

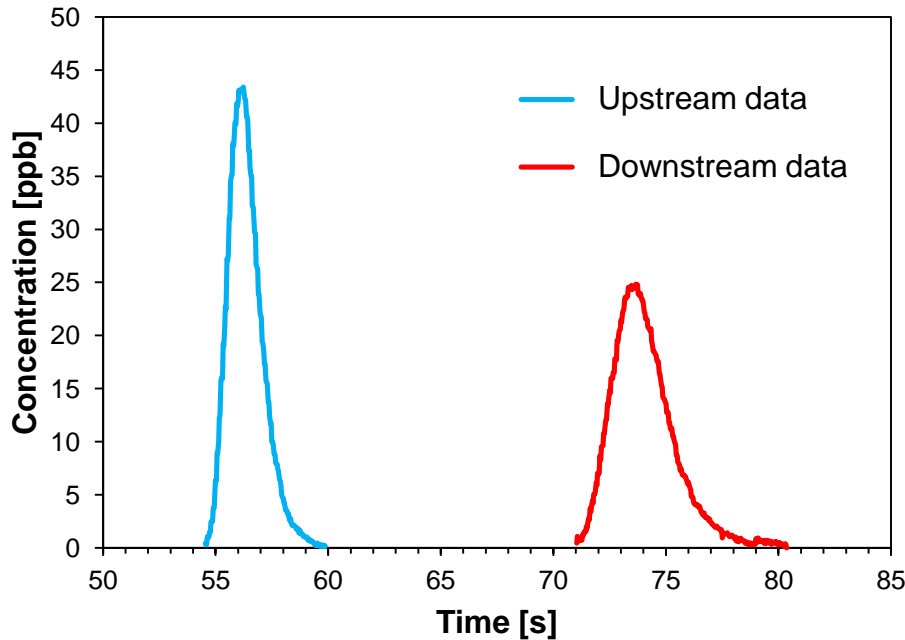
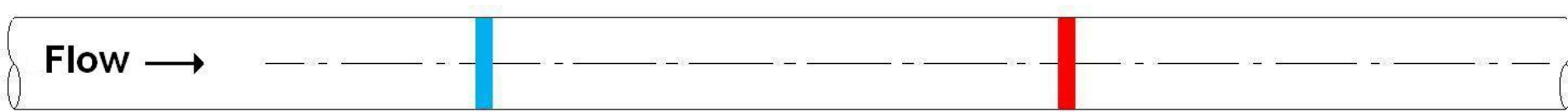
# Longitudinal Dispersion Coefficients within Turbulent and Transitional Pipe Flow

James Hart<sup>1</sup>, Ian Guymer<sup>1</sup>, Amy Jones<sup>1</sup> and Virginia Stovin<sup>2</sup>

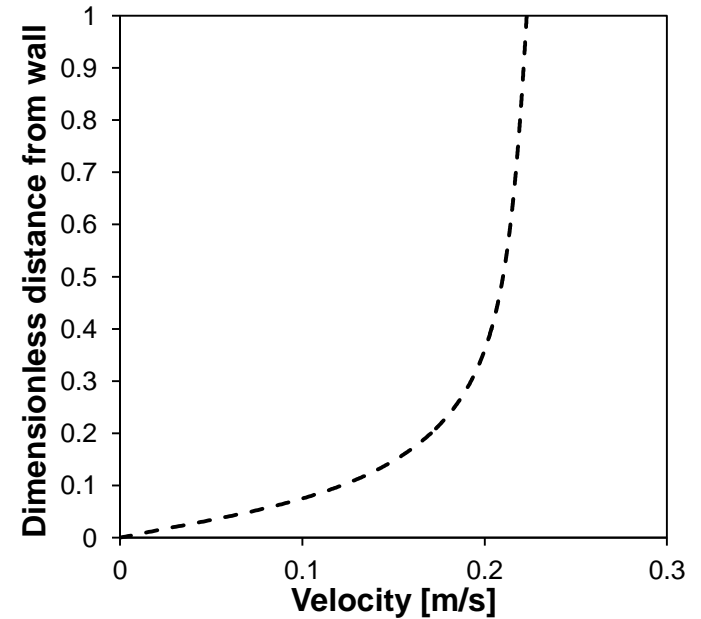
*<sup>1</sup>School of Engineering, University of Warwick*

*<sup>2</sup>Department of Civil and Structural Engineering, University of Sheffield*

# Introduction



**Longitudinal Dispersion:**  
Spreading of a solute along the longitudinal axis.



**Differential Advection:**  
Spreading due to spatial difference in velocity.

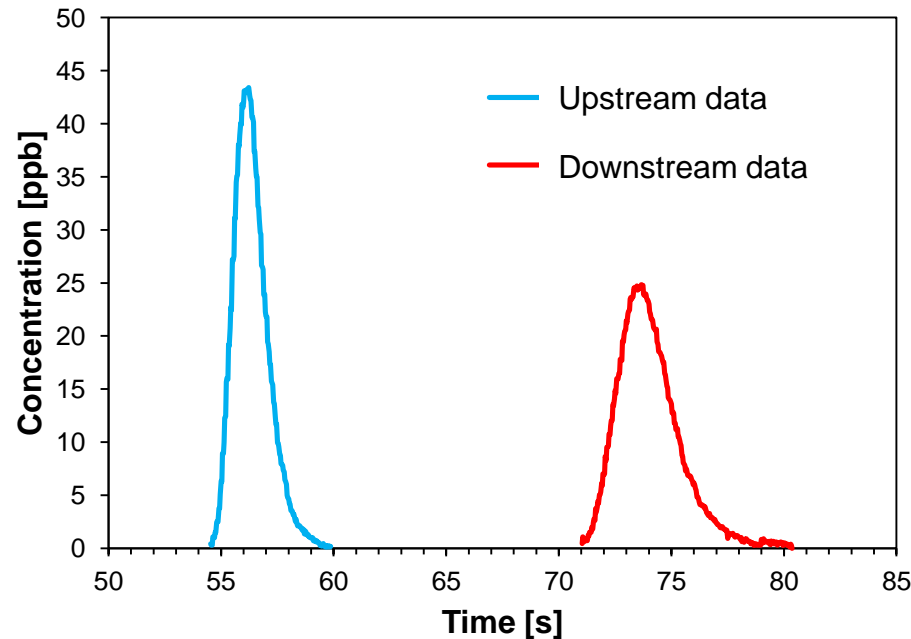
# Background – Taylor [1954]

Taylor's analysis:

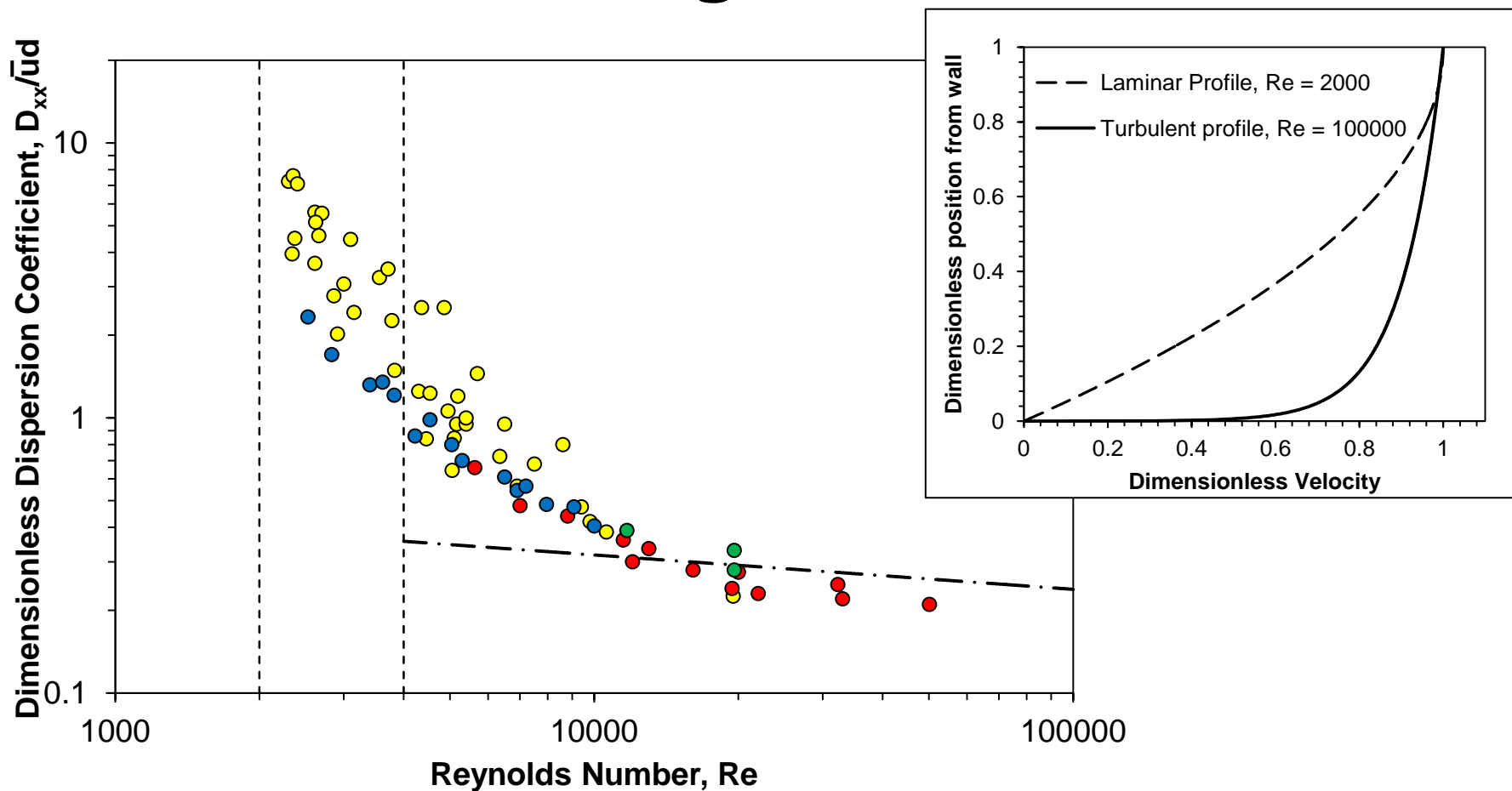
$$\frac{\partial c}{\partial t} = D_{xx} \frac{\partial^2 c}{\partial x^2} - u \frac{\partial c}{\partial x}$$

Taylor's Expression:

$$D_{xx} = 10au_*$$



# Background



● Fowler and Brown [1943]

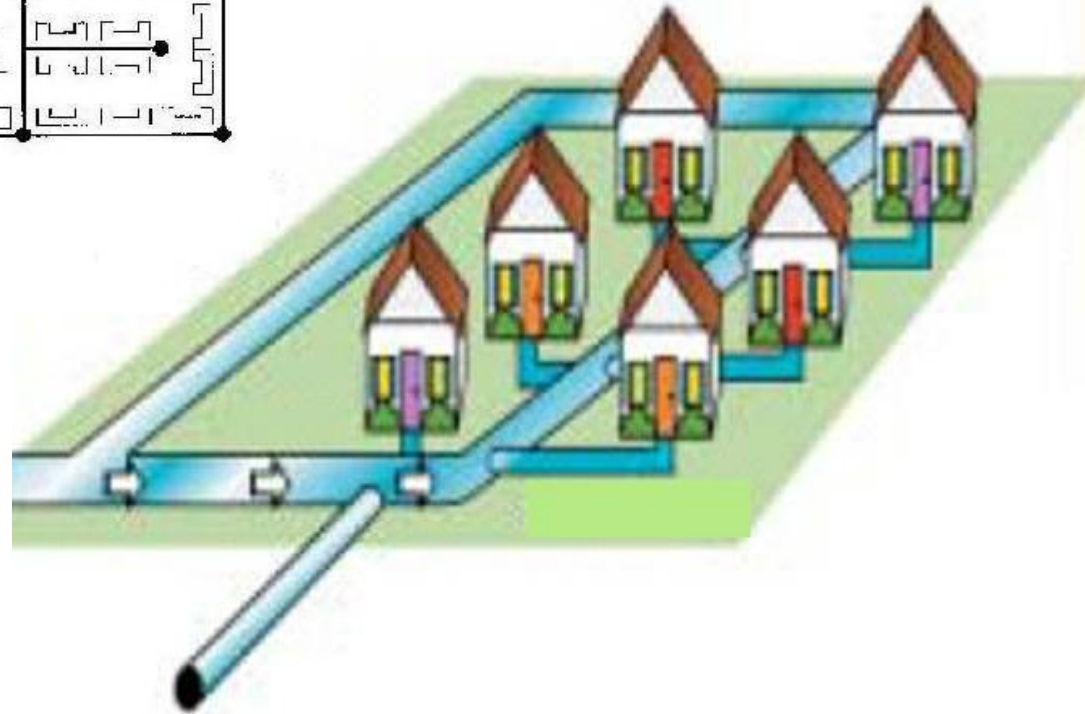
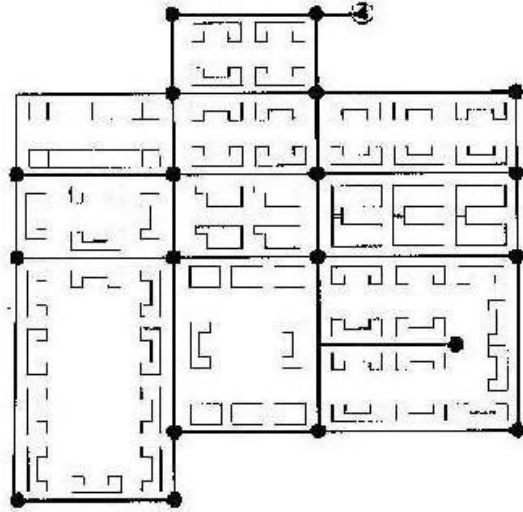
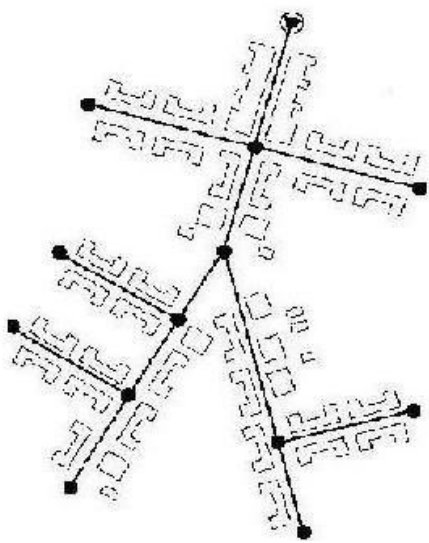
● Keys [1955]

— · — Taylor [1954]

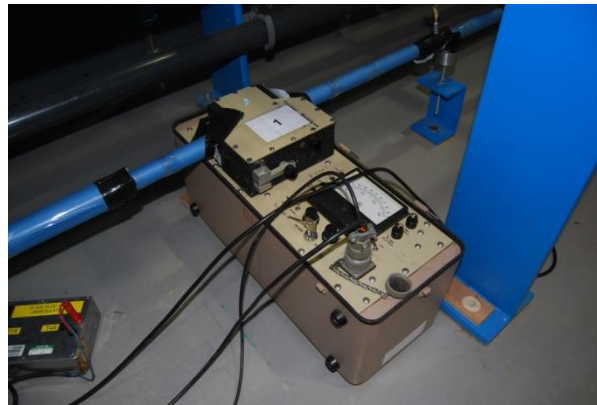
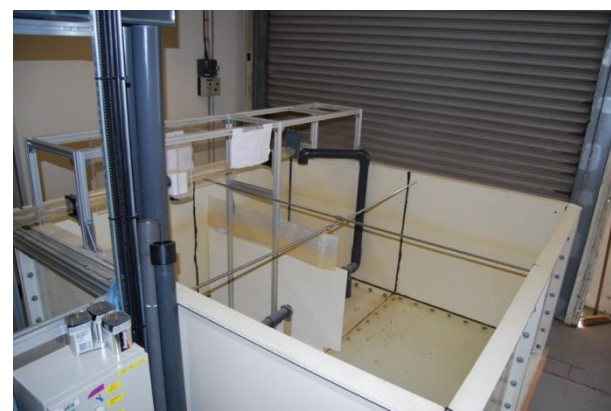
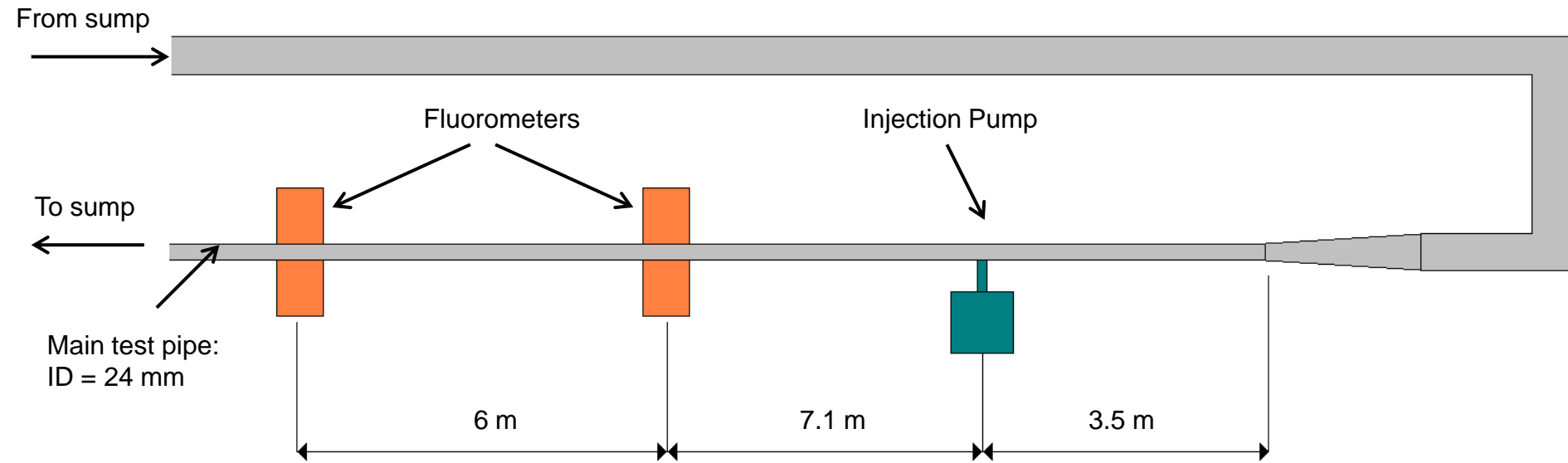
● Taylor [1954]

● Flint and Eisenklam [1969]

# Applications

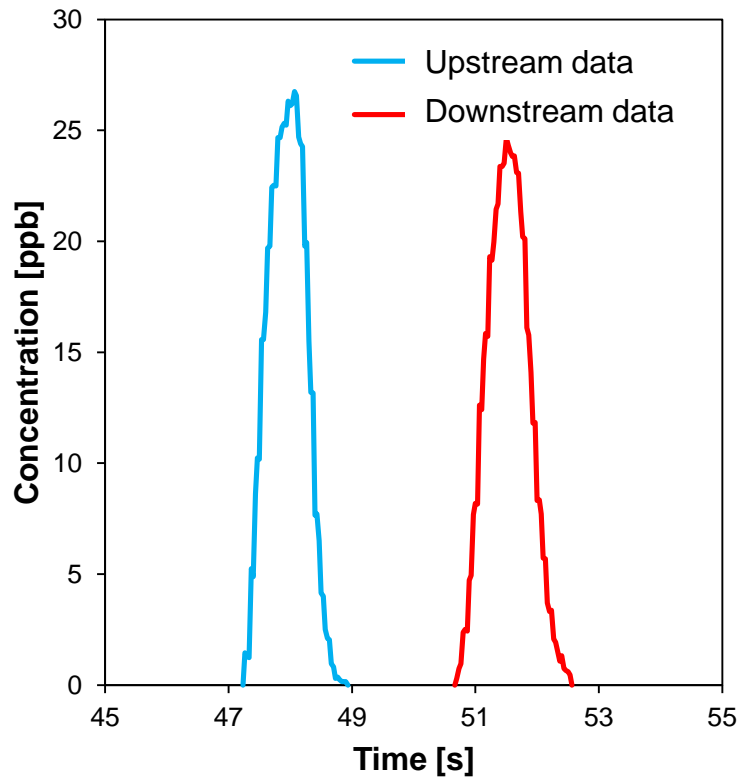


# Experimental Set-up

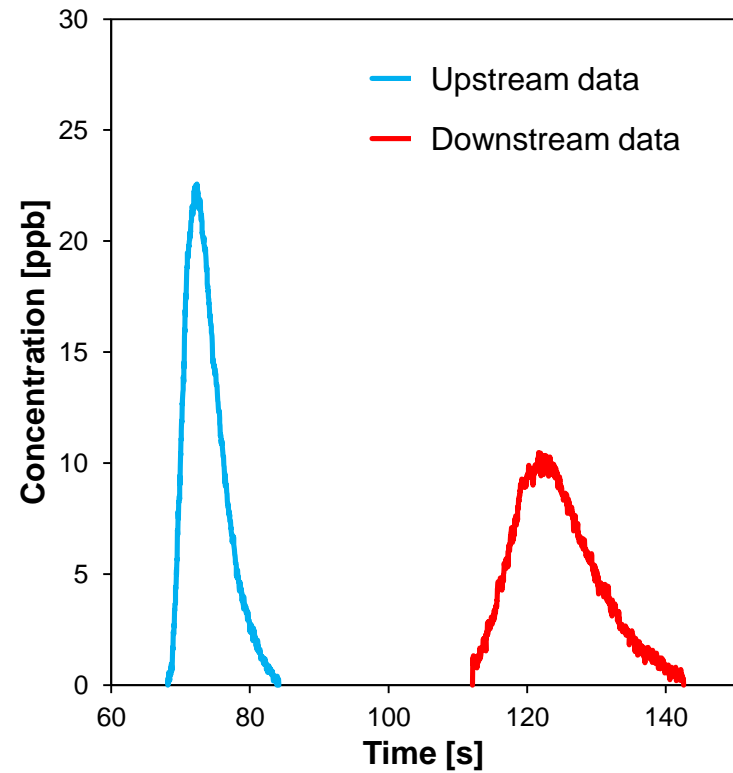


# Results

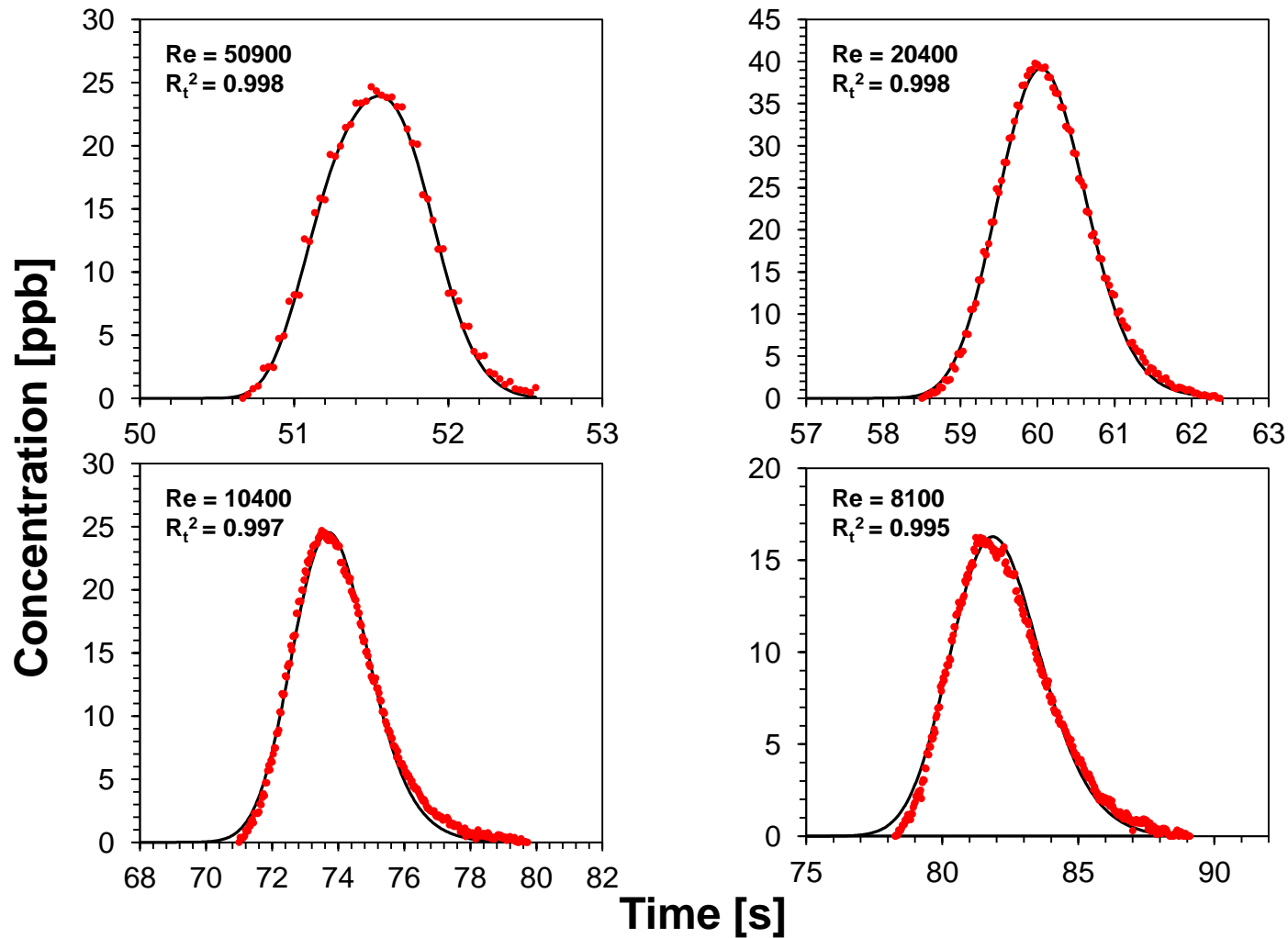
**Re = 50900**



**Re = 3800**



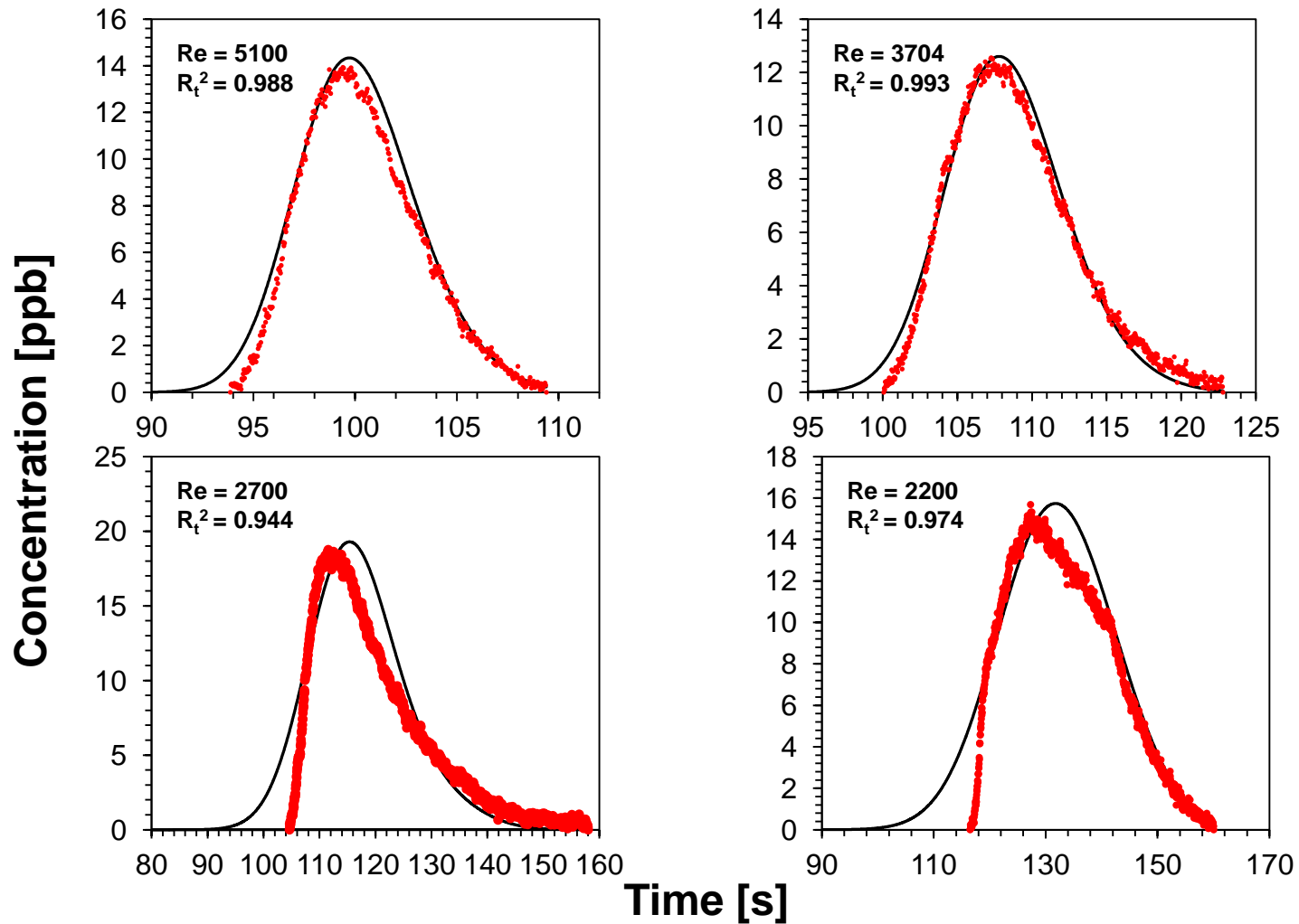
# Results and Analysis



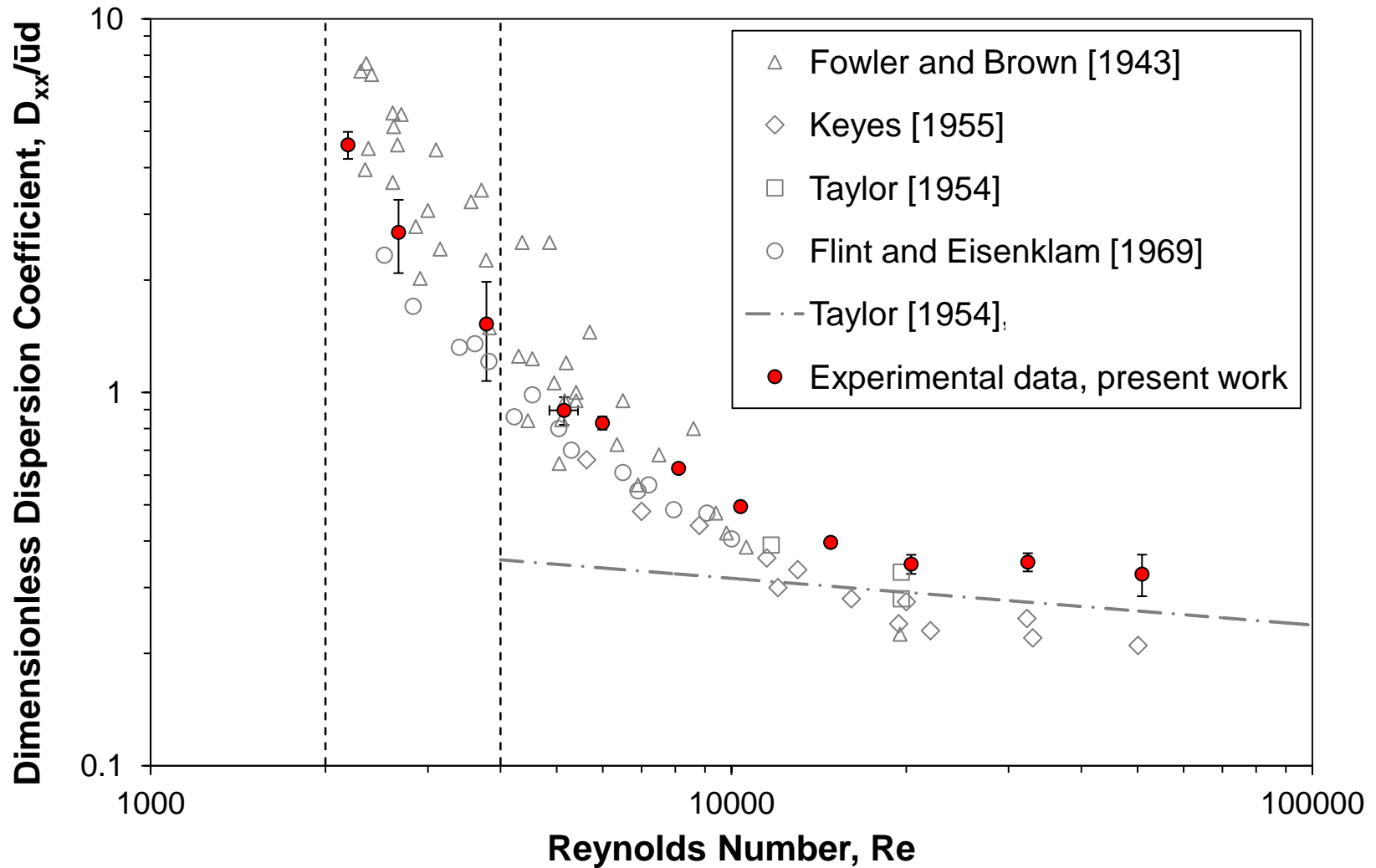
Downstream data • Optimised profile —



# Results and Analysis

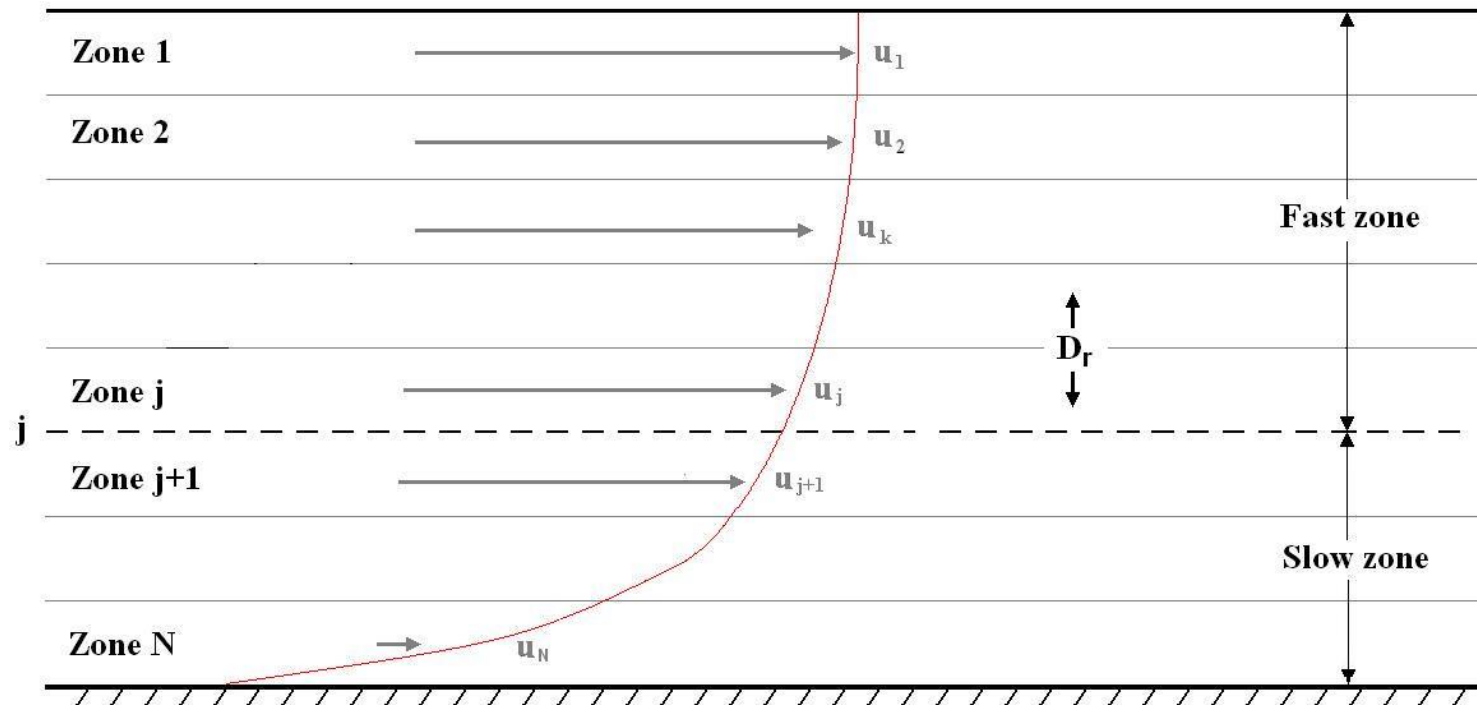


# Results and Analysis

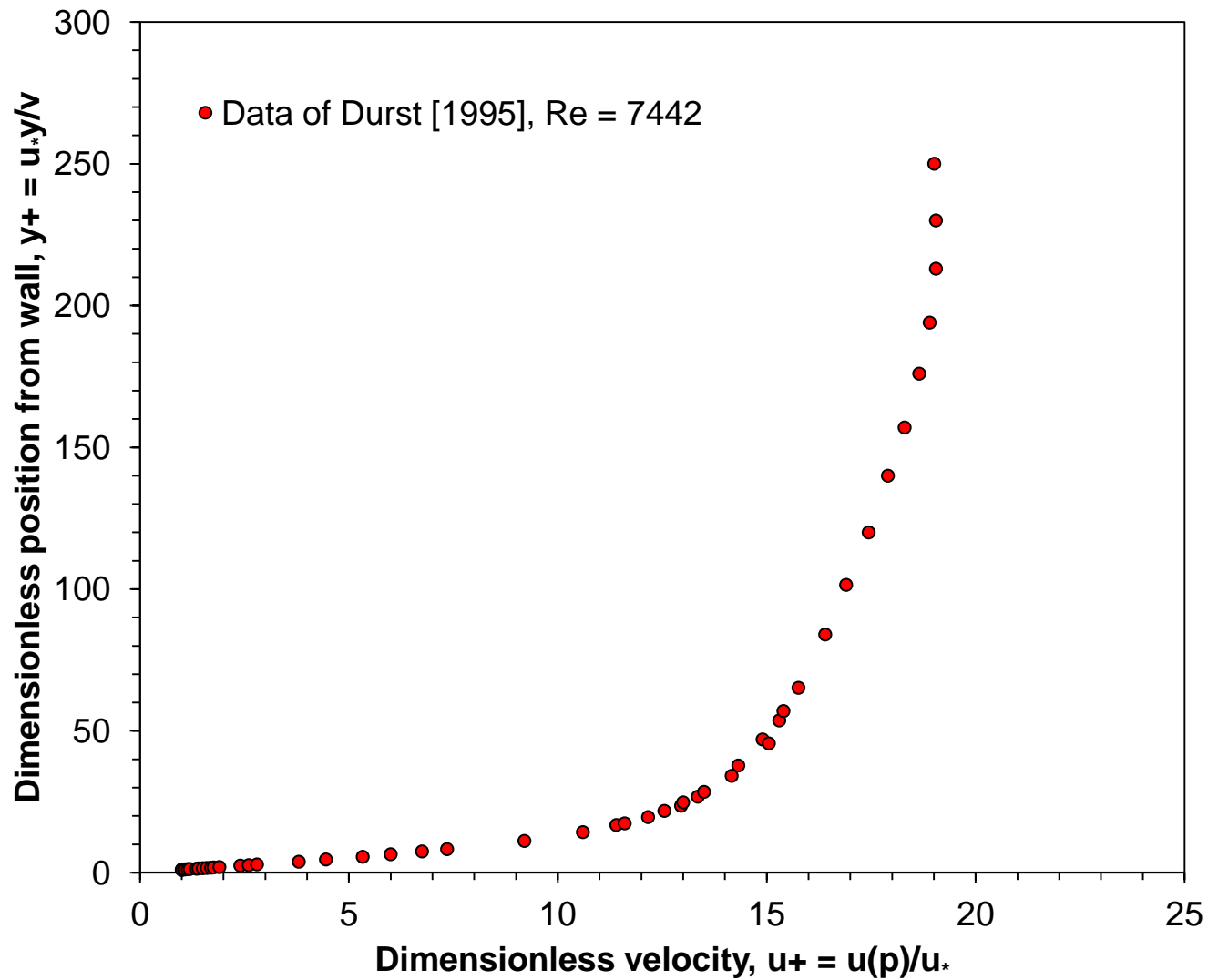


# Numerical Model – Chikwendu [1986]

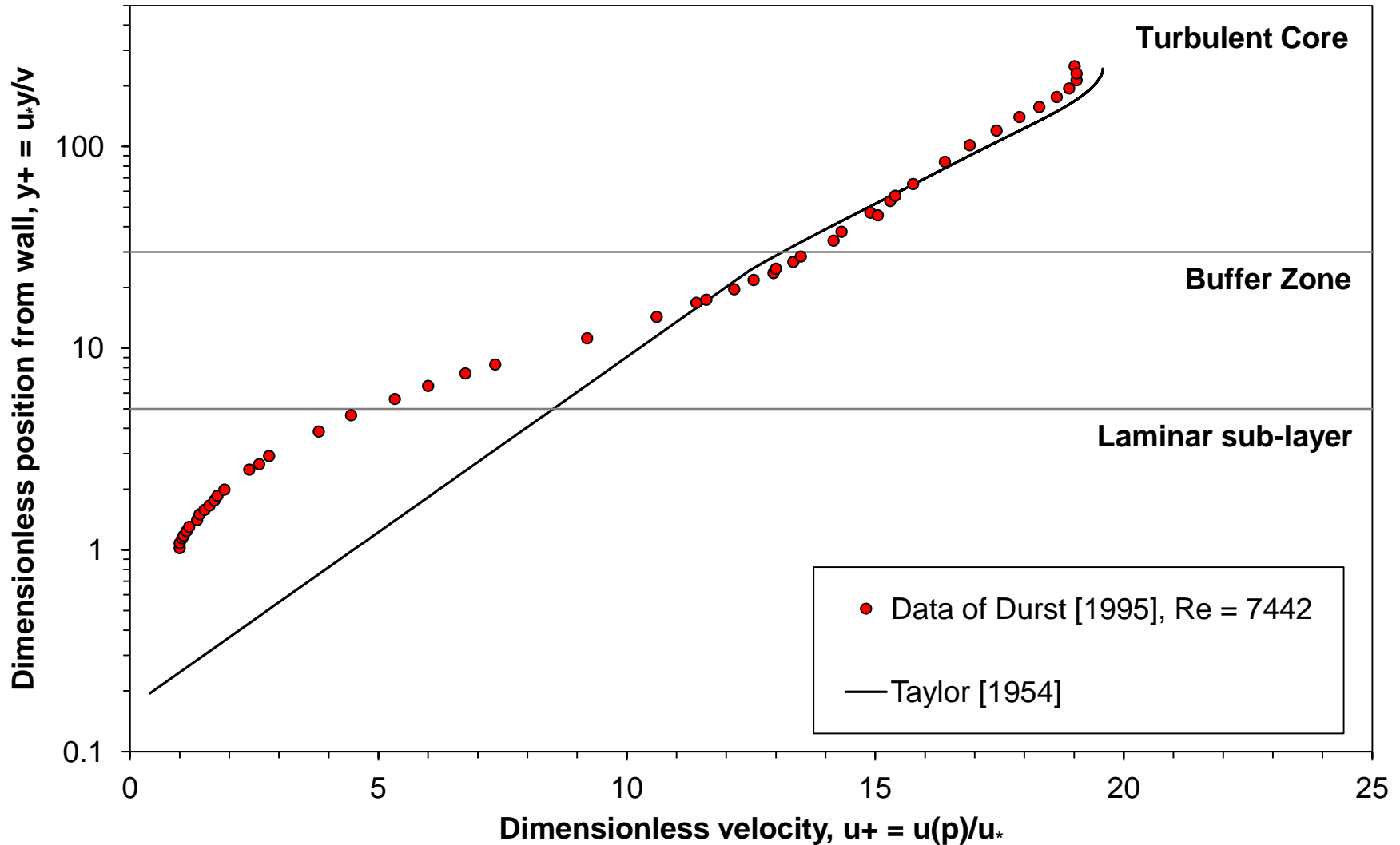
$$D_{xx} = \sum_{j=1}^{N-1} \frac{a^2 p_j^4 (1 - p_j^2)^2 [u_{f,1 \rightarrow j} - u_{s,j \rightarrow N}]^2}{4D_{r,j,(j+1)}} (W_j + W_{j+1})$$



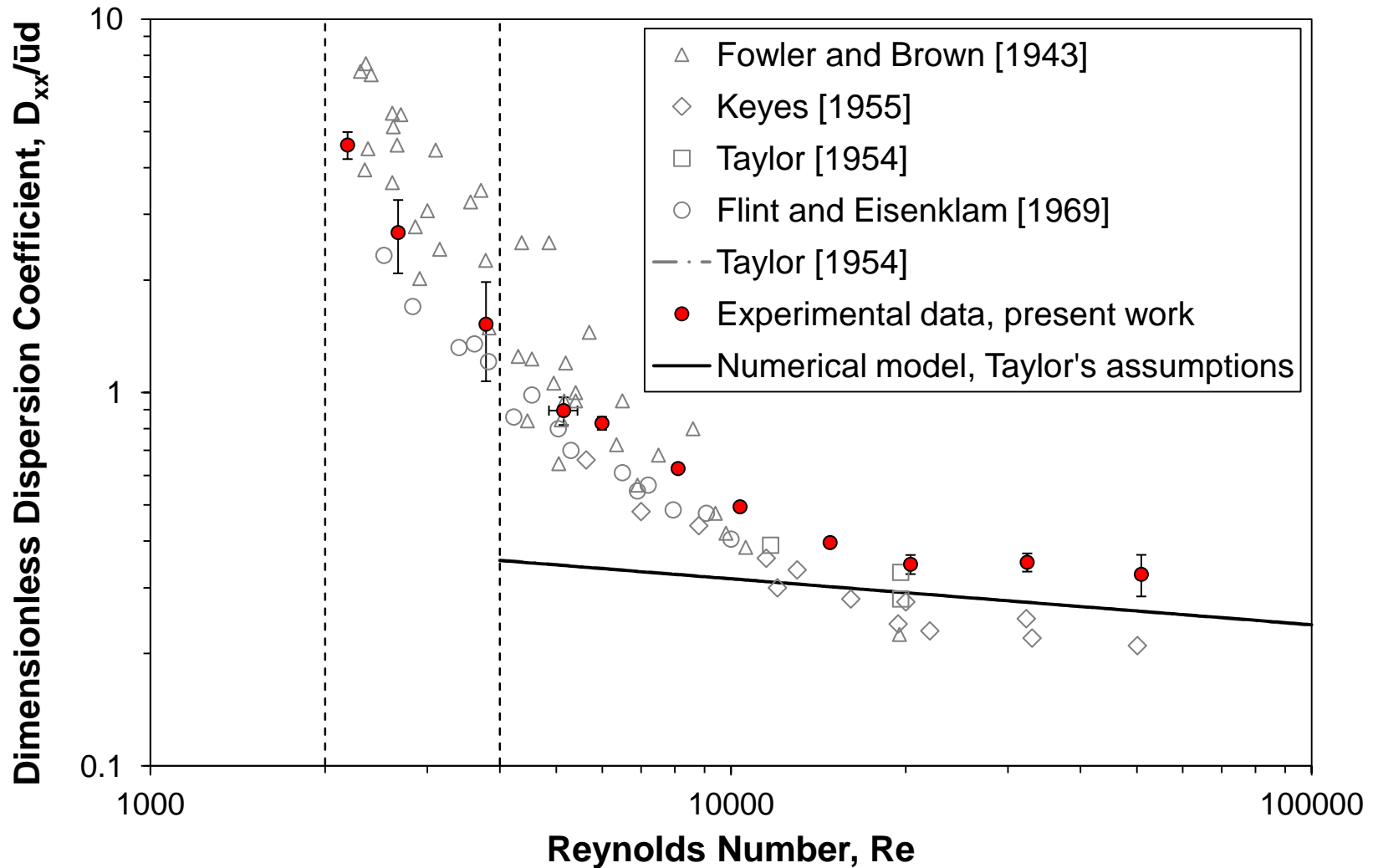
# Turbulent Flow



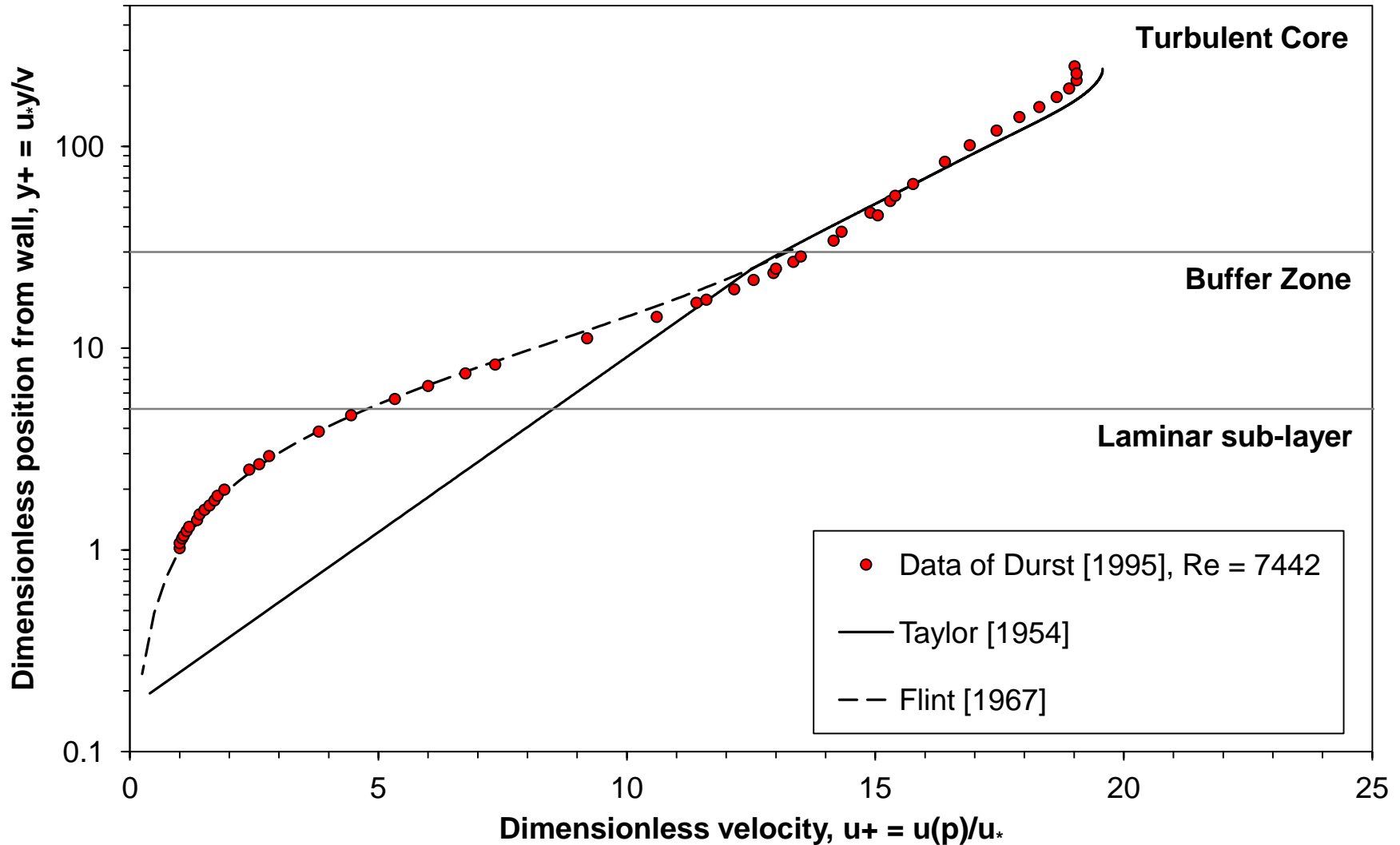
# Velocity Profile - Turbulent Flow



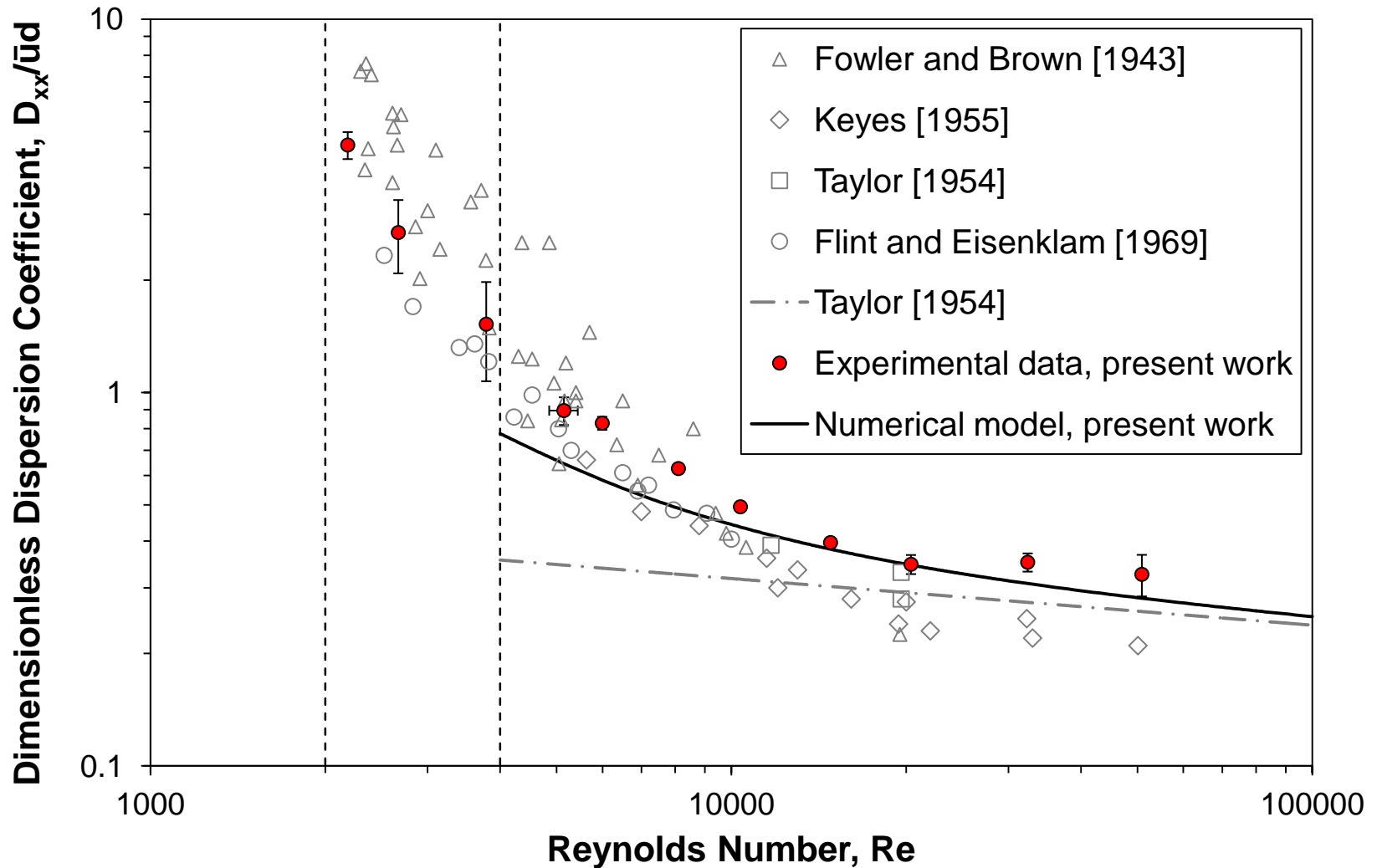
# Numerical Model – Turbulent Flow



# Velocity Profile - Turbulent Flow



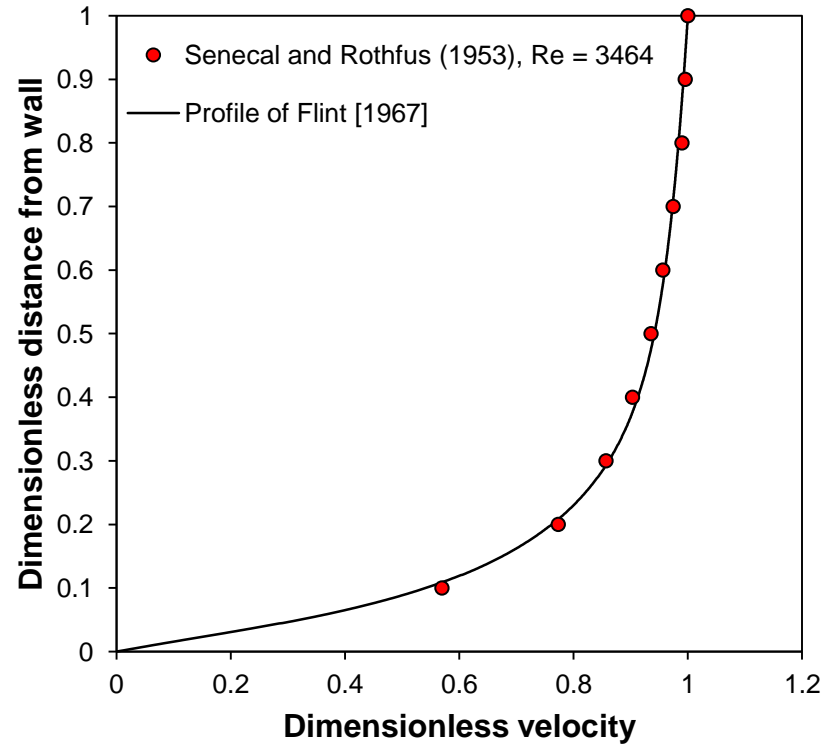
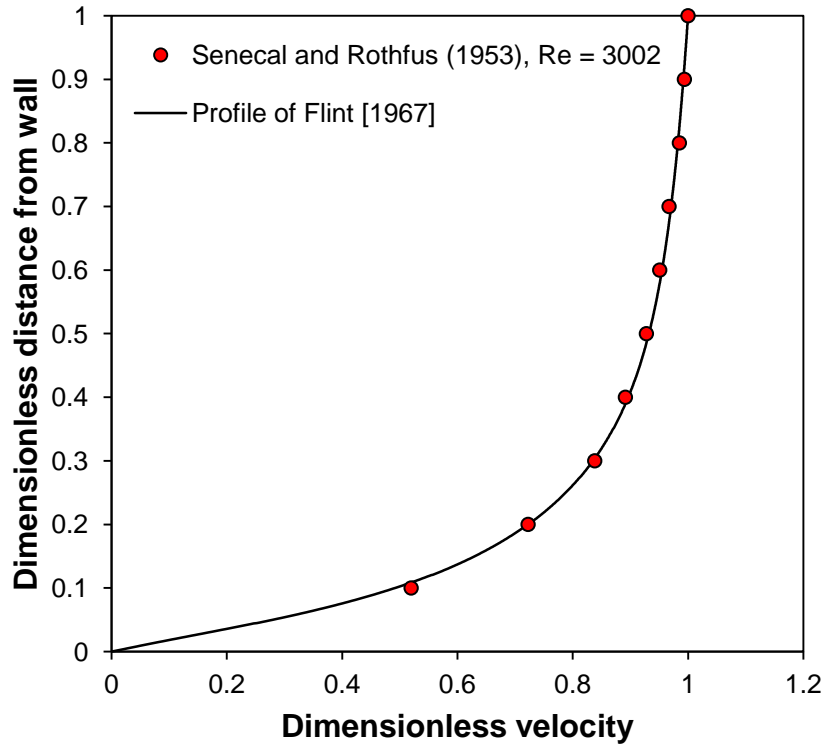
# Numerical Model – Turbulent Flow





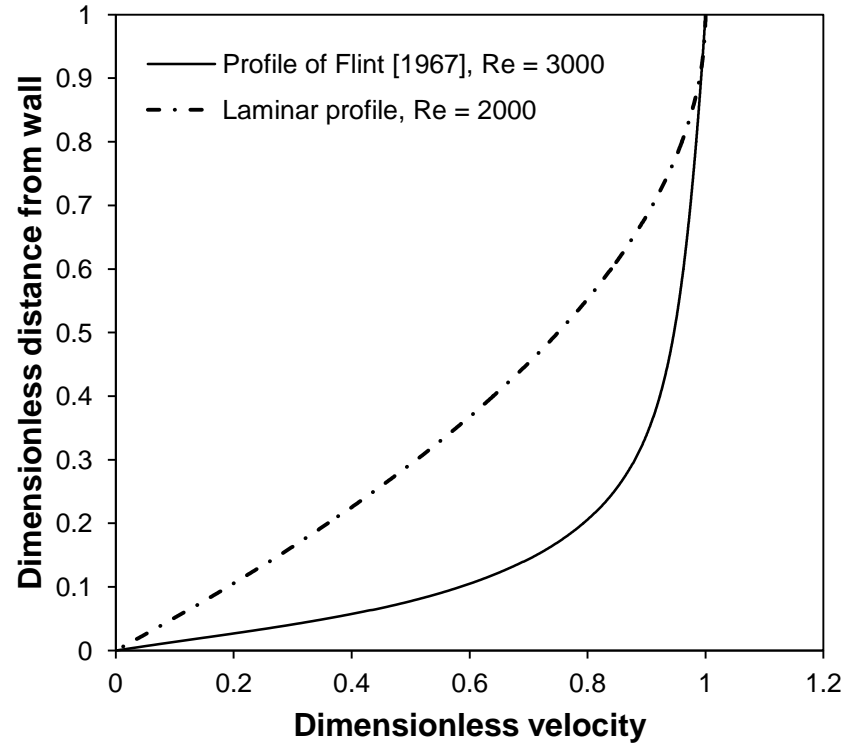
# Transitional Flow

**3000 < Re < 4000:** Profile of Flint [1967]



# Transitional Flow

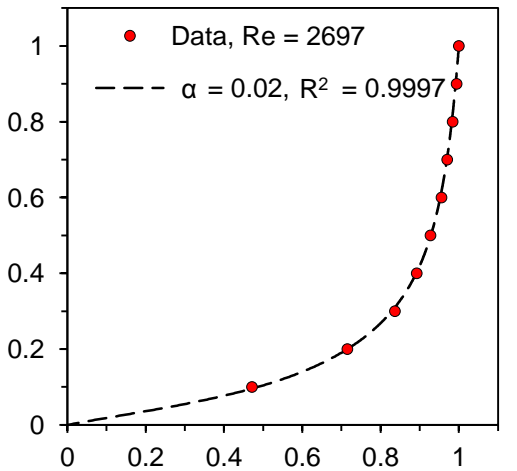
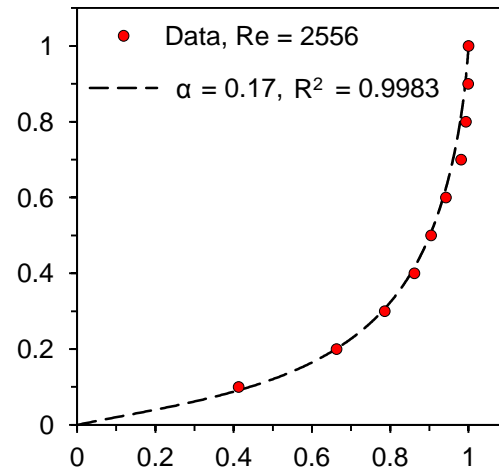
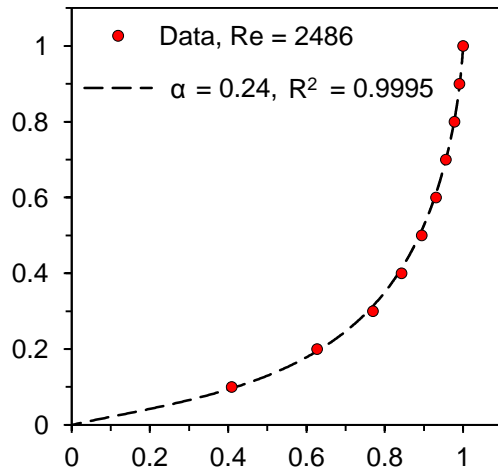
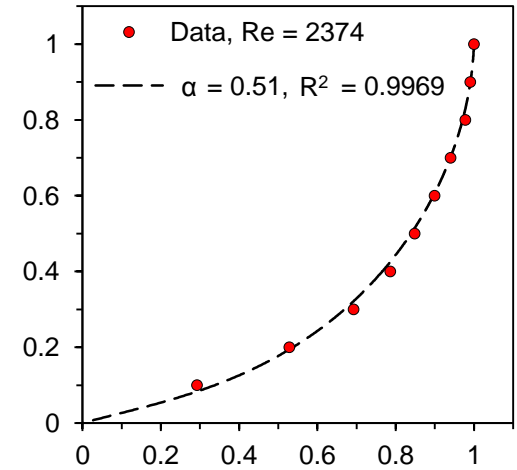
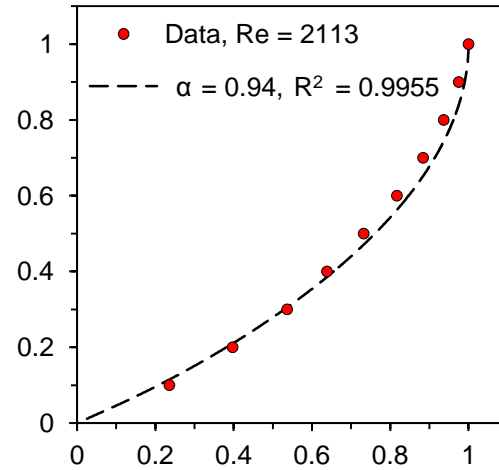
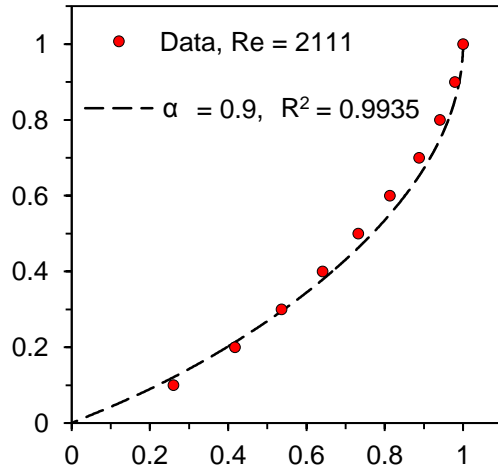
**2000 < Re < 3000:**



$$u(p) = \alpha u(p)_L + (1 - \alpha)u(p)_T$$

# Velocity Profile - Transitional Flow

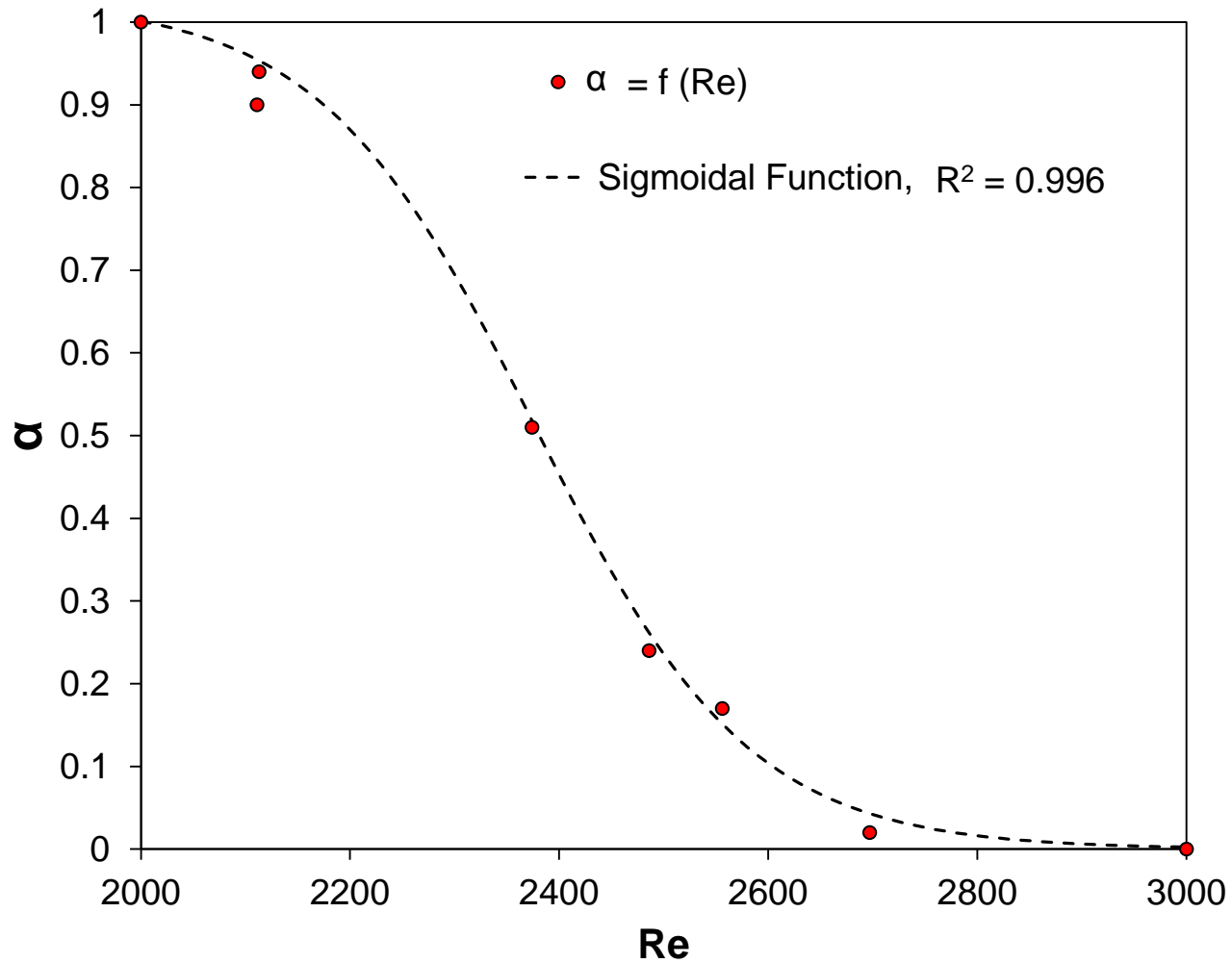
Dimensionless position from wall



• Data of Senecal and Rothfus [1953]

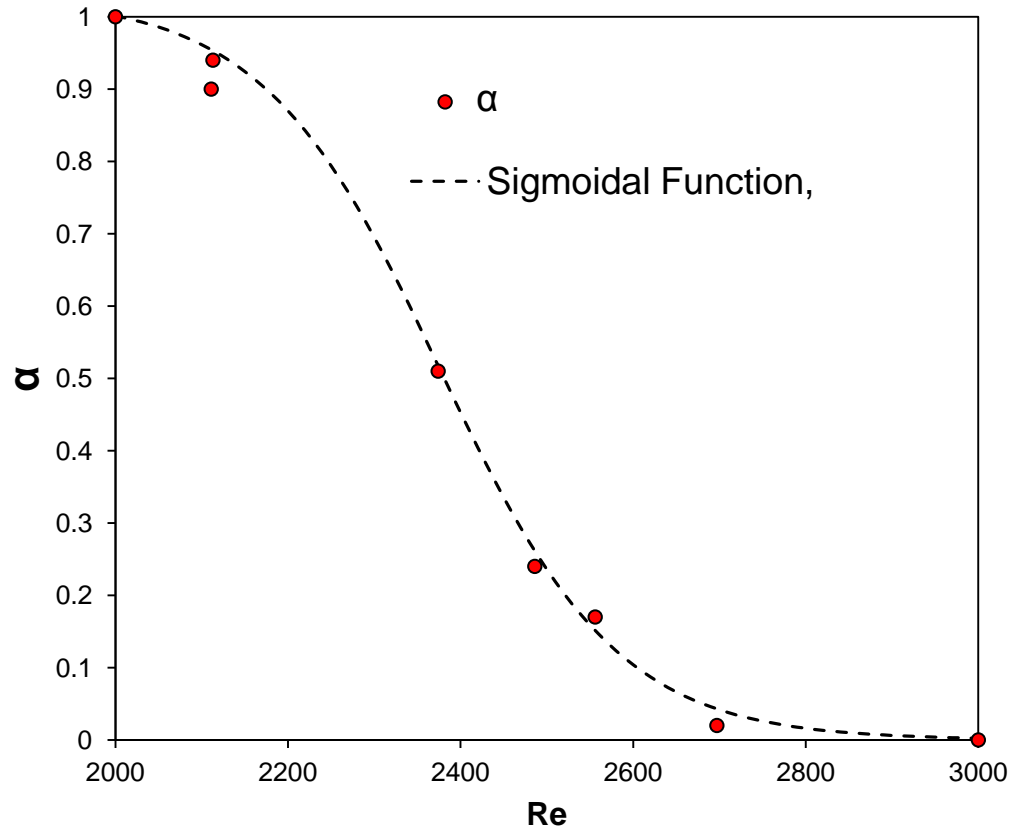
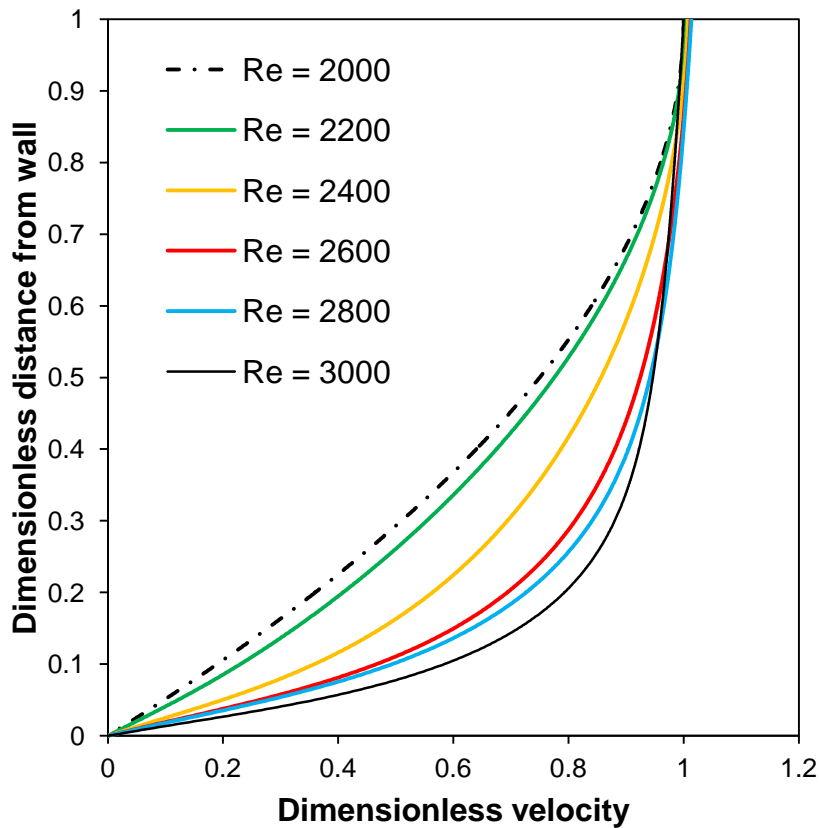
Dimensionless velocity

# Transitional Flow



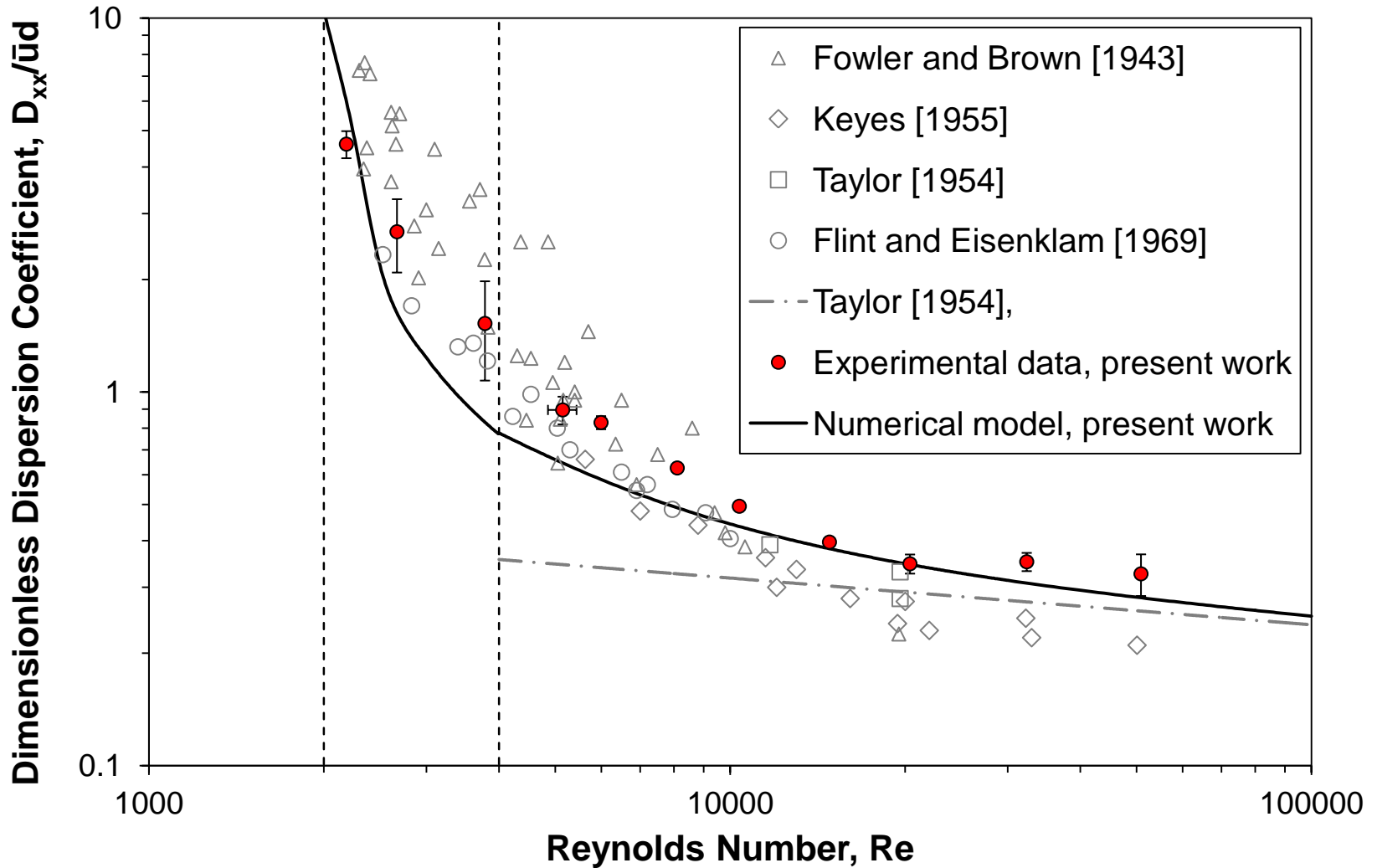
# Transitional Flow

2000 < Re < 3000:



$$u(p) = \alpha u(p)_L + (1 - \alpha)u(p)_T$$

# Numerical Model – Transitional Flow



# Questions ?