The uncertainty of measurements in river hydraulics –

evaluation of the friction velocity based on an unrepeatable experiment

Magdalena Mrokowska, Paweł Rowiński, Monika Kalinowska

Institute of Geophysics PAS

Aim of the presentation

The aim is to present the basis of uncertainty analysis of measurements and its application in practice.

River hydraulics deals with physical quantities, hence it is absolutely critical that it should follow a physical experimental methodology. Good practice in experimental work requires complementing a measured value by a quantified level of uncertainty.

- Uncertainty of direct and indirect measurements
- Statistical vs deterministic approach in estimating uncertainty
- An example of hydraulic experiment in natural settings
- Friction velocity – a physical variable masured indirectly

Aim of the presentation

The aim is to present the basis of uncertainty analysis of measurements and its application in practice.

River hydraulics deals with physical quantities, hence it is absolutely critical that it should follow a physical experimental methodology. Good practice in experimental work requires complementing a measured value by a quantified level of uncertainty.

- Uncertainty of direct and indirect measurements
- Statistical vs deterministic approach in estimating uncertainty
- An example of hydraulic experiment in natural settings
- Friction velocity – a physical variable masured indirectly

Aim of the presentation

The aim is to present the basis of uncertainty analysis of measurements and its application in practice.

River hydraulics deals with physical quantities, hence it is absolutely critical that it should follow a physical experimental methodology. Good practice in experimental work requires complementing a measured value by a quantified level of uncertainty.

- Uncertainty of direct and indirect measurements
- Statistical vs deterministic approach in estimating uncertainty
- An example of hydraulic experiment in natural settings
- Friction velocity – a physical variable masured indirectly

Aim of the presentation

The aim is to present the basis of uncertainty analysis of measurements and its application in practice.

River hydraulics deals with physical quantities, hence it is absolutely critical that it should follow a physical experimental methodology. Good practice in experimental work requires complementing a measured value by a quantified level of uncertainty.

- Uncertainty of direct and indirect measurements
- Statistical vs deterministic approach in estimating uncertainty
- An example of hydraulic experiment in natural settings
- Friction velocity – a physical variable masured indirectly

Aim of the presentation

The aim is to present the basis of uncertainty analysis of measurements and its application in practice.

River hydraulics deals with physical quantities, hence it is absolutely critical that it should follow a physical experimental methodology. Good practice in experimental work requires complementing a measured value by a quantified level of uncertainty.

- Uncertainty of direct and indirect measurements
- Statistical vs deterministic approach in estimating uncertainty
- An example of hydraulic experiment in natural settings
- Friction velocity – a physical variable masured indirectly

Uncertainty of direct measurements

How to estimate uncertainty effectively in the case of hydraulic measurements which are unrepeatable in nature?

Let X_i be a physical quantity, and x_i an estimate of X_i.

Deterministic approach

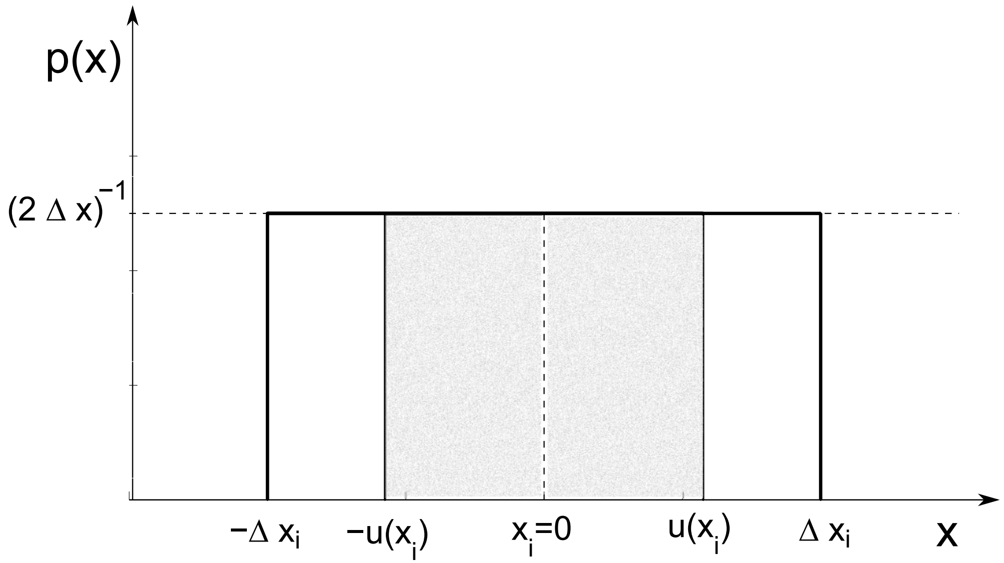
Maximum uncertainty of measured variable is evaluated as bounds of interval $[x_i - \Delta x_i, x_i + \Delta x_i]$.

Statictical approach (ISO)

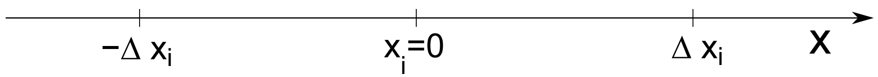
- Uncertainty quantified by standard deviation of x_i – standard uncertainty $u(x_i)$.
- Type B evaluation with assumed probability density based on scientific judgement.

Deterministic vs statistical approach

a)



b)



Standard uncertainty

$$u(x_i) = \frac{\Delta x_i}{\sqrt{3}}$$

Maximum deterministic uncertainty

$$\Delta x_i$$

Uncertainty of indirect measurements

Physical variable is determined through a functional relationship:

$$y = f(x_1, x_2, ..., x_n),$$

where: y – output quantity (dependednt variable), x_i – input quantities.

Uncertainty of indirect measurements – law of propagation of uncertainty

Maximum deterministic uncertainty

$$\Delta y_{max} \simeq \sum_{k=1}^{n} \left| \frac{\partial f}{\partial x_i} \right| \Delta x_i$$

Maximum combined standard uncertainty

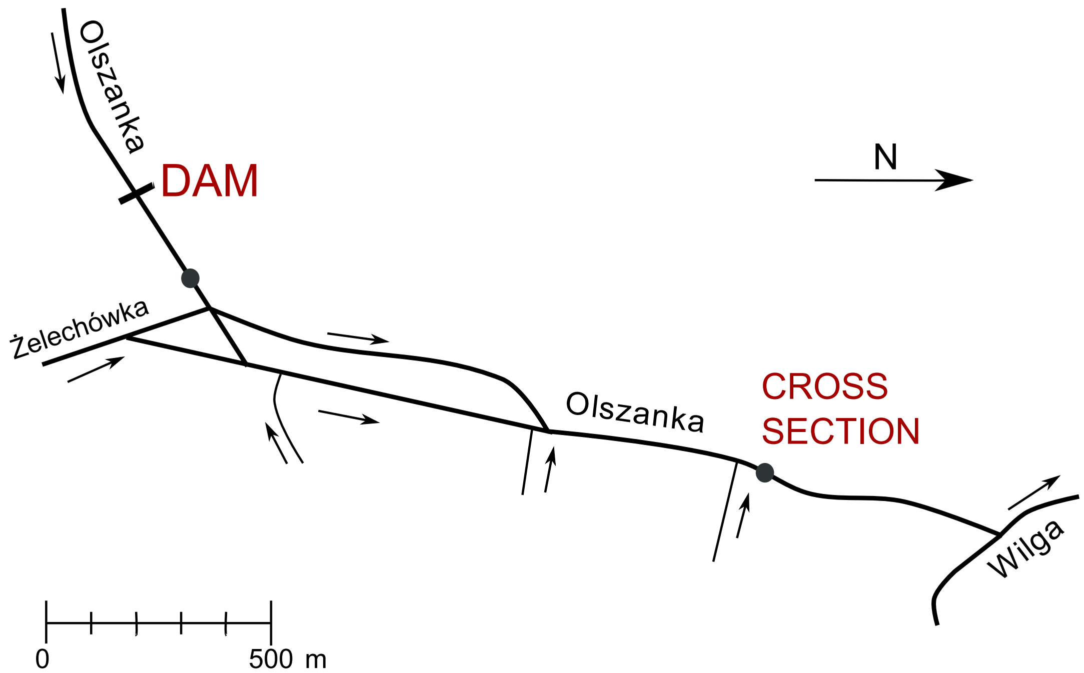
$$u_c(y)_{max} \simeq \sum_{k=1}^{n} \left| \frac{\partial f}{\partial x_i} \right| u(x_i)$$

Maximum combined standard uncertainty and maximum deterministic uncertainty are related as follows:

$$u_c(y)_{max} = \frac{1}{\sqrt{3}} \Delta y_{max}$$

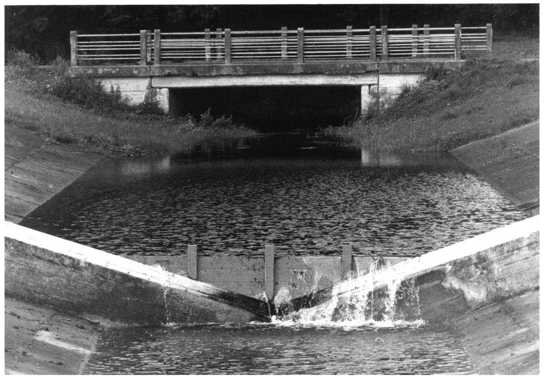
Field measurements

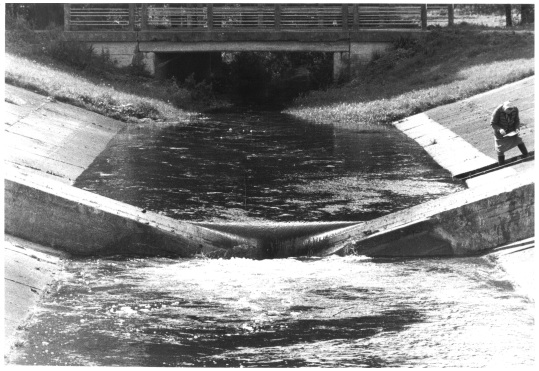
Dam-break like experiment performed on the Olszanka River (central Poland) in the 90's of 20th century.



Field measurements

Dam-break like experiment performed on the Olszanka River (central Poland) in the 90's of 20th century.

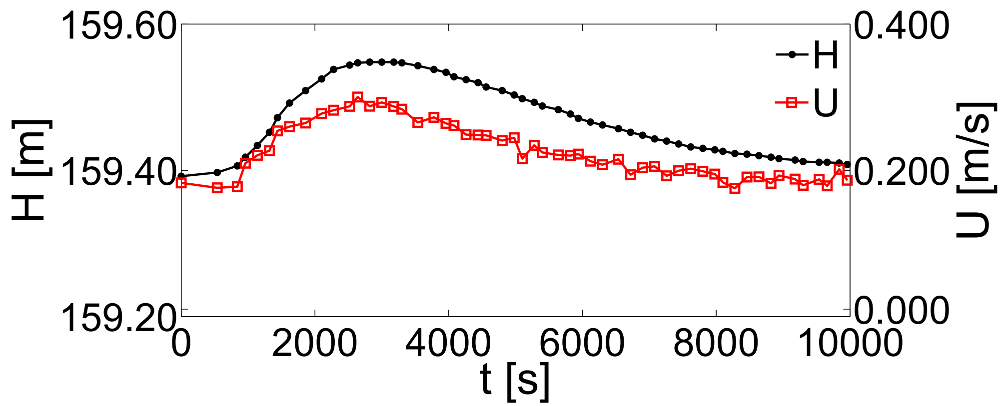




author: Jerzy Szkutnicki

Field measurements

Dam-break like experiment performed on the Olszanka River (central Poland) in the 90's of 20th century.



Friction velocity

The friction velocity expresses the bed shear stress in velocity units, and is defined as:

$$U_* = \sqrt{\frac{\tau_0}{\rho}},$$

where: τ_0 – bed shear stress [N/m^2], ρ – water density [kg/m^3].

Crucial in estimating hydraulic problems such as:

- resistance to flow
- bed load and suspended load transport
- pollutants transport

Friction velocity for trapezoidal channel

$$U_* = \left[gR \left(I + \frac{U}{g} \frac{b + mh}{bh + m\frac{h^2}{2}} \eta + \left(\frac{U^2}{g} \frac{b + mh}{bh + m\frac{h^2}{2}} - 1 \right) \vartheta - \frac{1}{g} \zeta \right) \right]^{\frac{1}{2}},$$

where:

I – bed slope [-],

U – mean cross-sectional velocity $[m/s]$,

R – hydraulic radius [m],

$\zeta = \frac{\partial U}{\partial t}$ – temporal derivative $[m/s^2]$,

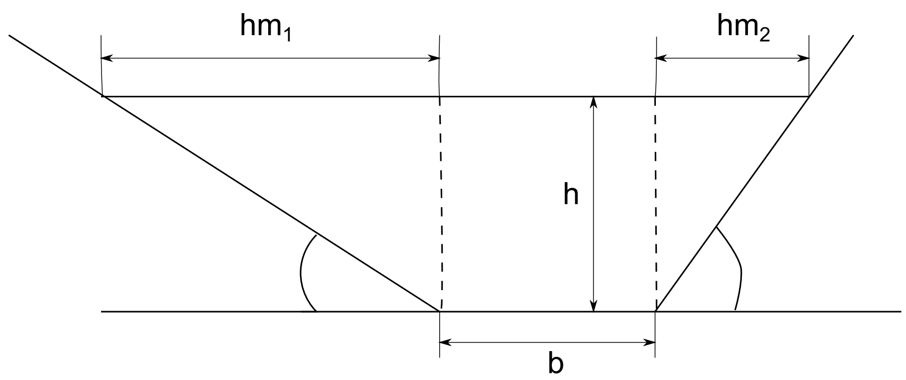
$\eta = \frac{\partial h}{\partial t}$ – temporal derivative $[m/s]$,

$\vartheta = \frac{\partial h}{\partial x}$ – spatial derivative [-],

Friction velocity for trapezoidal channel

$$U_* = \left[gR \left(I + \frac{U}{g} \frac{b + mh}{bh + m\frac{h^2}{2}} \eta + \left(\frac{U^2}{g} \frac{b + mh}{bh + m\frac{h^2}{2}} - 1 \right) \vartheta - \frac{1}{g} \zeta \right) \right]^{\frac{1}{2}},$$

b – river bed width [m],
h – the maximum depth of channel section [m],
$m = m_1 + m_2$,
m_1 and m_2 – side slopes [-],



The application of uncertainty analysis to the friction velocity evaluation

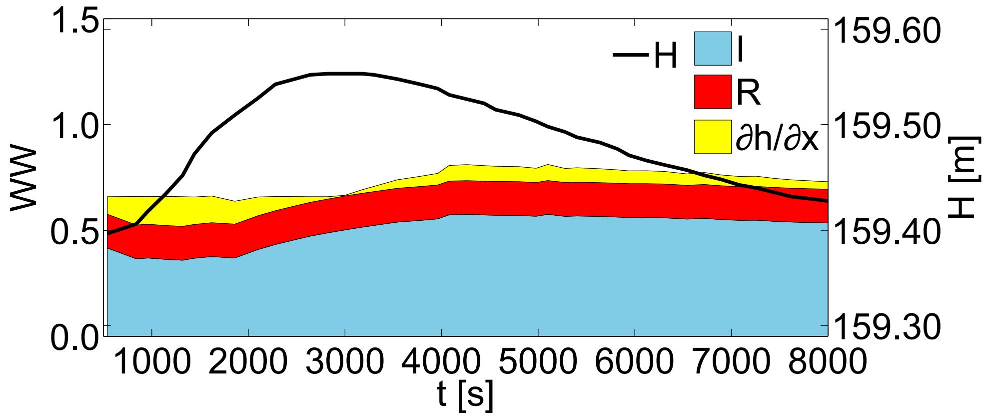
$$U_* = f(I, R, U, h, \zeta, \eta, \vartheta, b, m)$$

Sensitivity analysis provides the information which input variables x_i have the biggest impact on the uncertainty of an output variable y.

The effect of a small perturbation of input variable x_i on output variable y is assessed by a relative sensitivity coefficient:

$$WW_{x_i}^{y} = \frac{\partial y}{\partial x_i} \frac{x_i}{y}$$

Sensitivity analysis



There are three variables that have considerable impact on friction velocity:

- bed slope I
- hydraulic radius R
- spatial derivative $\frac{\partial h}{\partial x}$

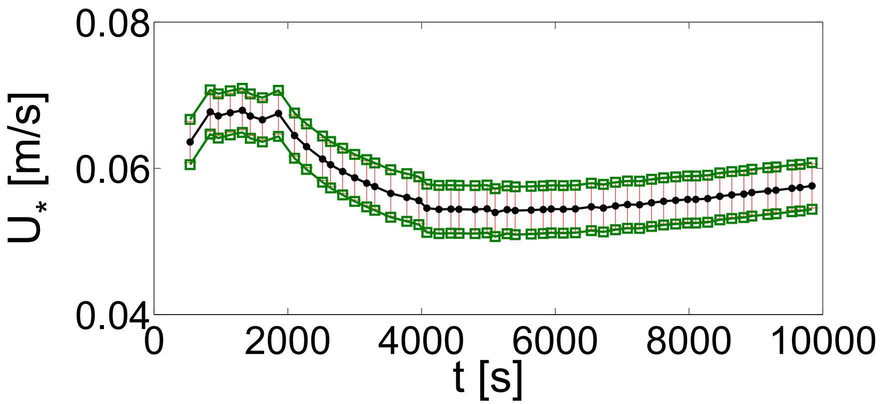
Evaluation of uncertainty

$$\Delta U_{*max} \simeq \left|\frac{\partial U_*}{\partial I}\right| \Delta I + \left|\frac{\partial U_*}{\partial R}\right| \Delta R + \left|\frac{\partial U_*}{\partial U}\right| \Delta U + \left|\frac{\partial U_*}{\partial h}\right| \Delta h +$$
$$+ \left|\frac{\partial U_*}{\partial \zeta}\right| \Delta \zeta + \left|\frac{\partial U_*}{\partial \eta}\right| \Delta \eta + \left|\frac{\partial U_*}{\partial \vartheta}\right| \Delta \vartheta + \left|\frac{\partial U_*}{\partial b}\right| \Delta b + \left|\frac{\partial U_*}{\partial m}\right| \Delta m$$

The maximum deterministic uncertainties of input variables:

- $\Delta h = 0.01$ m,
- $\Delta U = 10\%$ U,
- $\Delta R =$0.01 m,
- partial derivatives: $\zeta = 0.0001$ $[m/s^2]$, $\eta = 0.0001$ $[m/s]$, $\vartheta = 0.00001$ [-],
- $\Delta I = 0.0001$.

Evaluation of uncertainty



time [s]	U_*	ΔU_*	$u_c(U_*)$
0	0.064	0.003	0.0016
840	0.068	0.003	0.0015
9300	0.057	0.003	0.0018

The maximum deterministic uncertainty is between 4% - 5% of evaluated values of the friction velocity, and the maximum combined standard uncertainty is between 1% - 2%.

Summary

- The uncertainty analysis is a crucial element of measurements and data processing, which cannot be distregarded in river hydraulics.
- The uncertainty analysis provides the information about the quality of data, which improves the reliability of research results.
- The maximum deterministic uncertainty and the maximum combined standard uncertainty based on uniform distribution could be easily recalculated.
- The sensitivity analysis is a valuable addition to the uncertainty analysis, as it provides the information which input variables have the biggest impact on the uncertainty of an output variable.

Summary

- The uncertainty analysis is a crucial element of measurements and data processing, which cannot be distregarded in river hydraulics.
- The uncertainty analysis provides the information about the quality of data, which improves the reliability of research results.
- The maximum deterministic uncertainty and the maximum combined standard uncertainty based on uniform distribution could be easily recalculated.
- The sensitivity analysis is a valuable addition to the uncertainty analysis, as it provides the information which input variables have the biggest impact on the uncertainty of an output variable.

Summary

- The uncertainty analysis is a crucial element of measurements and data processing, which cannot be distregarded in river hydraulics.
- The uncertainty analysis provides the information about the quality of data, which improves the reliability of research results.
- The maximum deterministic uncertainty and the maximum combined standard uncertainty based on uniform distribution could be easily recalculated.
- The sensitivity analysis is a valuable addition to the uncertainty analysis, as it provides the information which input variables have the biggest impact on the uncertainty of an output variable.

Summary

- The uncertainty analysis is a crucial element of measurements and data processing, which cannot be distregarded in river hydraulics.
- The uncertainty analysis provides the information about the quality of data, which improves the reliability of research results.
- The maximum deterministic uncertainty and the maximum combined standard uncertainty based on uniform distribution could be easily recalculated.
- The sensitivity analysis is a valuable addition to the uncertainty analysis, as it provides the information which input variables have the biggest impact on the uncertainty of an output variable.

Summary

- The uncertainty analysis is a crucial element of measurements and data processing, which cannot be distregarded in river hydraulics.
- The uncertainty analysis provides the information about the quality of data, which improves the reliability of research results.
- The maximum deterministic uncertainty and the maximum combined standard uncertainty based on uniform distribution could be easily recalculated.
- The sensitivity analysis is a valuable addition to the uncertainty analysis, as it provides the information which input variables have the biggest impact on the uncertainty of an output variable.

Thank you for attention

This study has been financed by National Science Centre.
Grant no. DEC-2011/01/N/ST10/07395