Application of Lattice Boltzmann Method for generation of flow velocity field over river bed–forms: a comparison of numerical model simulations with field observations

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  - may be used for complex geometry;
- Comparison of the numerical results with the field measurements.

Lattice Boltzmann Equation

Fundamental principle of Lattice Boltzmann Methods is to:

- construct simplified molecular dynamics that incorporates the essential characteristics of physical microscopic processes
- the macroscopic averaged properties have to satisfy the desired macroscopic equations

Lattice Boltzmann Equation

Starting point for LBM is Boltzmann equation:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f = C(f, f)$$

where $f = f(\vec{x}, \vec{v}, t)$ is distribution function at time t particles with velocity $\vec{v}$ around position $\vec{x}$ and $C(f, f)$ is collision operator.

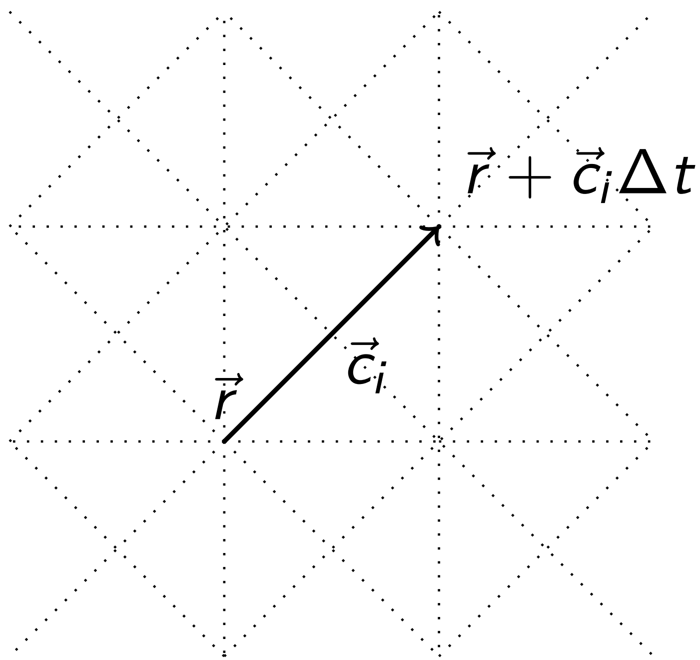
Lattice Boltzmann Equation

Next, we restrict set of posible velocity vectors $\{\vec{c}_i\}_{i=0\ldots b}$ to finit number of values $b+1$.

The Lattice Boltzmann Equation reads as follows:

$$f_i(\vec{r}+\vec{c}_i\Delta t, t+\Delta t) = f_i(\vec{r}, t)+\Omega_i(\vec{f}(\vec{r}, t))$$

where $f_i(\vec{r}, t)$ represents the probability of finding a particle at position $\vec{r}$ and time t with velocity $\vec{v} = \vec{c}_i$.



Macroscopic quantities

The main fluid quantities are obtained by simple summation upon the dicrete speeds.

Density:

$$\rho(\vec{r}, t) = \sum_i f_i(\vec{r}, t)$$

Velocity:

$$\vec{u}(\vec{r}, t) = \frac{1}{\rho(\vec{r}, t)} \sum_i f_i(\vec{r}, t)\vec{c}_i$$

Bhatnagar-Gross-Krook (BGK) approximation

Lattice BGK equation form:

$$f_i(\vec{r} + \vec{c}_i \Delta t, t + \Delta t) = f_i(\vec{r}, t) + \underbrace{\frac{1}{\tau}\left(f_i^{eq}(\vec{r}, t) - f_i(\vec{r}, t)\right)}_{\Omega_i\left(\vec{f}(\vec{r}, t)\right)}$$

where:

τ – relaxation time which is a function of kinematic viscosity

f_i^{eq} – local equilibrium distribution function

BGK approximation

Local equilibrium distribution function:

$$f_i^{eq} = w_i \rho \Big(1 + \frac{u_a c_{ia}}{\vec{c}_s^2} + \frac{u_a u_b Q_{iab}}{2 \vec{c}_s^4} \Big)$$

where:

$\vec{c}_s$ – model sound speed: $c_s^2 = \sum_i w_i c_i^2$

$\{w_i\}_i$ – set of weights normalized to unity

$Q_{iab} = c_{ia} c_{ib} c_s^2 - \delta_{ab}$

$\vec{c}_i = (c_{ix}, c_{iy})$ – discrete velocity vector

$\vec{u}_i = (u_{ix}, u_{iy})$ – macroscopic velocity vector

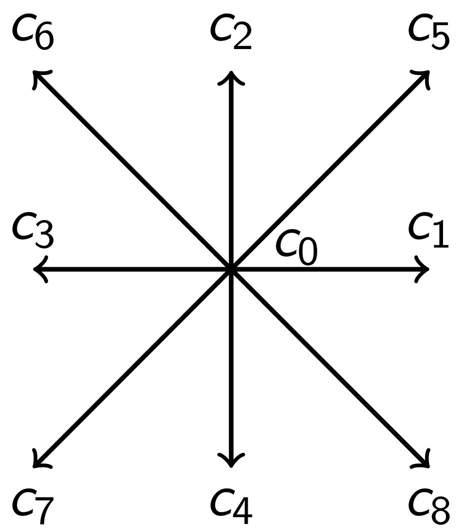
Lattice Boltzmann Equation

By using the Chapman-Enskog expansion, the LBGK microdynamics can recover the governing fluid equations (incompressible N-S Equations) if Δx and Δt are small enough.

D2Q9 discretization

D2Q9 dicret velocity vectors:

$$
\begin{aligned}
c_0 &= (0,0), & c_3 &= (-1,0), & c_6 &= (-1,1) \\
c_1 &= (1,0), & c_4 &= (0,-1), & c_7 &= (-1,-1) \\
c_2 &= (0,1), & c_5 &= (1,1), & c_8 &= (1,-1)
\end{aligned}
$$

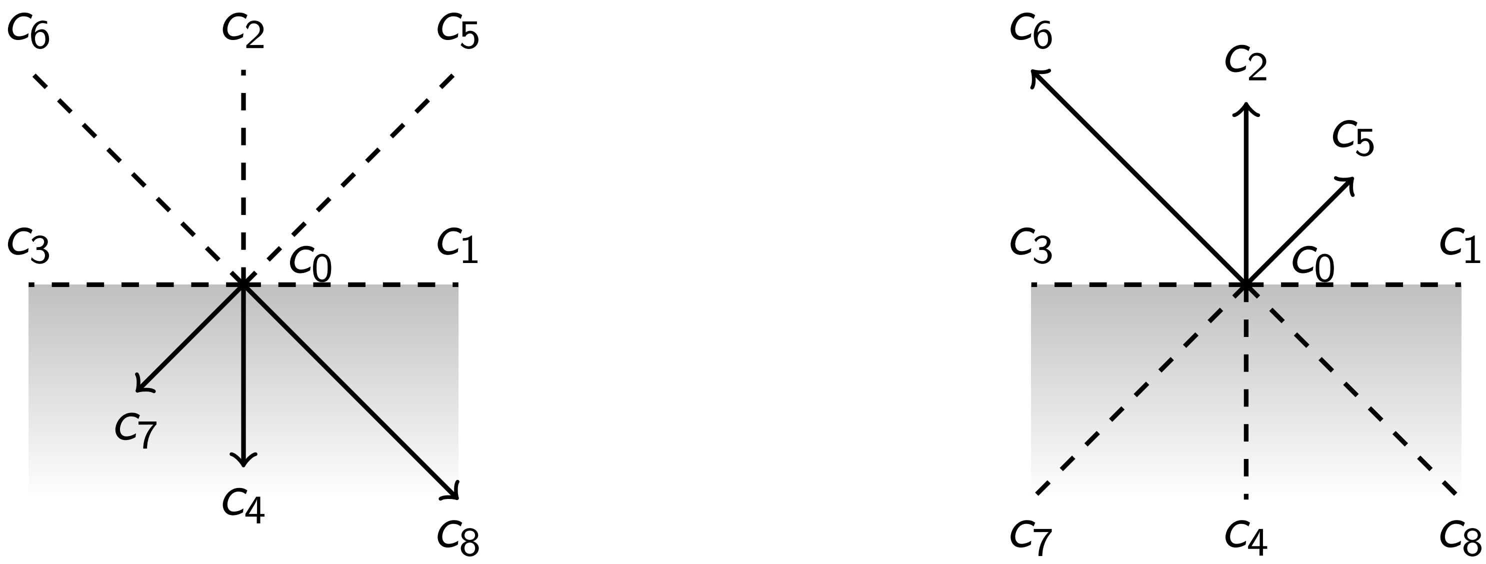


D2Q9 set of weights:

$$
w_i = \begin{cases} 4/9, & i = 0 \\ 1/9, & i = 1,2,3,4 \\ 1/36, & i = 5,6,7,8 \end{cases}
$$

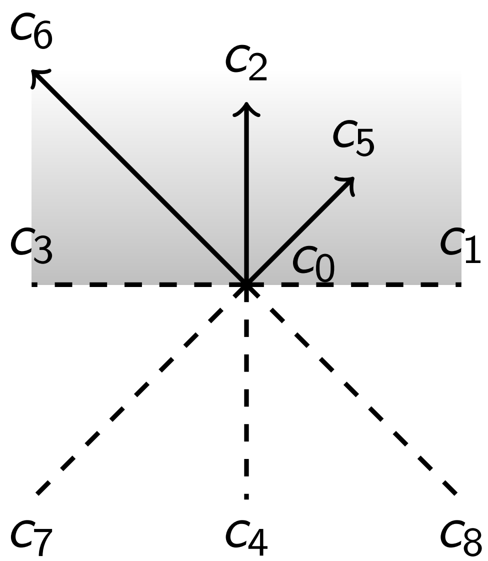
Boundary conditions

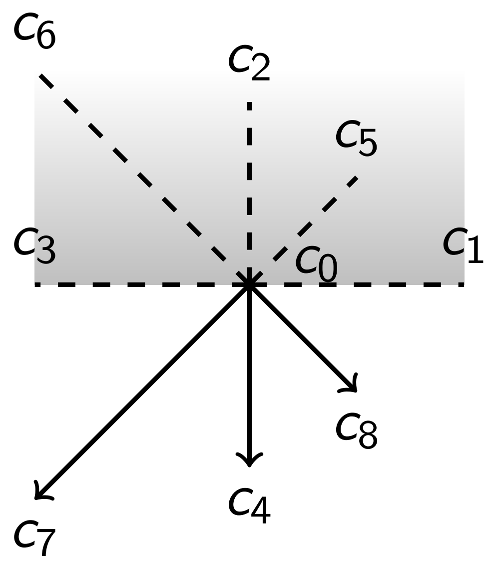
No-slip boundary condition(bounce-back boundary condition)



Boundary conditions

Free-slip boundary condition:





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Inflow boundary condition For given u_{in}, ρ_{in} we assume that $\vec{f}$ is in local equilibrium state $f_i = f_i^{eq}(u_{in}, \rho_{in})$.

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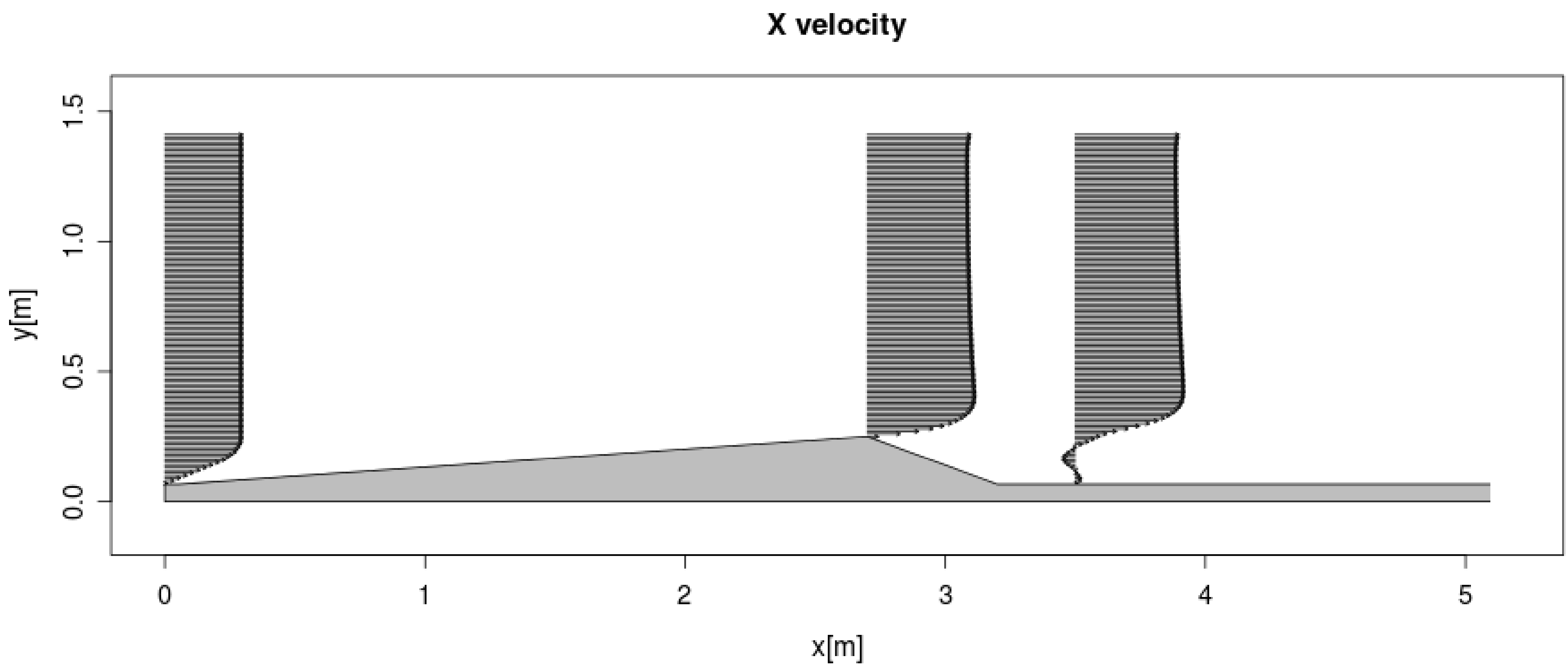
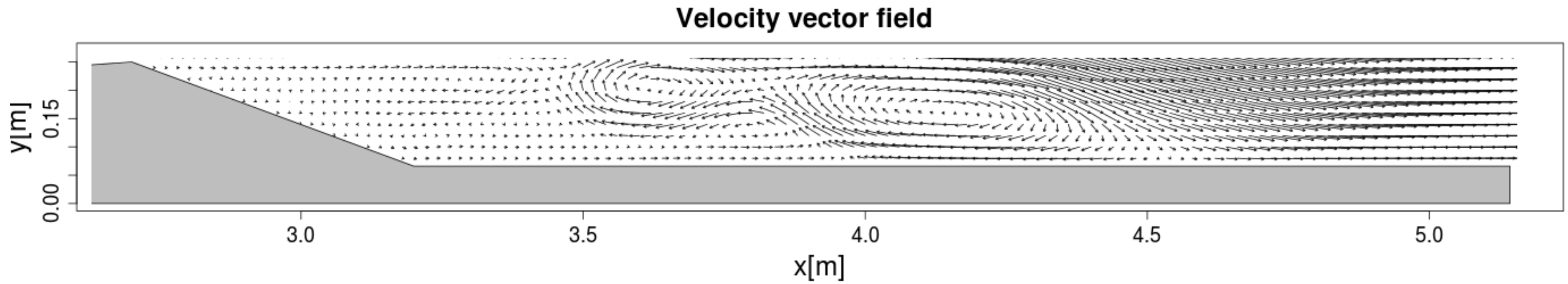
Outflow boundary condtion We assume then $f_i(x_{out}, t + \Delta t) = 2f_i(x_{out} - \Delta x, t + \Delta t) - f_i(x_{out} - 2\Delta x, t + \Delta t)$

Numerical simulation





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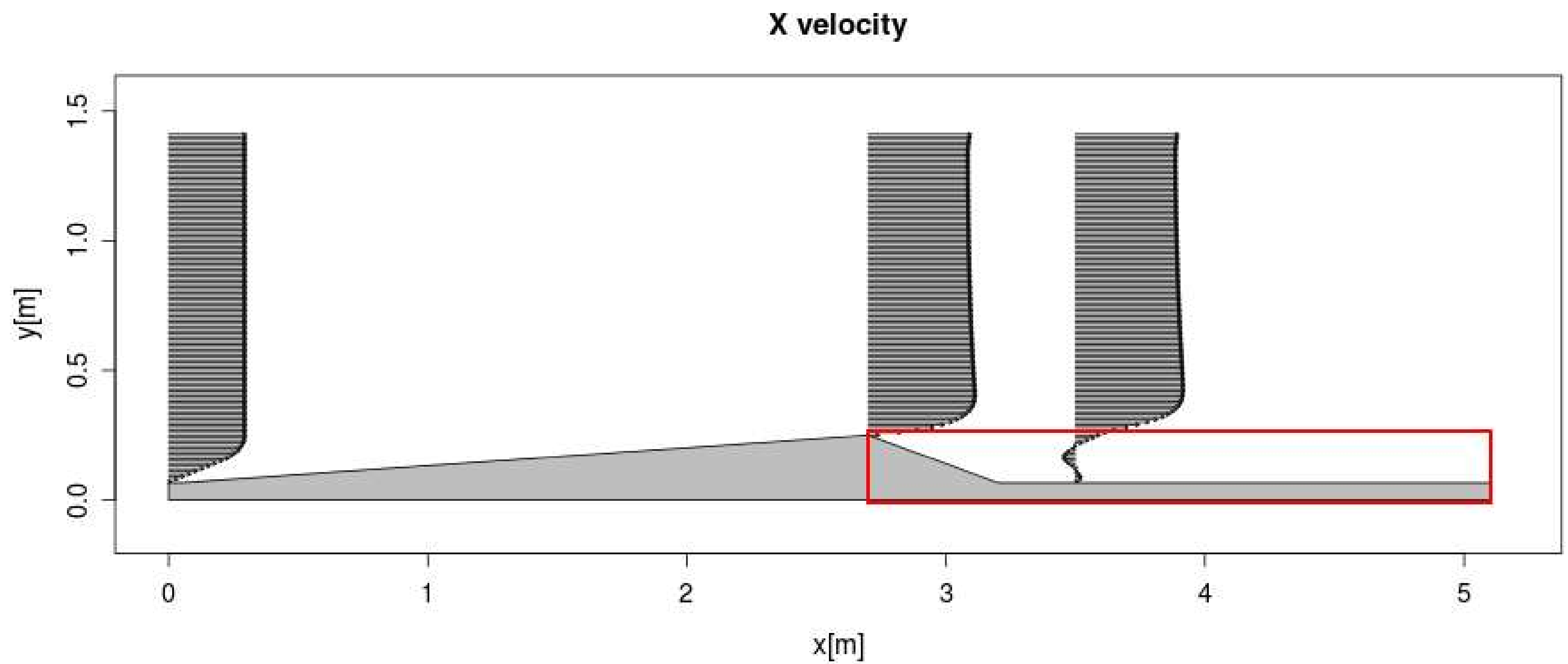
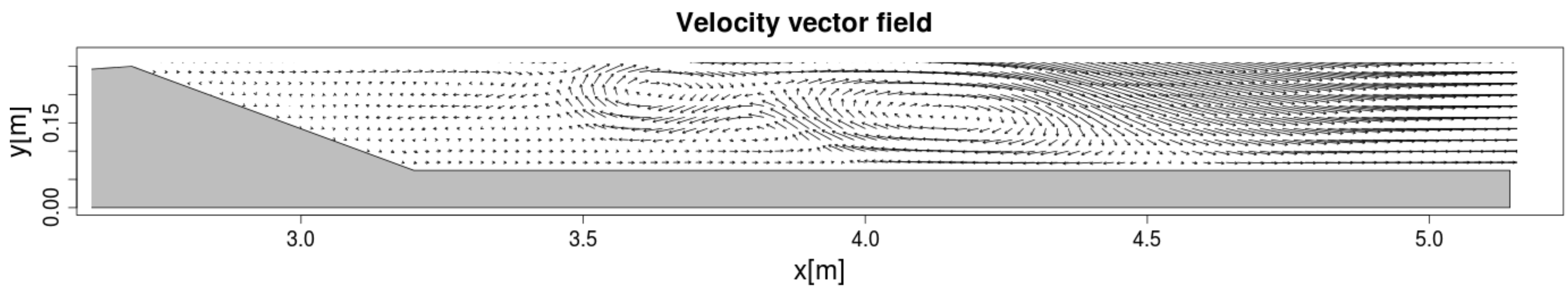


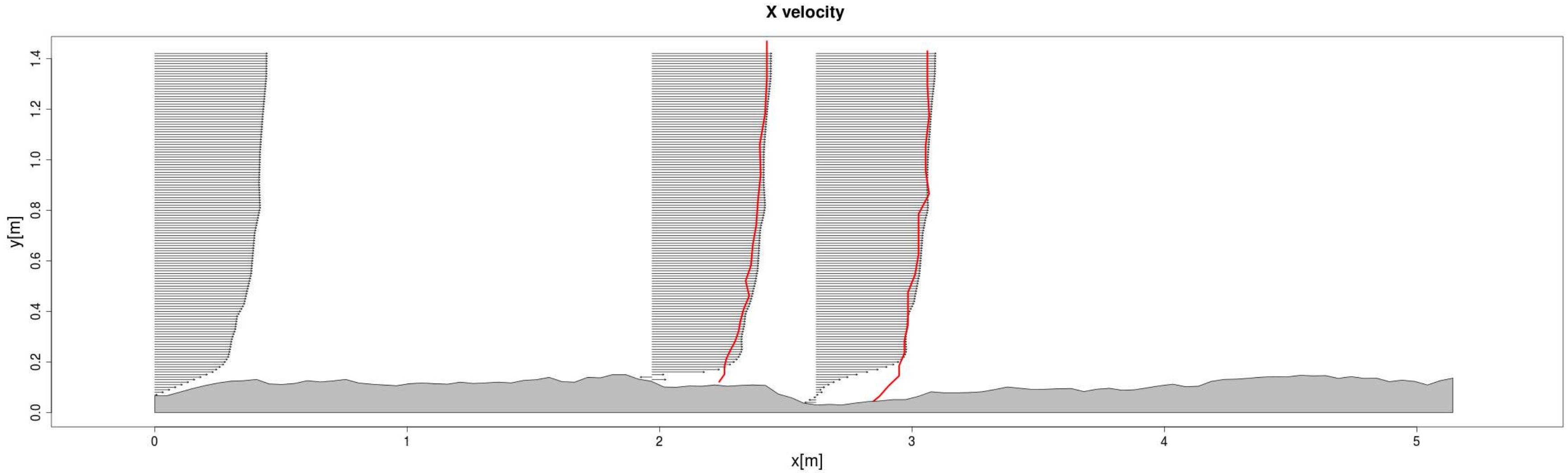


Spree, NE Germany

- discharge $Q = 13 - 17 m^3/s$,
- mean velocity $U = 0.5 - 0.7 m/s$,
- mean flow depth $h = 1.5 - 2.0 m$,
- Reynolds number $Re = 1.4 \times 10^6$.

Kleeberg A., Sukhodolov A., Sukhodolova T., Köhler J. (2010). Dynamic of riverine matter deposition, resuspension and respective phosphorus entrainment within a vegetation mosaic.- Freshwater Biology 55: 326-345

Numerical simulation–results



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   - Fluid–particle interaction over sand dunes