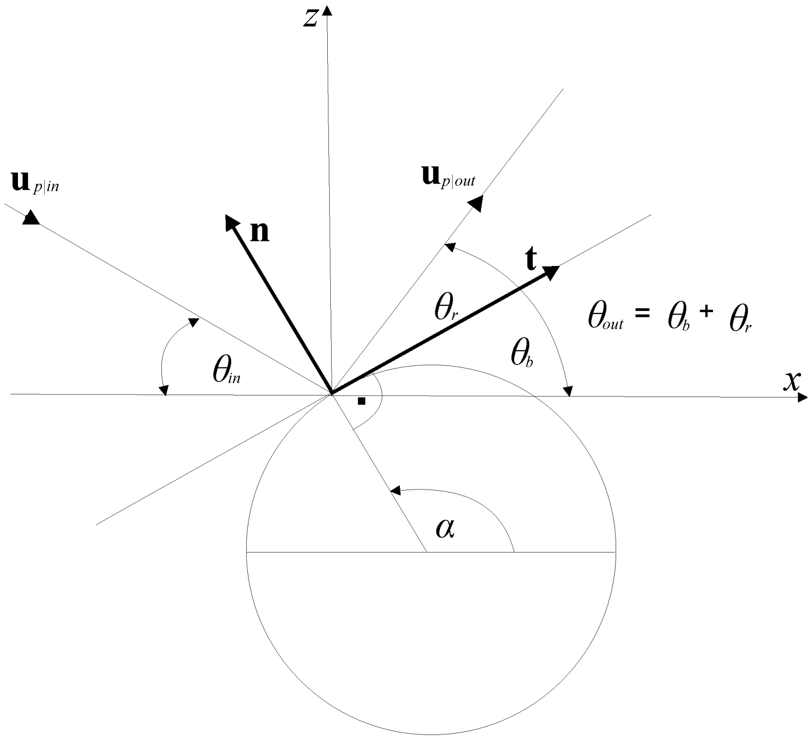
# Collision with the bed (Niño and Garcia 1994)




$$u_{p|out} = f\sqrt{\left(u_{p|in}^2 + v_{p|in}^2\right)}\cos\left(\theta_{in} + \theta_b\right)\frac{\cos\left(\theta_r + \theta_b\right)}{\cos\theta_r}$$

$$v_{p|out} = f\sqrt{\left(u_{p|in}^2 + v_{p|in}^2\right)}\cos\left(\theta_{in} + \theta_b\right)\frac{\cos\left(\theta_r + \theta_b\right)}{\sin\theta_r}$$

$$\tan\theta_r = \frac{e}{f}\tan\left(\theta_{in} + \theta_b\right)$$




$$\mathbf{v}^1_{\mathbf{out}} = \mathbf{v}^1_{\mathbf{in}} - (\mathbf{n} - f\mathbf{t})(\mathbf{n} \cdot \mathbf{V}^0)\frac{(1+e)m_2}{(m_1+m_2)}$$

$$\mathbf{v}^2_{\mathbf{out}} = \mathbf{v}^2_{\mathbf{in}} - (\mathbf{n} - f\mathbf{t})(\mathbf{n} \cdot \mathbf{V}^0)\frac{(1+e)m_1}{(m_1+m_2)}$$

where:

$$\mathbf{n} = (cos\alpha, sin\alpha), \qquad \mathbf{t} = (-sin\alpha, cos\alpha).$$

- How the velocity of channel bed affects the average characteristics of saltation (length and height)?

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- Is the ratio $m_2/m_1$ (coefficient of cohesion) can be successfully used to determine the amount of momentum transferred at the moment of particle collision with the channel bottom?

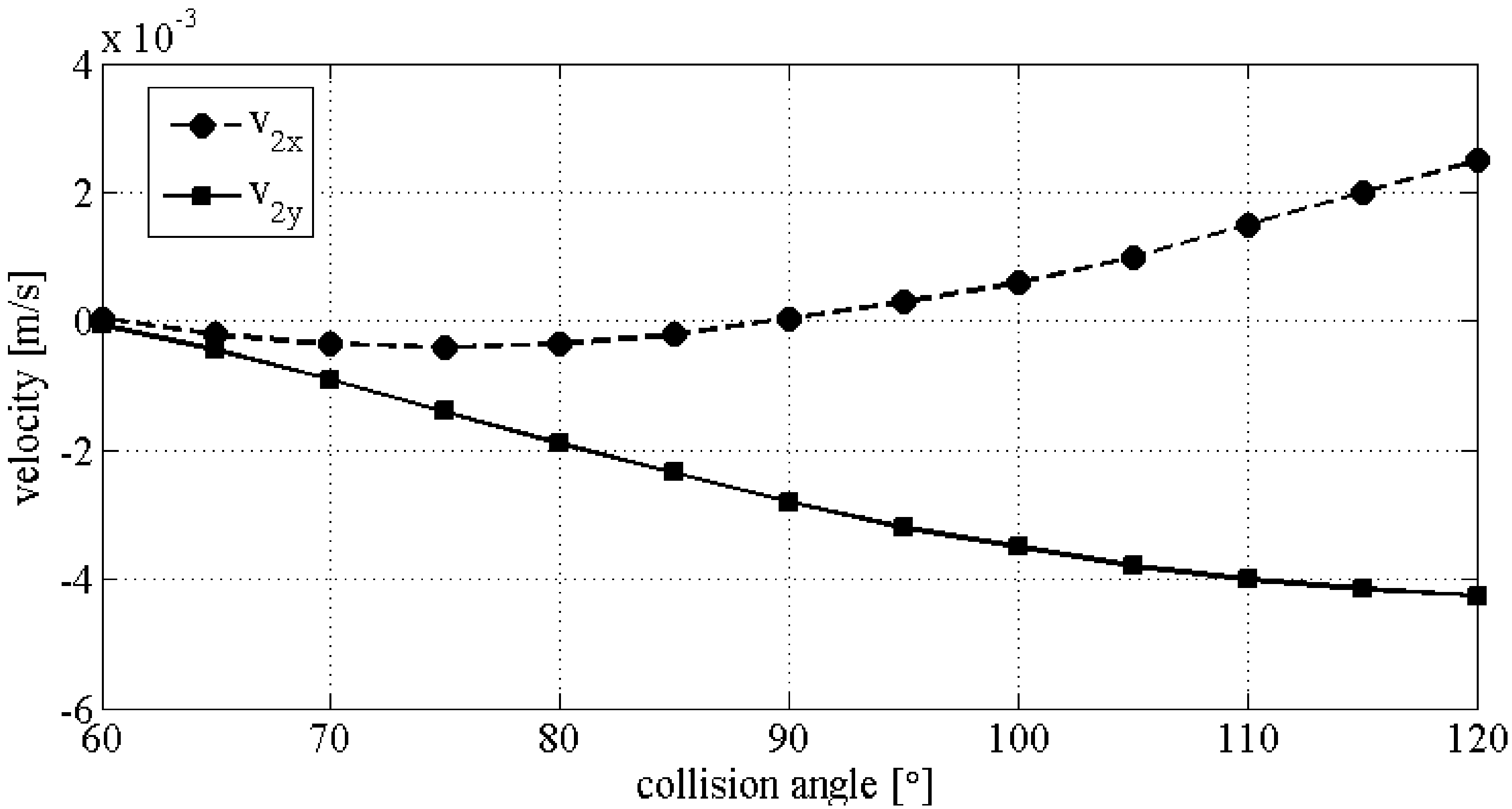



Figure: Particle velocity of the surface layer vs. collision angle, for the following data: $V_1$ = (0.074 m/s, -0.043 m/s), $\theta_{in}$ = 30° $m_2/m_1 = 500$

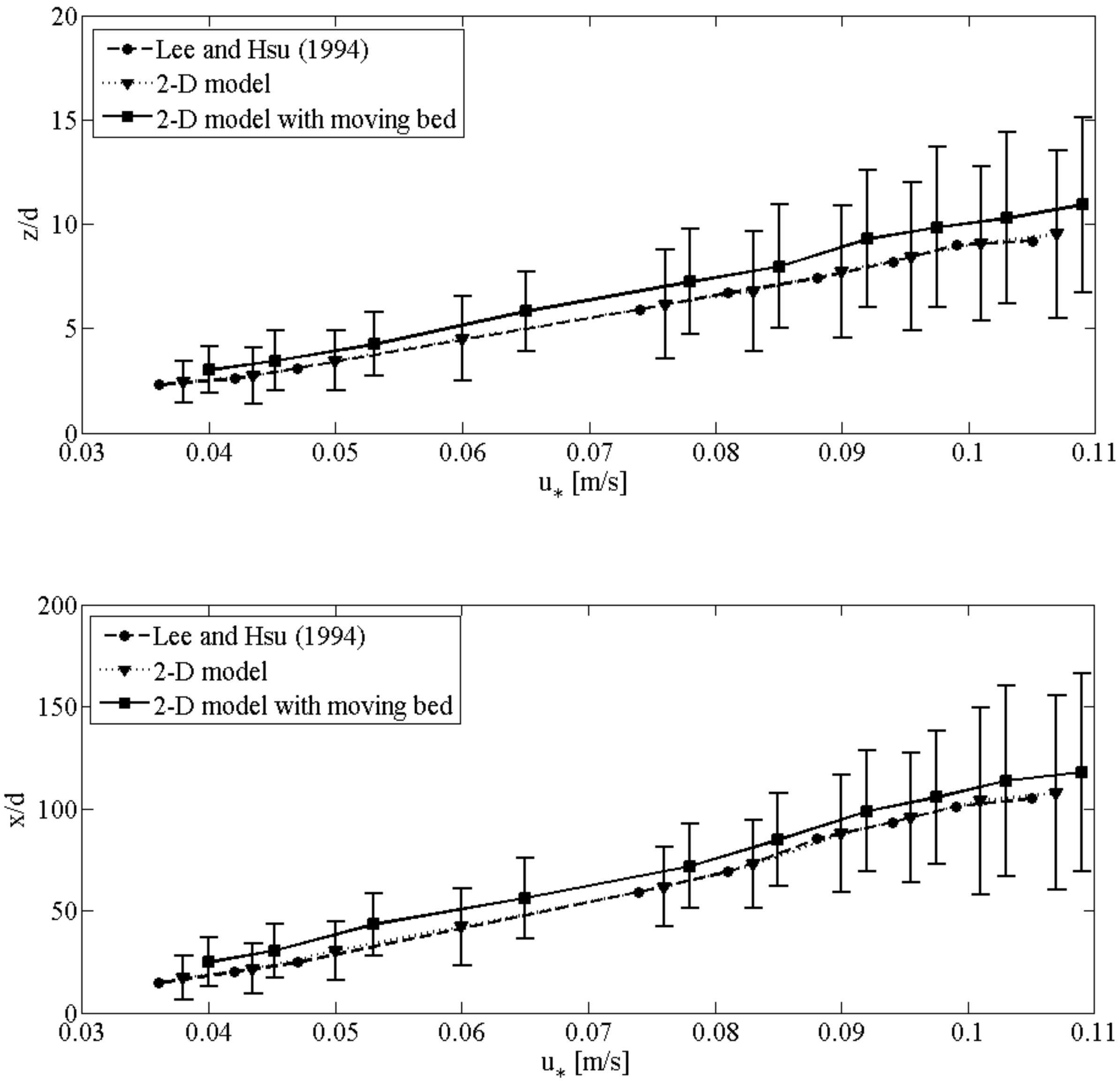



Figure: Comparisons of numerical simulations with experimental data of Lee and Hsu (1994) (a) height and (b) length

$$\mathbf{v}^1_{\mathbf{out}} = \mathbf{v}^1_{\mathbf{in}} - (\mathbf{n} - f\mathbf{t})(\mathbf{n} \cdot \mathbf{V}^0)[w_1]$$
$$\mathbf{v}^2_{\mathbf{out}} = \mathbf{v}^2_{\mathbf{in}} - (\mathbf{n} - f\mathbf{t})(\mathbf{n} \cdot \mathbf{V}^0)[w_2]$$

where:

$$[w_1] = \frac{(1+e)m_2}{m_1+m_2}, \qquad [w_2] = \frac{(1+e)m_1}{m_1+m_2}$$

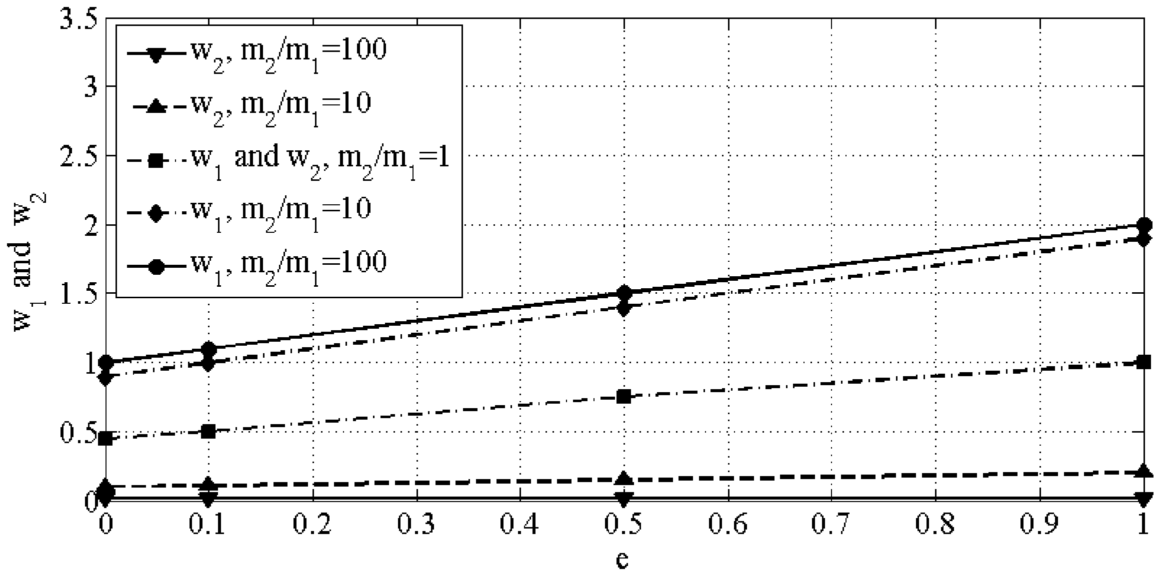
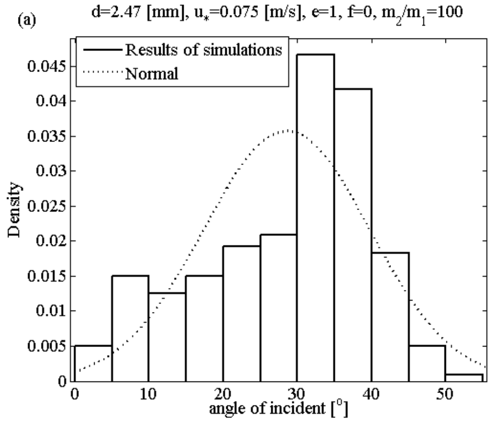
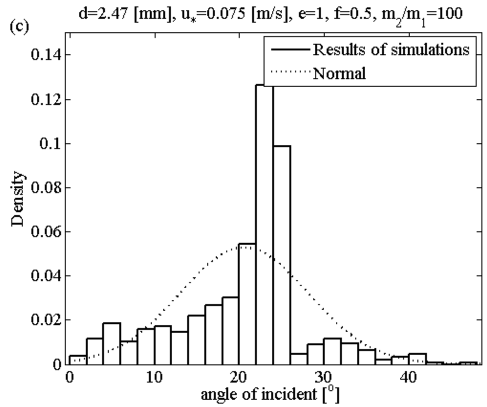


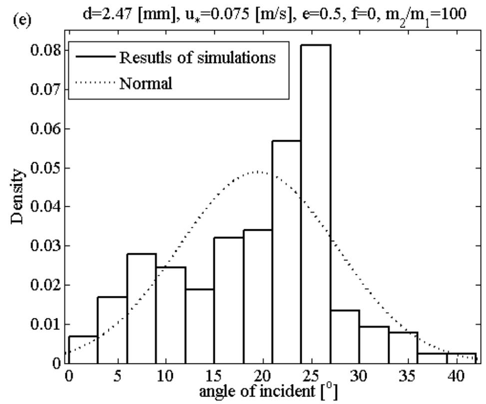

Figure: Values of the factors $[w_1]$ and $[w_2]$ as a function of the friction $f$ and restitution $e$ coefficients for the three selected ratio of $m_2/m_1 =$ 1, 10 and 100



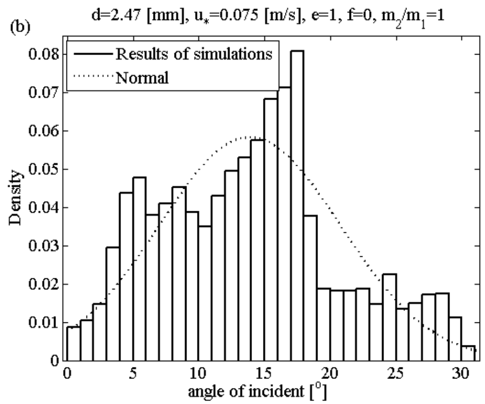

(a) e=1, f=0, $m_2/m_1 = 100$



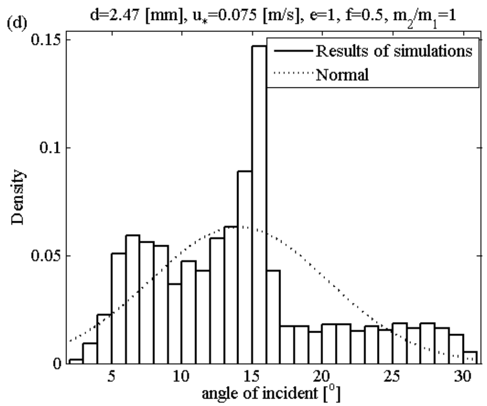

(b) e=1, f=0.5, $m_2/m_1 = 100$



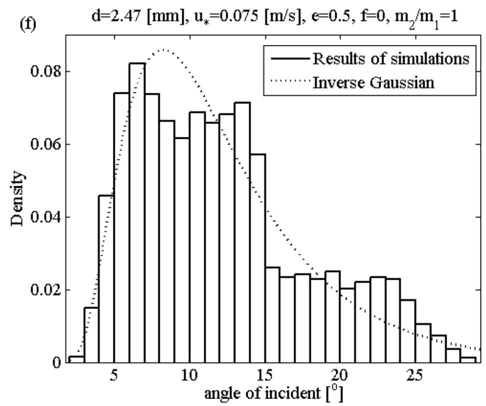

(c) e=0.5, f=0, $m_2/m_1 = 100$




(d) e=1, f=0, $m_2/m_1 = 1$




(e) e=1, f=0.5, $m_2/m_1 = 1$




(f) e=0.5, f=0, $m_2/m_1 = 1$

Figure: Probability distributions of angles of incidence for particles of diameter d = 2.47 [mm], $u_* = 0.075$ [m/s] for different ratio $m_2/m_1$

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- The numerical results of saltaion over moving (with averaged velocity) bed show small overestimation in the mean saltation height and length;
- The ratio $m_2/m_1$ may be successfully used in the Lagrangian models to set the links between the saltating grains and the moving bed;
- Generally, the presented results may be the first step to build the model of bed load transport that will consist of two regimes: saltation and sheet flow.

# Discussions and cooperation with:

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