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Evidence of non-universality of von Kármán's κ

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3. Flows with Bed-Load Transport
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Introduction

- ✓ The *von Kármán constant* κ relates the $u(y)$ profile along y in a wall-bounded shear flow to the shear stress at the bed surface.

- ✓ The universal logarithmic law of time-averaged streamwise velocity by Theodore von Kármán in 1930 is:

$$\frac{u(y)}{u_\tau} = \frac{1}{\kappa} \ln\left(\frac{y}{y_0}\right)$$

u_τ = shear velocity;

y_0 =zero-velocity level;

κ=von Kármán’s constant, having a universal value of 0.41.

- ✓ The zero-velocity level y_0 is dependent on smooth, transitional and rough flow regimes governed by the shear Reynolds number R_* and/or k_s;

$$R_* = u_\tau k_s / \upsilon$$

k_s = Nikuradse equivalent sand roughness;
υ = kinematic viscosity of fluid.

- ✓ The expressions of y_0 for different flow regimes are (van Rijn 1993):

$$y_0(R_* \le 3) = \frac{\upsilon}{9.1 u_\tau} \quad \text{for smooth regime}$$

$$y_0(3 < R_* \le 70) = \frac{\upsilon}{9.1 u_\tau} + \frac{k_s}{30} \quad \text{for transitional regime}$$

$$y_0(R_* > 70) = \frac{k_s}{30} \quad \text{for rough regime}$$

- ✓ There have been some attempts to compute κ mathematically for an idealized flow over a smooth rigid wall or for homogeneous turbulence. For instance:

1) for an idealized flow, Long et al. (1993) analyzed the turbulent flow in smooth pipes with an eddy viscosity closure, obtaining $\kappa = 0.408 \pm 0.004$;

2) Lo et al. (2005) estimated von Kármán’s κ for wall-bounded turbulence based on the similarity with homogeneous constant-shear turbulence, obtaining $\kappa \approx 0.42$;

3) using an energy argument and the mathematical symmetry for an idealized flow, Hughes (2007) argued that $\kappa = 0.414$ for smooth wall.

✓ Some considerations were made to set the *u*-profiles over rough beds and for low submergence by fixing the y_0 value.

✓ Koll (2006) described the parameterization of the *u*-profiles over rough beds by identifying three layers:

1) the roughness layer, in which the *u*-profile is influenced by wall roughness;
2) the log-law layer, in which the log-law velocity profile exits;
3) the outer layer, which covers up to the free surface.

✓ The first two layers form the wall region.

✓ The *u*-profile holds, in the log-law layer, as

$$u/u_\tau = \kappa^{-1}\ln[(y - d)/(y_\mathrm{R} - d)] + (u_\mathrm{R}/u_\tau),$$

d = zero-plane displacement height;
u_R = velocity at the top of the roughness layer y_R.

✓ Nikora et al. (2000) put forward a concept that the large-scale turbulent eddies are in the order of the zero-plane displacement height d, where the mixing length l vanishes.

$$l = \kappa(y - d)$$

✓ The parameters κ, d, y_R and u_R/u_τ can be predicted from the fitting of $u(y)$-profiles using experimental data.

- ✓ For low submergence, the wakes produced by the roughness layer reduce severely or eliminate the log-law and the outer layers, implying that κ does not hold.

- ✓ The evaluation of κ from the log-law loses its significance if the roughness layer destroys the log-law or the outer layer.

- ✓ Considering that the roughness layer extends up to $4k_s$ on the top of the roughness elements, the non-existence of the log-law layer is found up to $h \approx 4k_s$, (h = flow depth).

- ✓ As a consequence, κ loses its implication and cannot be evaluated.

✓ This topic has attracted the attention not only of hydraulicians but also of sedimentologists.

✓ In the literature, there remain many examples of the non-universality of von Kármán's κ in flows over sediment beds:

For instance, von Kármán's κ behaves as a variable in flows with:
(1) low relative submergence, S_r (= h/k_s);
(2) bed-load transport;
(3) suspended-load transport.

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Rand (1953):

✓ He was the first to conduct a flume experiment to study the effect of relative submergence on von Kármán’s κ with artificial bottom roughness.

✓ He used 63 mm square bars spaced at 25.3 mm. He observed a value of $\kappa = 0.3$ for $S_r = 3.3$.

Bayazit (1976):

✓ He reported a gradually increasing κ (greater than 0.4) as the relative submergence S_r decreases below 2.5.

✓ He evaluated κ by fitting the velocity data to the logarithmic law having the position of virtual bed level at $0.33k_s$ below the roughness crest for all experiments.

Pokrajac et al. (2006):

- ✓ Pokrajac et al. (2006) reanalyzed the experimental data of Bayazit (1976) with the roughness crest shear velocity instead of the bed shear velocity.

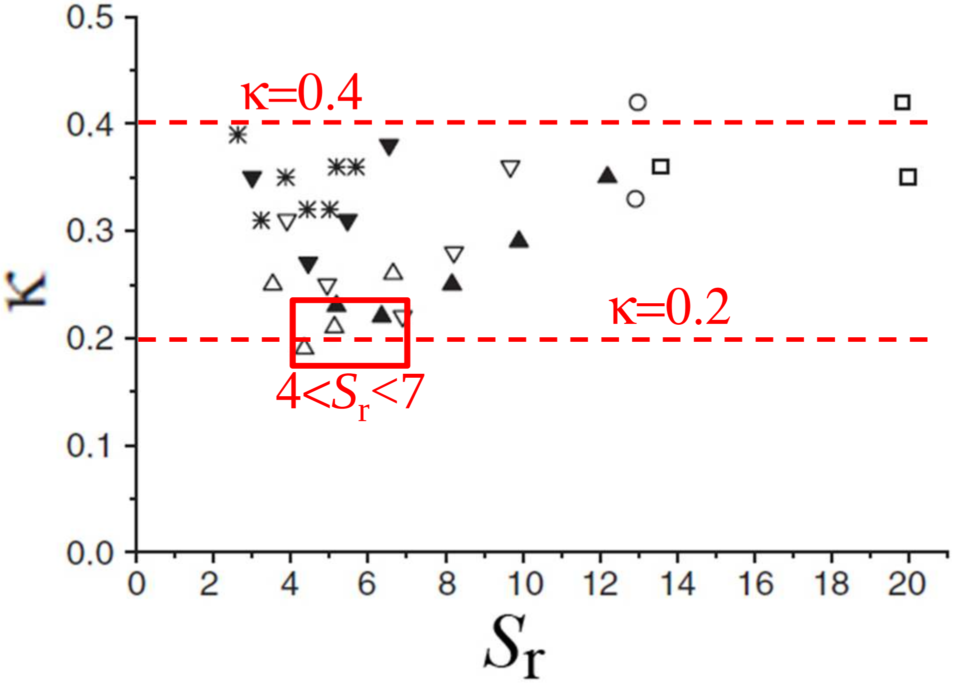
- ✓ Pokrajac et al. (2006) obtained modified values of κ. However, the values of κ remained greater than 0.41 for $S_r < 2.5$.

Dittrich and Koll (1997), Koll (2002) and Koll (2006):

- ✓ They showed that κ is non-universal and depends on both the irregularity of the surface and S_r.

- ✓ These parameters influence the formation and the evolution of the turbulence coherent structures, and in turn the velocity gradient.

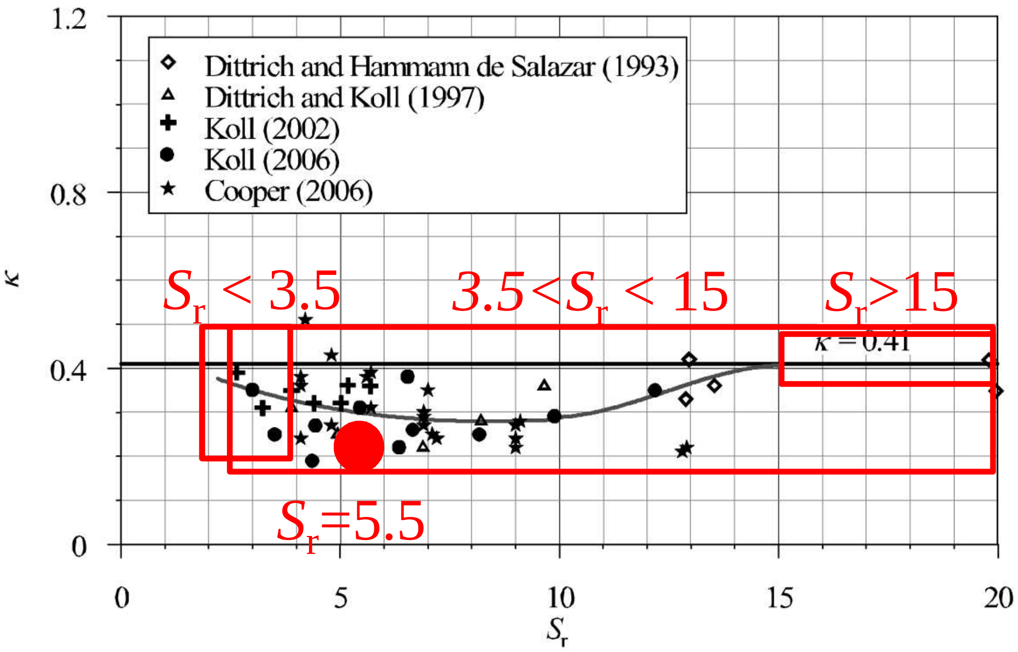
- ✓ Koll (2006) observed that κ approaches a value of 0.4 for regular surfaces and large values of S_r.
- ✓ It decreases significantly down to 0.2 if the bed roughness becomes large enough to decrease the flow depth relative to the roughness height.
- ✓ The value of κ reaches a minimum value within the range $4 < S_r < 7$ and increases again with a decrease in S_r.



Relation between κ and relative submergence S_r (Koll 2006)

Cooper (2006)

- ✓ also observed the values of κ lower than 0.41 for lower values of S_r ($4 < S_r < 13$).



Dependency of von Kármán's κ on the relative submergence S_r

- ✓ The figure shows that κ increases drastically for $S_r < 3.5$;
- ✓ It reaches the minimum value $\kappa = 0.25$ at $S_r = 5.5$;
- ✓ It becomes universal for $S_r > 15$;
- ✓ The curve κ versus S_r is sagging within $3.5 < S_r < 15$.

- ✓ Now the questions concerned with the experiments and the estimations of the non-universality of κ are:

1) if Nikuradse's equivalent sand roughness k_s used for scaling h were the same as those obtained from the velocity profiles, and

2) if roughness layer influences the value of κ.

- ✓ The latter is the major decisive factor towards the non-universality of κ;

- ✓ the former provides the scale up to which κ is a function of S_r.

- ✓ Additionally, hyporheic exchange is often controlled by subsurface advection.
- ✓ It is driven by the interaction of the fluid flow with sedimentary pore water, as macro-rough beds are permeable.
- ✓ The nature and magnitude of the induced hyporheic exchange flow influences the main stream.
- ✓ This influence occurs by changing the velocity and Reynolds shear stress profiles and possibly κ as well.

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Gust and Southard (1983)

- ✓ They observed a decrease in κ with an increase in bed-load transport.

- ✓ They suggested that, after a transitional regime related to the entrainment threshold of sediments, κ adjusted to a constant value of 0.32 ± 0.04 for all the bed-load experiments in which the transport rate varied by a factor 10.

- ✓ This implies that the uncertainties associated with the measured bed-load transport rate had a minimal effect on the result that κ is reduced by 25% from its universal value.

Best et al. (1997)

- ✓ They used a phase Doppler anemometer to differentiate the characteristics of the fluid from those of the sediment particles.

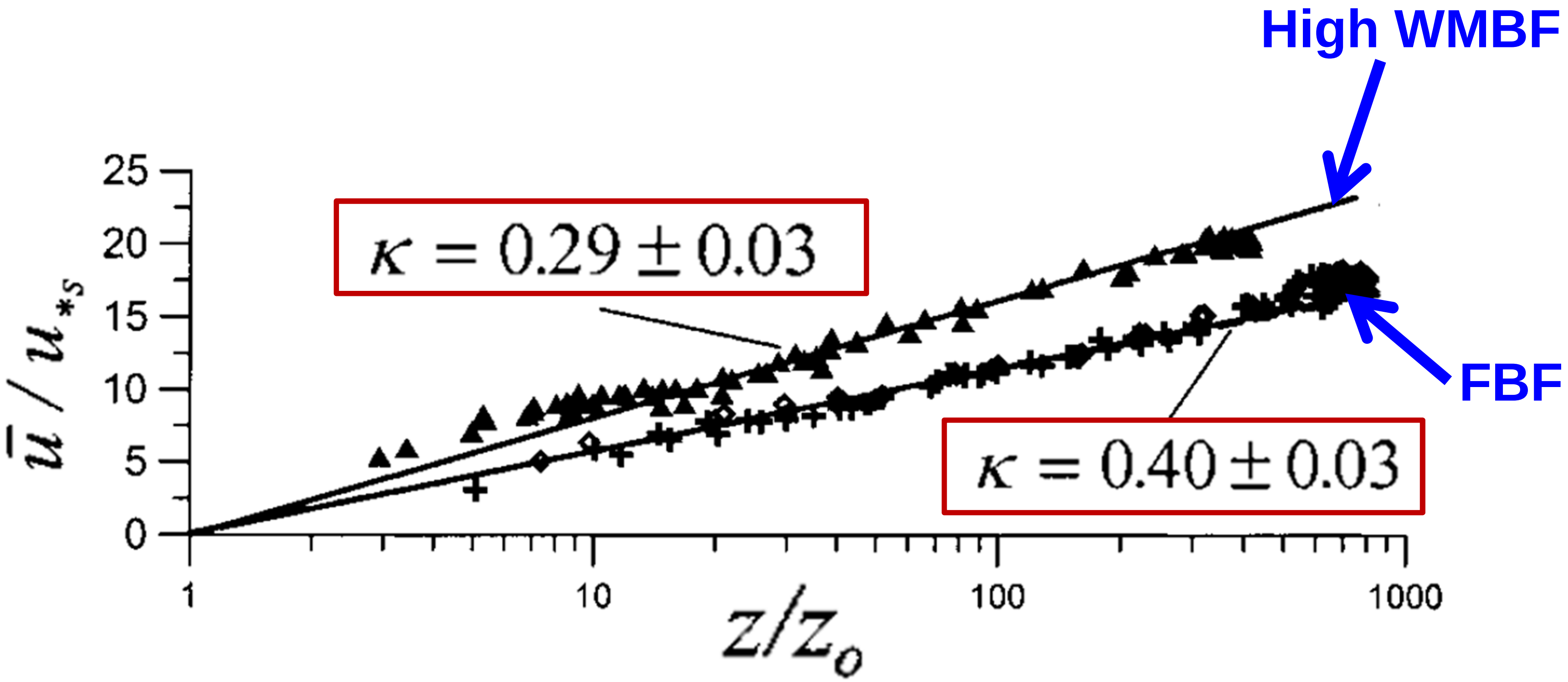
- ✓ They observed that the average value of κ was 0.385 in presence of bed-load transport.

Nikora and Goring (2000)

- ✓ They reported a study on the characteristics of turbulent structure of high Reynolds number in quasi-two-dimensional flows.

- ✓ They performed three sets of measurements taken with an acoustic Doppler velocimeter in an irrigation field canal for two bed conditions: (1) fixed-bed flow (FBF) and weakly mobile-bed flow (WMBF).

- ✓ Nikora and Goring (2000) obtained $\kappa \approx 0.29$ for the WMBF being significantly smaller than $\kappa \approx 0.4$ for the FBF.



$\bar{u} / u_{*s}$: Local mean velocity/Friction velocity from turbulent stress distribution

z/z_o : Distance from the bed/Roughness length

- ✓ Bennett and Bridge (1995), Nikora and Goring (1999) and Gallagher et al. (1999) have also revealed an appreciable decrease in κ under bed-load transport conditions.

- ✓ Nikora and Goring (1999) imagined that the reduction in κ might reflect the special turbulence characteristics within a rather narrow range of the Shields parameter when the bed shear stress is approximately equal to the critical shear stress.

- ✓ In Nikora and Goring (2000), the drag reduction effects were expressed as decreased values of κ.

- ✓ The general concept is that the drag reduction prevails when the spacing between turbulent bursting events increases in comparison to the spacing in flows with no sediment (Tiederman et al. 1985).

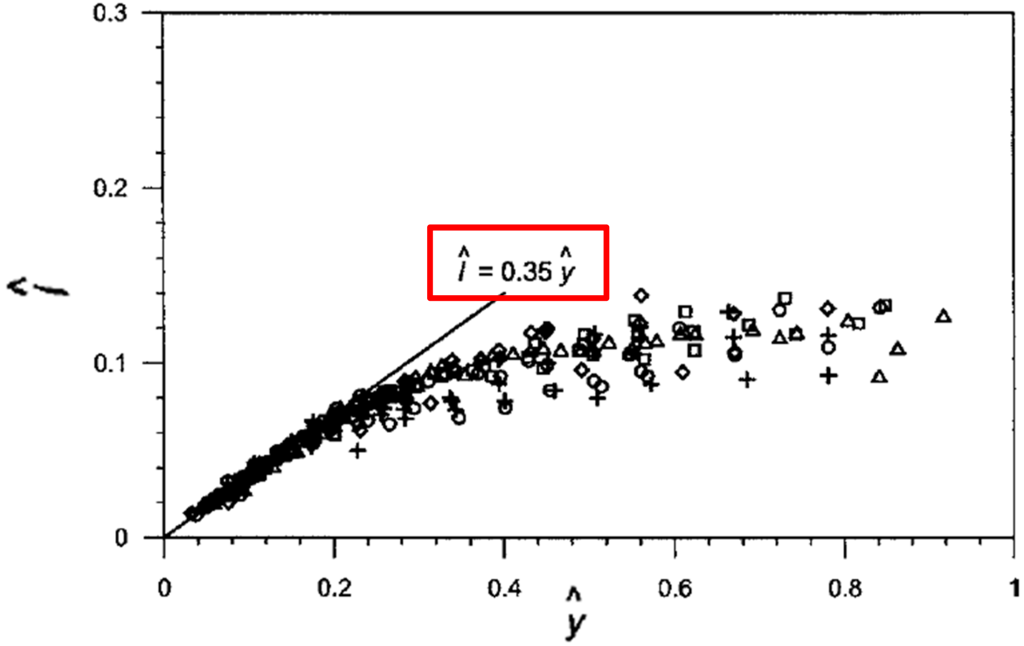
- ✓ However, it is revealed that κ reduces when spanwise (lateral) spacing between bursting events increases, while streamwise spacing remains unchanged (Hetsroni et al. 1997).

- ✓ Nikora and Goring (2000) found that the streamwise spacing between bursting events was approximately the same for both WMBF and FBF, referring to an increase in spanwise spacing for WMBF.

Dey and Raikar (2007):

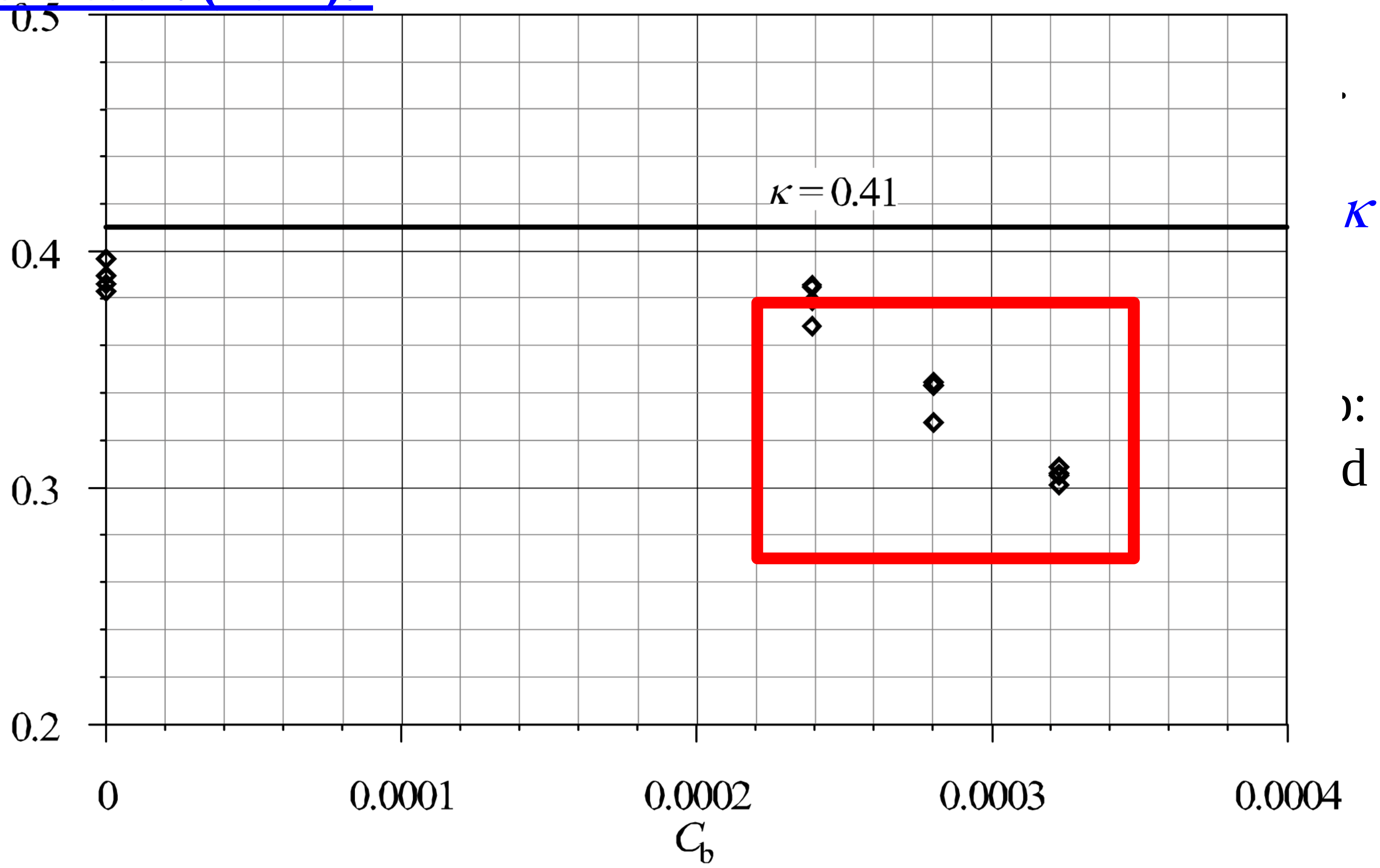
✓ They reported the laboratory experimental results on the turbulent flow characteristics measured by an acoustic Doppler velocimeter.

✓ They observed that the variation of the mixing-length is considerably linear with the elevation above the bed within the inner-layer and obtained von Kármán’s κ=0.35.



Mixing length as a function of flow depth

Gaudio et al. (2011):



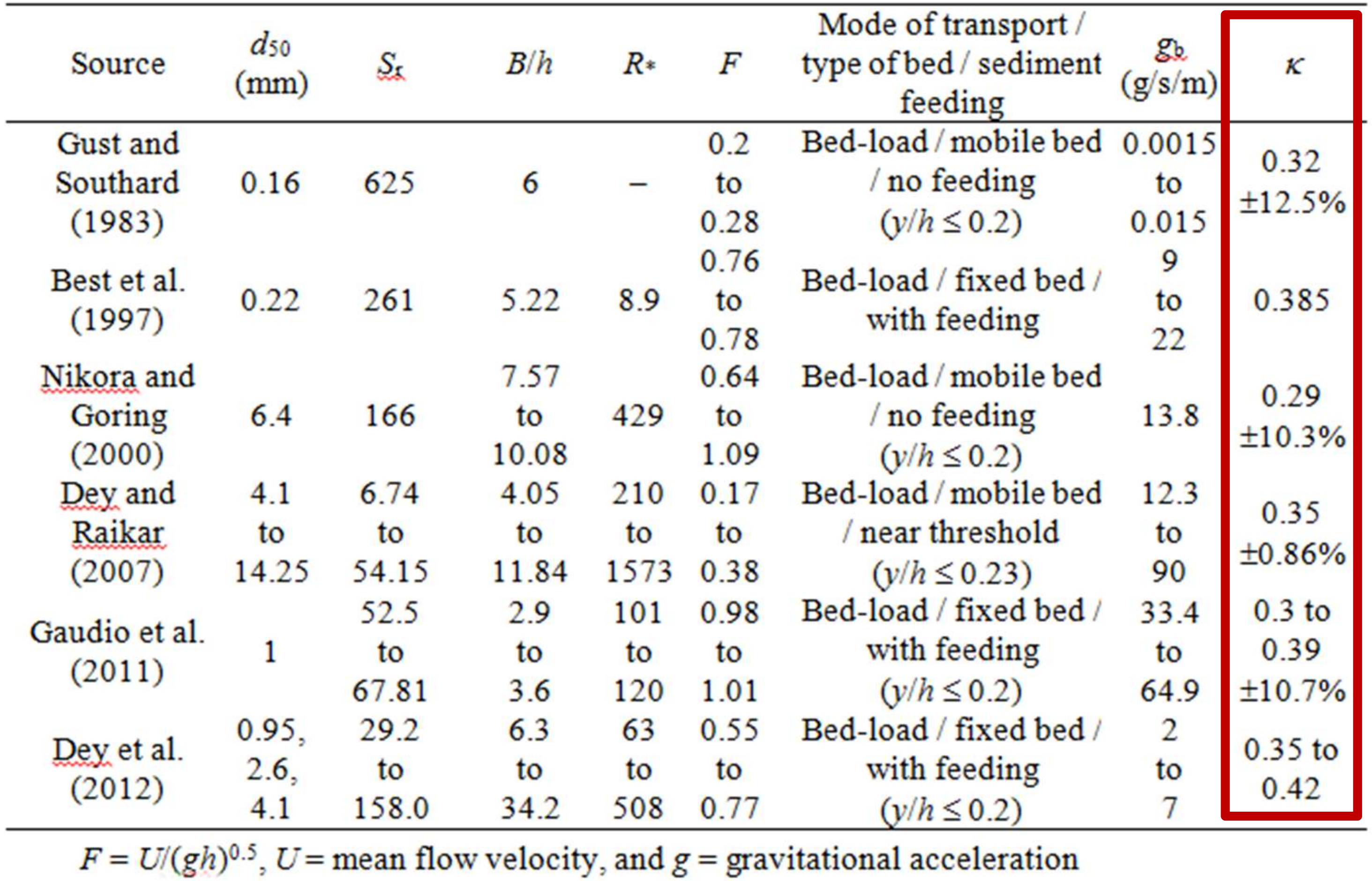
✓ ,

✓ κ

✓):
d

- ✓ Further, Dey et al. (2012) fitted a log-low for mobile-bed flows to obtain $\kappa = 0.37$.

Summary of experimental data and results for κ in flows with bed-load transport



Source	d_{50} (mm)	S_r	B/h	R_*	F	Mode of transport / type of bed / sediment feeding	g_b (g/s/m)	κ
Gust and Southard (1983)	0.16	625	6	–	0.2 to 0.28	Bed-load / mobile bed / no feeding ($y/h \leq 0.2$)	0.0015 to 0.015	0.32 ±12.5%
Best et al. (1997)	0.22	261	5.22	8.9	0.76 to 0.78	Bed-load / fixed bed / with feeding	9 to 22	0.385
Nikora and Goring (2000)	6.4	166	7.57 to 10.08	429	0.64 to 1.09	Bed-load / mobile bed / no feeding ($y/h \leq 0.2$)	13.8	0.29 ±10.3%
Dey and Raikar (2007)	4.1 to 14.25	6.74 to 54.15	4.05 to 11.84	210 to 1573	0.17 to 0.38	Bed-load / mobile bed / near threshold ($y/h \leq 0.23$)	12.3 to 90	0.35 ±0.86%
Gaudio et al. (2011)	1	52.5 to 67.81	2.9 to 3.6	101 to 120	0.98 to 1.01	Bed-load / fixed bed / with feeding ($y/h \leq 0.2$)	33.4 to 64.9	0.3 to 0.39 ±10.7%
Dey et al. (2012)	0.95, 2.6, 4.1	29.2 to 158.0	6.3 to 34.2	63 to 508	0.55 to 0.77	Bed-load / fixed bed / with feeding ($y/h \leq 0.2$)	2 to 7	0.35 to 0.42

$F = U/(gh)^{0.5}$, U = mean flow velocity, and g = gravitational acceleration

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- ✓ Also in flows with suspended sediment load von Kármán's κ has been a long disputed parameter in the data analysis of the logarithmic velocity profile fittings.

- ✓ During 1946-1961, pioneering research was conducted by Vanoni (1946), Einstein and Chien (1955) and Elata and Ippen (1961) to examine the effect of suspended sediment concentration C_v on the velocity profile.

- ✓ One of the key findings was that κ decreases with an increase in C_v, although Coleman (1981, 1986) expressed a strong dissatisfaction on this issue.

- ✓ Coleman believed that Vanoni's finding was an artifact of the erroneous technique of evaluating κ that was generally accepted at the time when he did this work.

- ✓ According to Coleman, the experimental studies by Einstein and Chien (1955) were also unacceptable because the measurements were taken only over the lower 40% of the flow depth in the experimental channel that they used.

- ✓ Thus, it was difficult to determine the boundary layer thickness, the maximum velocity or any general information about the flow.

- ✓ The experiments by Elata and Ippen (1961) were done using virtually neutrally buoyant polystyrene particles to simulate the sediment suspension.

- ✓ The velocity profiles were presented in a velocity defect form; and the apparent decrease of κ was due to its incorrect evaluation.

- ✓ However, Coleman (1981) used the wake-law to study velocity profiles in sediment-laden flows and suggested that the wake coefficient rather than κ is affected by the existence of sediment suspension.

- ✓ The wake-law that describes the logarithmic velocity profile only in the vicinity of the bed has the same value of κ as that in clear-water flow ($\kappa = 0.41$).

- ✓ Lyn (1986) noted that the use of pure log-law profile leads to a decreased value of κ with C_v, while the use of log-wake-law leads to a universal one.

- ✓ Cioffi and Gallerano (1991) determined experimentally the velocity and sediment concentration profiles on a mobile flat bed.

- ✓ without assessing κ in the inner region, Cioffi and Gallerano (1991) only verified that the measured velocity profiles were reasonably interpolated with $\kappa = 0.4$ for $y/h < 0.15$.

- ✓ Cellino and Graf (1999) carried out an experimental study to investigate the influence of suspended sediments in the flow under non-capacity and capacity conditions.
- ✓ They accepted $\kappa = 0.4$ as a reasonable value for describing the velocity profiles through the velocity defect law.
- ✓ After Muste (2002), a constant value for $\kappa = 0.41$ would be appropriate for the sediment-laden flows involving only small C_v (smaller than 0.05).
- ✓ The aforementioned discussion is given on the investigators who opposed the changed value of κ from its universal value.

- ✓ On the other hand, other investigators believed that κ is non-universal in sediment-laden flows.

- ✓ Many other researchers during 1960s and 1970s also reported that κ diminishes as C_v increases (Vanoni and Nomicos 1960; Hino 1963; Paintal and Garde 1964; Bohlen 1969; Ippen 1971).

- ✓ Such decrement is primarily due to increasing gradient of the velocity profiles in presence of sediment suspension.

Nouh (1989):

- He reported that the variation of κ with the average concentration C_{av} of suspended sediments in straight open-channel flows depends on the flow Reynolds number, $Re = 4Uh/\upsilon$.

- As C_{av} increases, κ decreases in flows with $Re < 7\times10^5$, but increases in flows with $Re > 7\times10^5$.

- The variation of κ with C_{av} is insignificant in flows with Re equal to about the critical value $Re_c = 7\times10^5$.

- He hypothesized that the turbulence level close to the boundary decreases for $Re < Re_c$ and increases for $Re > Re_c$, as C_{av} increases.

- ✓ For $Re < Re_c$, C_{av} affects the turbulence level more than Re, and vice versa for $Re > Re_c$. These two effects are balanced for $Re \approx Re_c$.

- ✓ Nouh (1989) also observed that the increased values of κ in flows with fine suspended sediments are larger than those in flows with relatively coarse suspended sediments, and also those in flows with high C_{av} are larger than those in flows with low C_{av}.

- ✓ In fact, he explained that, for a given C_{av}, coarse suspended sediments produce a larger reduction in turbulence level than fine suspended sediments.

- ✓ In a clear-water flow, universal value of κ is invariant of Re (for $4\times10^5 < Re < 2\times10^6$).

Wang and Qian (1992):

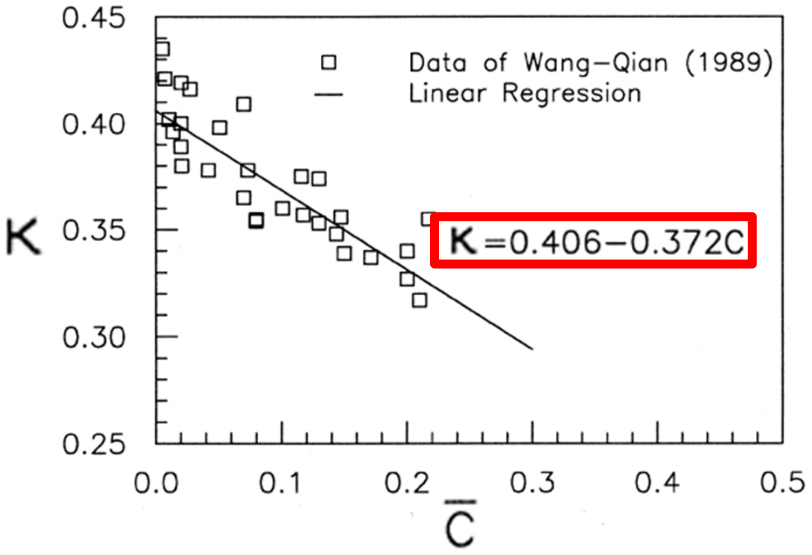
- ✓ Wang and Qian (1992) showed that, in the lower-flow region, the values of κ in sediment-laden flows are less than those in clear-water flows.

Guo and Julien (2001):

- ✓ They argued that the reduction of κ in sediment-laden flows is governed by C_{av} and the mass density gradient given by the Richardson number.

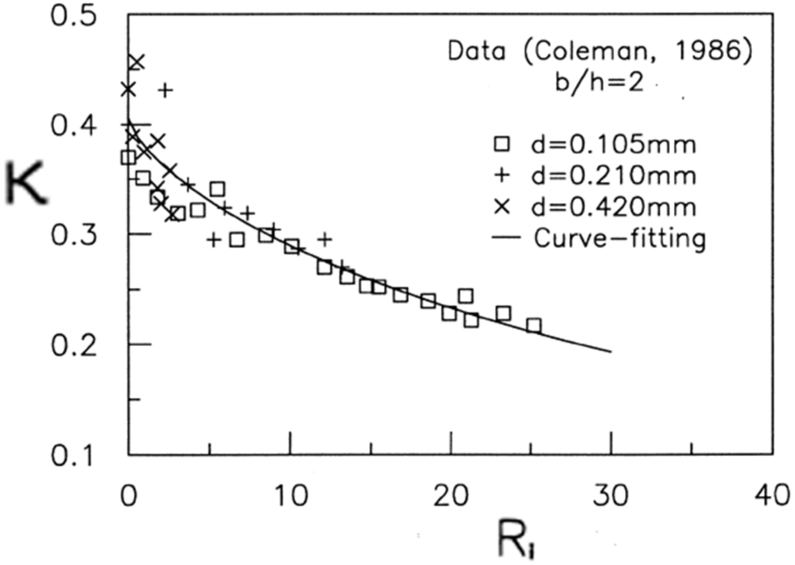
Analysis by Guo and Julien (2001):

Average concentration effect on von Kármán’s *k*



$\bar{C}$:Time-averaged volumetric sediment concentration

Density gradient effect on von Kármán’s *k*

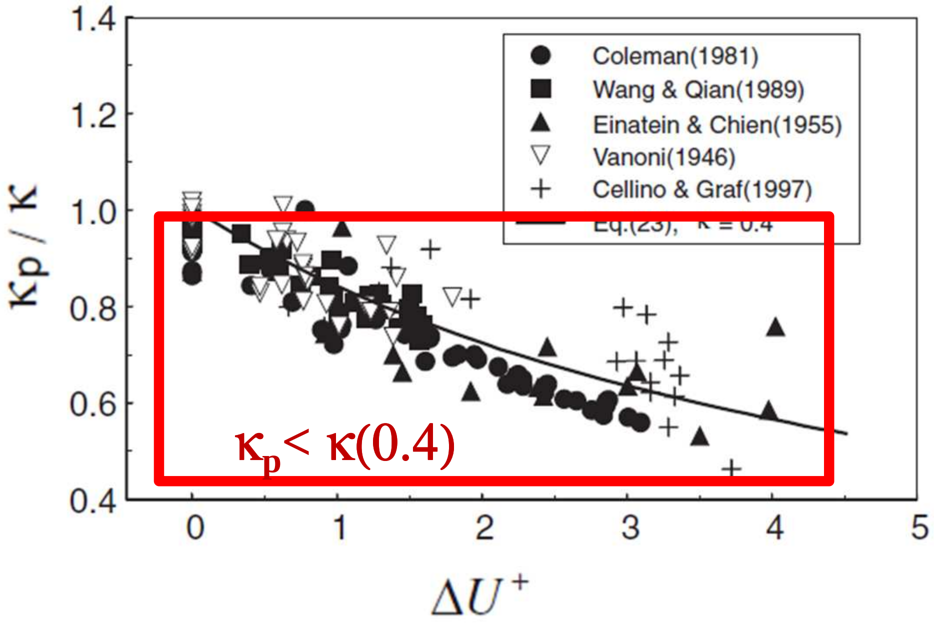


b/h: Channel aspect ratio
d: Particle diameter
R_i: Richardson number

Wang et al. (2001):

- ✓ In presence of sediment suspension, they modified von Kármán's κ_p for the log-law and κ_w for the wake-law separately.

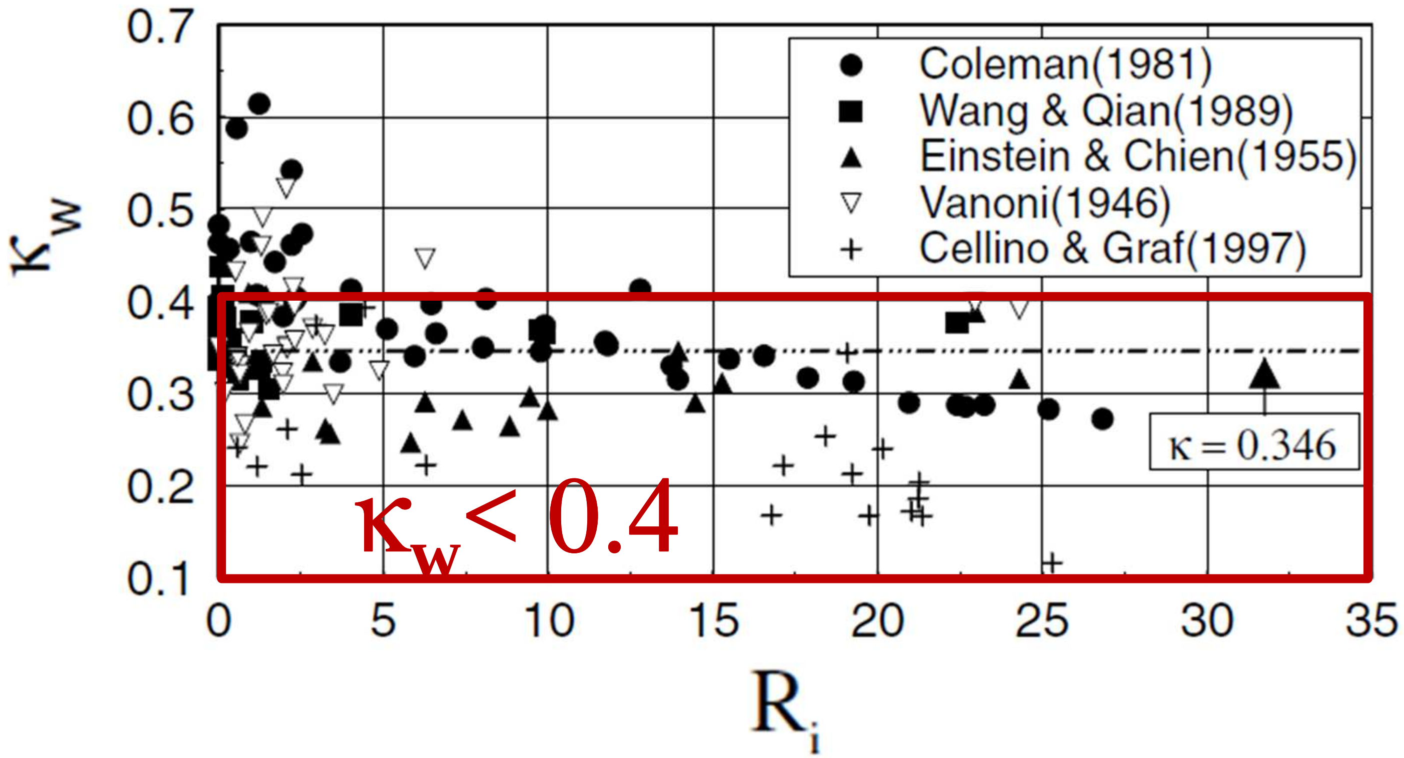
- ✓ For the log-law, they proposed $\kappa_p = 2.08\kappa/(\Delta U^+\kappa + 2.08)$, where ΔU^+ is $(u_c - u_s)/u_\tau$, and u_c and u_s are the velocities at $y/h = 0.05$ in clear-water and sediment-laden flows, respectively.



The relationship of κ_p and ΔU^+

Wang et al. (2001):

- ✓ For the wake-law, they obtained the average value κ_w=0.346 with high data scattering.



Relationship of κ_w and R_i

Wang et al. (2001):

✓ They conducted simultaneous measurements of both the suspended particles and water in particle-laden flows.

✓ They used a discriminator particle-tracking velocimeter, observing that κ decreases with an increase in C_{av}.

Reanalyzing Coleman’s data:

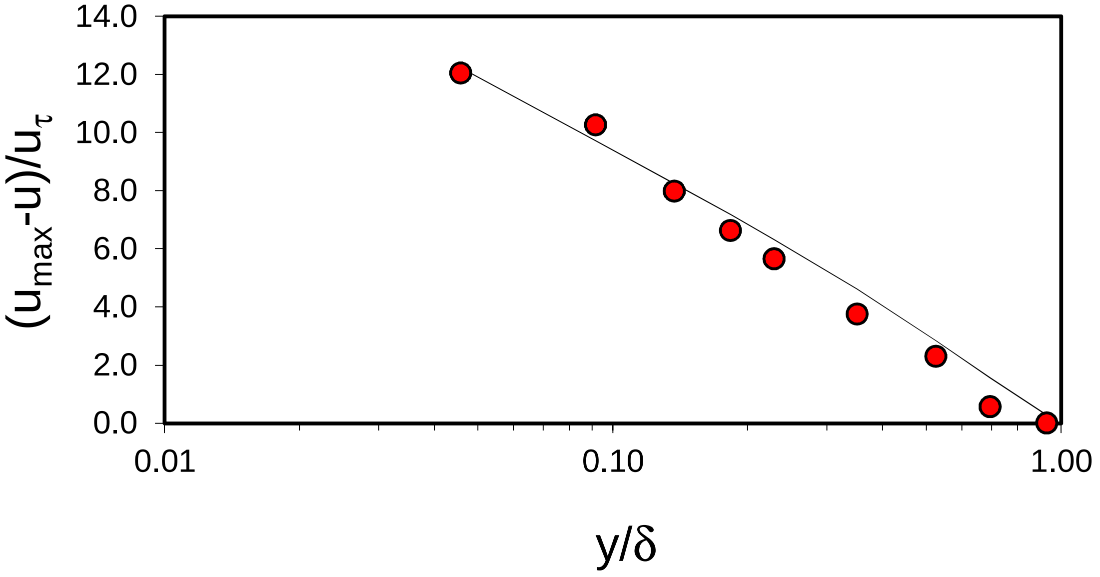
✓ In fact, the investigators supporting the universality of κ also in sediment-laden flows referred to Coleman’s (1981, 1986) data and analysis.

✓ Owing to the available evidence that κ varies in the presence of suspended sediments, it seems to be necessary to reanalyze Coleman’s dataset in order to verify the proposed constancy of κ.

- ✓ By adopting the velocity defect law as in the original work, the dimensionless data $(u_{max} - u)/u_\tau$ were plotted as a function of y/δ in a semi-log graph;

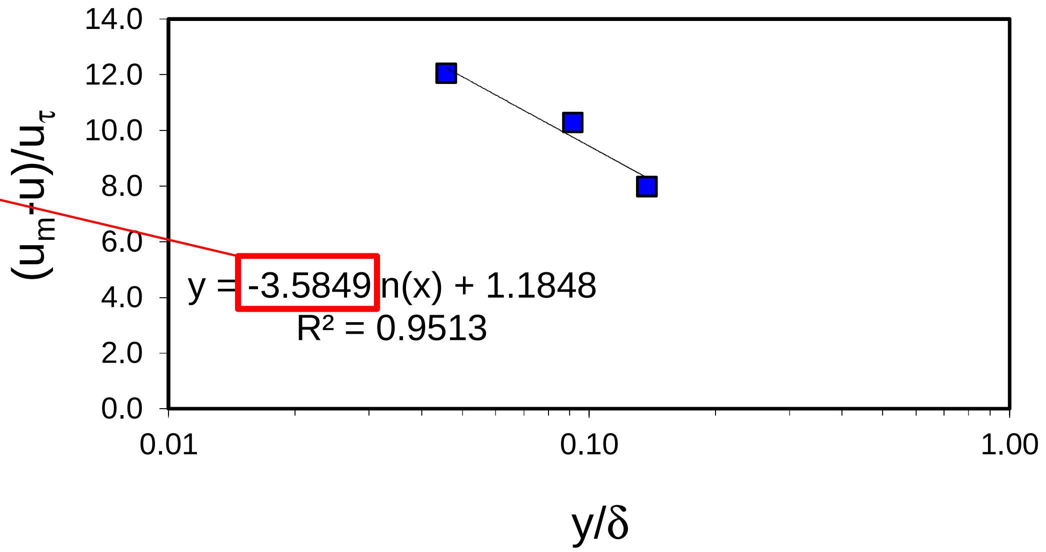
- ✓ δ, the boundary layer thickness, is the distance above the bed at which the time-averaged streamwise velocity reaches its maximum value u_{max}.

Example: Coleman's Run 19



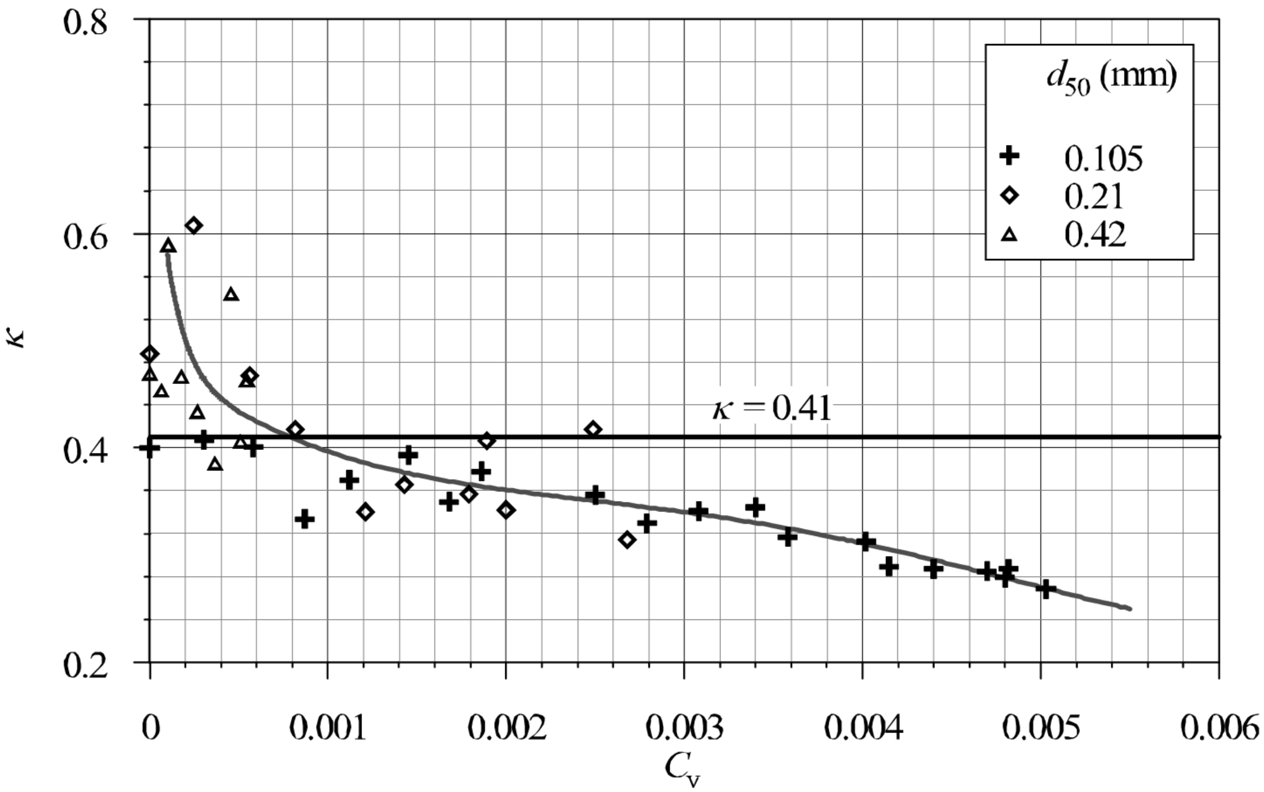
$-\kappa^{-1} = -3.5849$

$\kappa = 0.279$



- ✓ The slopes $-\kappa^{-1}$ were evaluated from the regression analysis fitting straight lines within the log-law layer in the inner flow region ($y \leq 0.15\delta$).

- ✓ We obtained values of κ different from those given in Coleman (1981, 1986).



Variation of κ with C_v obtained by reanalyzing Coleman's data (1981, 1986)

✓The mean trend (solid gray line) shows progressively diminishing values of κ as C_v increases.

✓In general, for $C_v > 0.0008$, the values of κ are less than its clear-water value 0.41.

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✓ Reasons of the variability of κ are recognized as follows:

1. Sediment particles during the bed-load transport interact with both the flow that accelerates them and the rough bed that decelerates them (Gyr and Schmid 1997).

2. Collisions cause momentum extraction from the mean flow in the near-bed region, resulting in reduction of local streamwise velocities and increase of velocity gradients (Owen 1964; Smith and McLean 1977).

3. In low submergence, κ was found to depend on the irregularity of the surface and the relative submergence, which influence the formation and the expansion of the coherent turbulent flow structures, and thus the velocity gradient.

- ✓ In case of bed-load transport, it was not even possible to provide any solid relationship between κ and the bed-load transport rate, although Gaudio et al. (2011) provided an empirical relationship expressing κ as a function of C_b and R_b/d_{50}.

- ✓ The von Kármán κ is given by l/y, where l is the Prandtl mixing length, $[-\overline{u'v'}/(du/dy)^2]^{0.5}$, and u' and v' are the fluctuations of the streamwise and vertical velocity components, u and v, respectively.
- ✓ Thus, κ is dependent on the Reynolds shear stress $-\overline{u'v'}$ relative to the mass density of fluid and on the velocity gradient du/dy.

- ✓ Hence, the variable values of κ depend on the prevailing effect between turbulence intensity and velocity gradient.
- ✓ For instance, if turbulence intensity increases and/or velocity gradient decreases, then κ increases and vice versa.
- ✓ In order to assess the non-universality of κ, future researchers have to keep an eye on the variations of these two parameters with vertical distance within the logarithmic law layer.

Thank you for your kind attention

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Thank you for your kind attention