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STEP LENGTH INFLUENCE IN MODELLING ADVECTION AND DIFFUSION OF BED-LOAD PARTICLES

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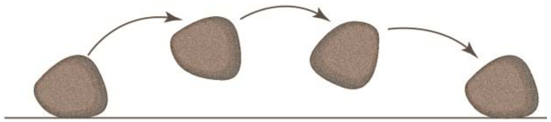
HYTECH Aquatic Interfaces = Hydrodynamic Transport in Ecologically Critical Heterogeneous Interfaces

Rivers bed = water-sediment interface

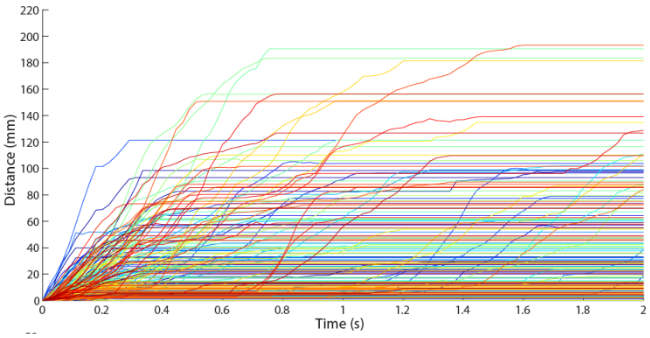
- Exchanging processes
- Potential contamination

HOW SEDIMENT MOVES?

- Field & lab experiments with bed load tracers
- Heterogeneity along travelling path



- From a deterministic approach → to a more appropriate stochastic analysis of motion
- Diffusion emerges due to turbulence and bed roughness [1,2,3]

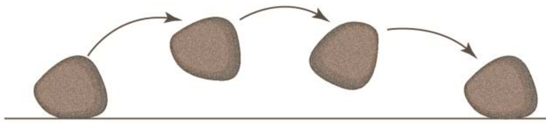


[1] Nikora et al., Water Resour. Res. 2002;
[2] Ganti et al., J. Geophys. Res. 2010;
[3] Furbish et al., J. Geophys. Res. 2012.

OBJECTIVE:

investigate characteristics of bedload sediment motion to model advective and diffusive transport behaviour

By studying particles’ step length = travel distance from pick up to deposition



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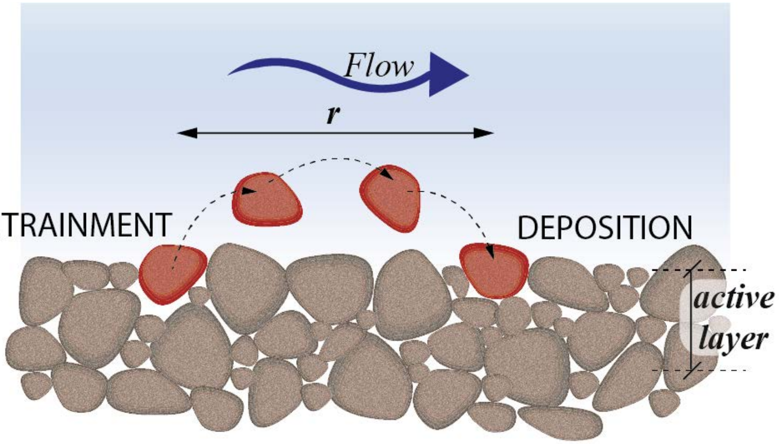
Exner’s equation in entrainment E and deposition D terms

$$(1-\lambda_p)\frac{\partial\eta(x,t)}{\partial t}=D(x,t)-E(x,t)$$

η = local bed elevation
λ_p = bed porosity



$$D(x,t)=\int_0^\infty E\left(x-r,t-{}^{r}\!/\!_{v_p}\right)f_S(r)dr$$

Stochastically represented

r/v_p = particle travelling time

- Using tracers as representative of entire population
  f_T = fraction of tracers

$$\begin{cases} E_T(x,t) = E(x,t)\,f_T(x,t) \\ D_T(x,t) = \int_0^\infty E\left(x-r,\, t-{}^{r}\!/_{v_p}\right) f_T\left(x-r,\, t-{}^{r}\!/_{v_p}\right) f_S(r)\,dr \end{cases}$$

- Bed in equilibrium: only active layer L_a exchanges sediment with bed load transport [4]

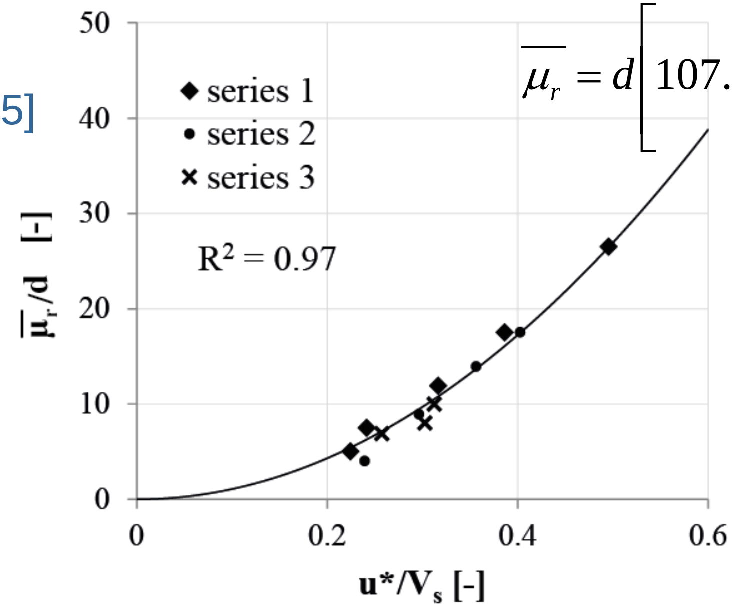
$$(1-\lambda_p)L_a \frac{\partial f_T(x,t)}{\partial t} = D_T(x,t) - E_T(x,t)$$

$$(1-\lambda_p)\frac{L_a}{E}\frac{\partial f_T(x,t)}{\partial t} = \int_0^\infty f_T\left(x-r,\, t-{}^{r}\!/_{v_p}\right) f_S(r)\,dr - f_T(x,t)$$

[4] Hirano, M. Proc. Jpn, Soc. Civ. Eng. 1971.

Step length→ probability density function $f_s(r)$ depending on:

FLOW INTENSITY[5]



$$\overline{\mu_r} = d\left[107.8\left(\frac{u^*}{V_s}\right)^2\right]$$

+ BED ROUGHNESS

$$\left({}^{d}\!/\!{}_{\sigma_b}\right)^2$$

u^*=shear velocity
V_s=settling velocity
σ_b=standard deviation of bed elevation

$f_s(r)$ lognormally distributed

$$\begin{cases} \mu_r = \overline{\mu_r}\left(\dfrac{d}{\sigma_b}\right)^2 \\ \sigma_r = k\mu_r \end{cases}$$

[5] Lajeunesse et al., J. Geophys. Res. 2010.

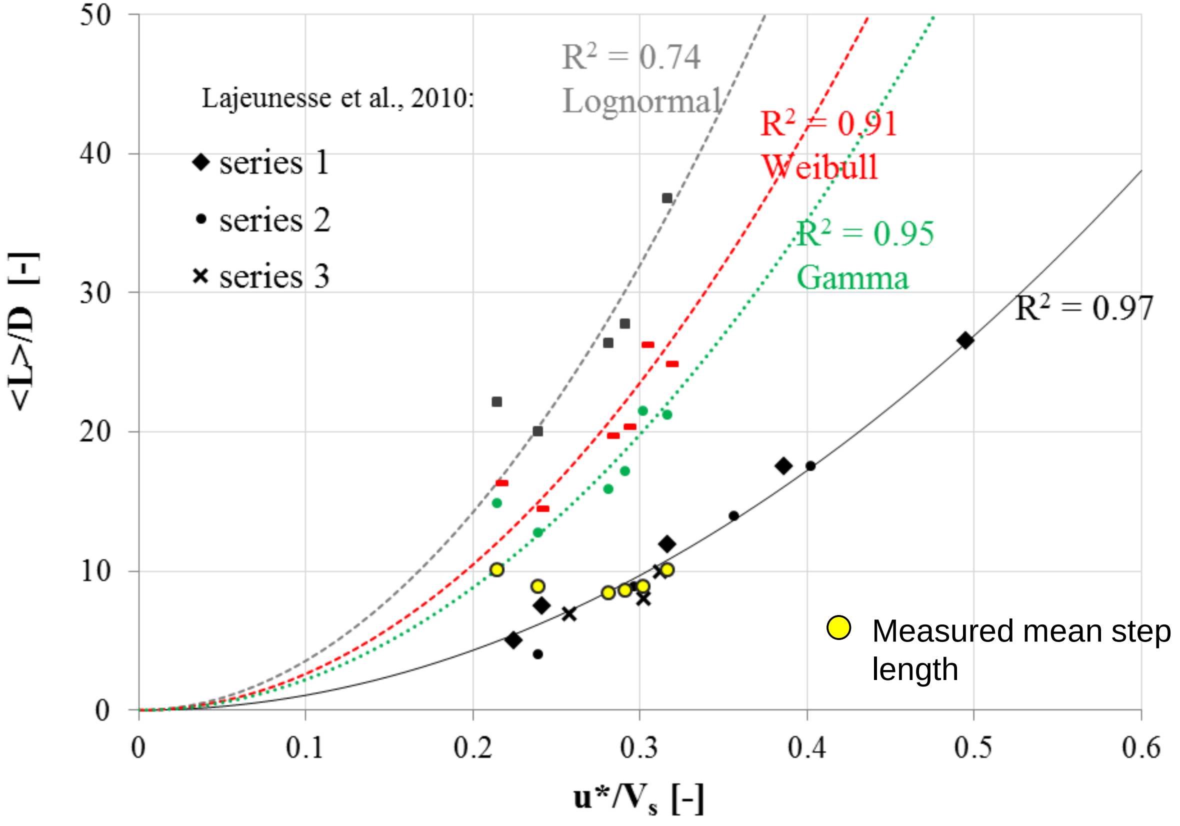
Step length→ comparison with other database:

Experiments 2009 [6]:

- Uniform bed d_{50} = 5 mm
- Increasing $\tau^* = 0.06\text{-}0.09$
- Measured step length over 220x80 mm → truncated pdf

$$\overline{\mu_r} = d\left[k\left(\frac{u^*}{V_s}\right)^2\right]$$

Lajeunesse: k = 107
Gamma fit: k = 220.4
Weibull fit: k = 261.7
Lognormal: k = 355.8



[6] Tregnaghi et al., J. Geophys. Res. 2012.

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Unpublished data representative of a medium river gravel at a reasonable transport rate[7]:

- Sediment bed: d_{16} = 0.13 mm, d_{84} = 4.8 mm, σ_s= 1.9 mm
- Tracer size: d_T = 0.8 mm
- u^* = 0.06 m/s, τ = 3.5 N/m^2, u^*/u^*_c = 2.15

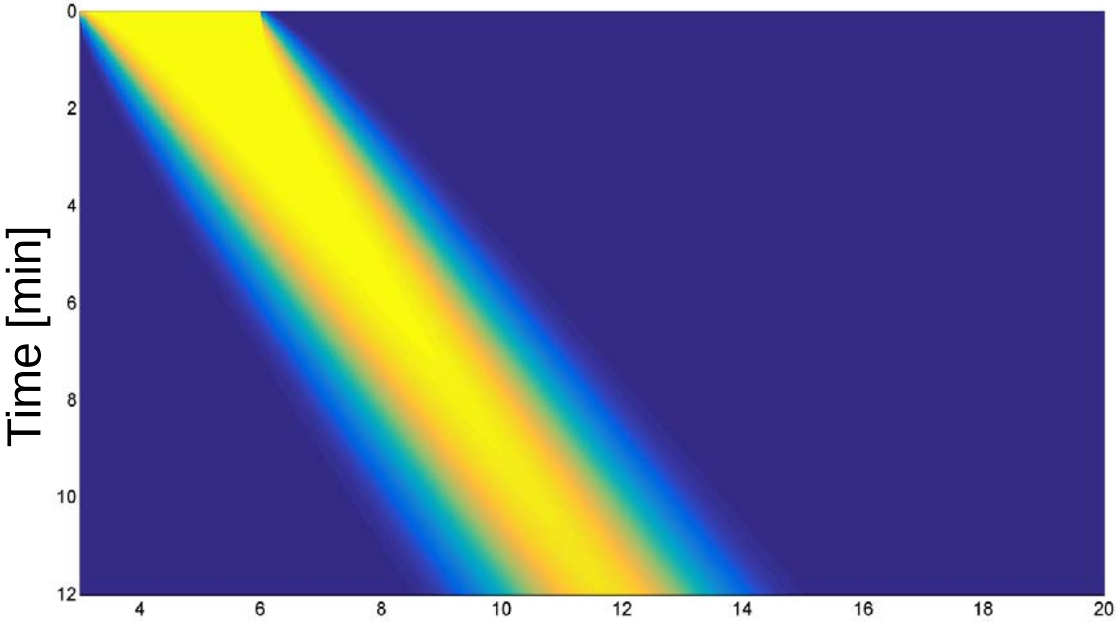
- $L_a = 1.62\ \sigma_b$ [8]

- Volumetric entrainment rate E [9]

$$W_i^* = \frac{(s-1)gq_{bi}}{F_i(u^*)^3}$$

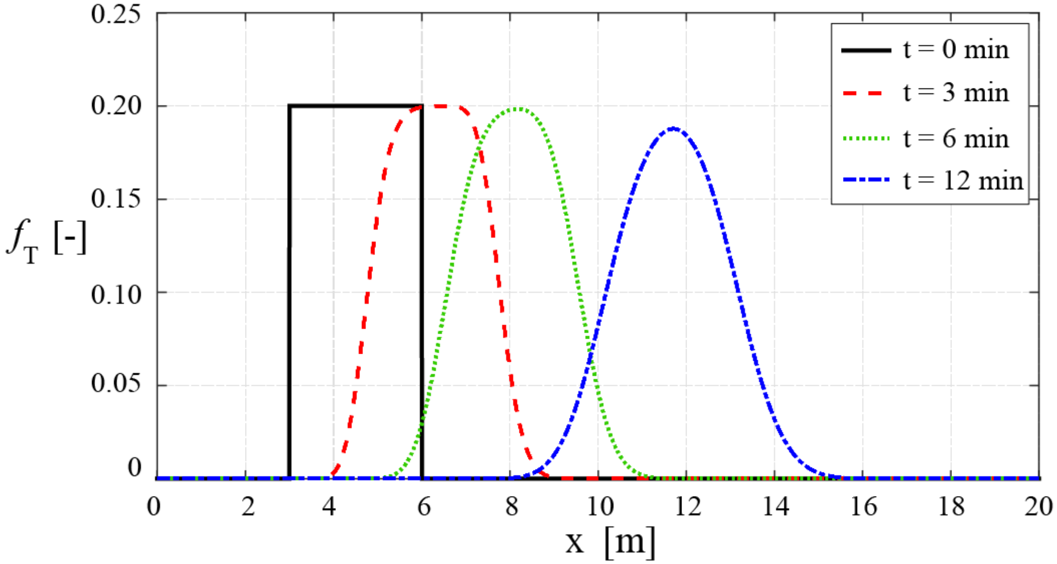
- v_p = particle's velocity at $z = 0.5d$ [10]

[7] Marion, Ph.D. thesis, Univ. of Padua, Italy, 1995.
[8] Wong et al., Water Resour. Res, 2007.
[9] Wilcock and Kenworthy, Water Resour. Res, 2002.
[10] Nikora et al., J. Hydraul. Eng., 2001



Advection and diffusion of an initial plateau concentration of tracers

Snapshots after
3, 6 & 12 minutes



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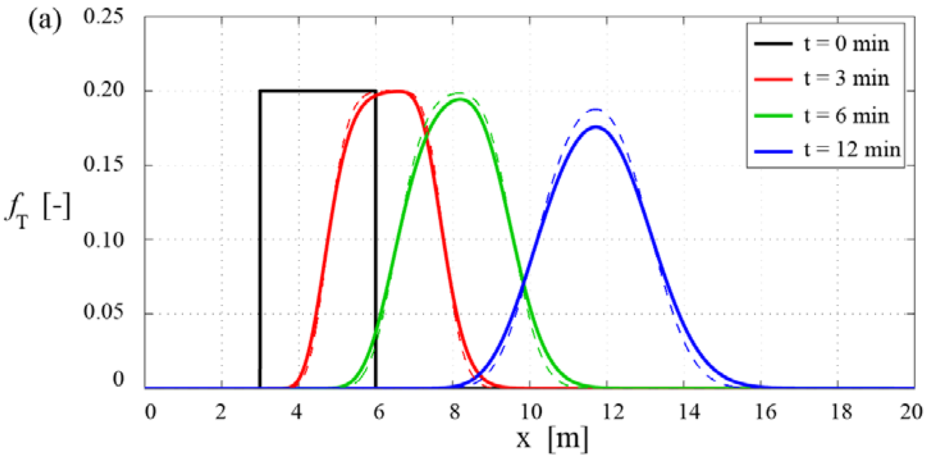
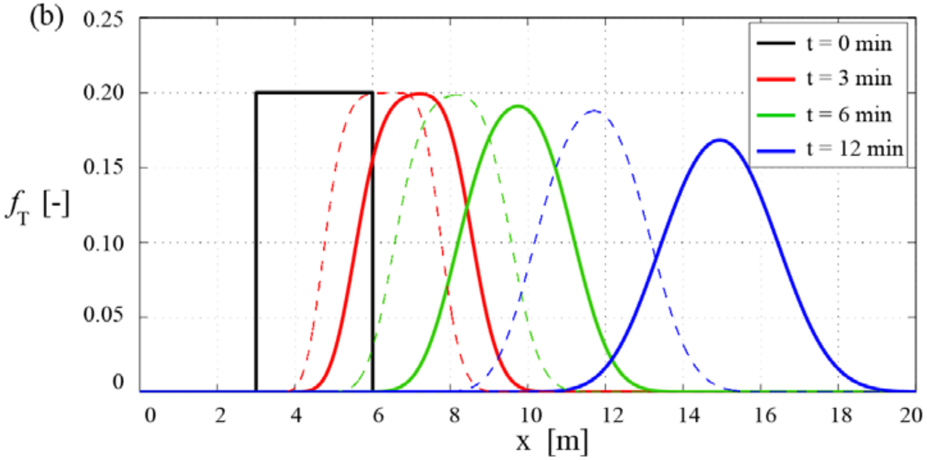
Influence of particle’s trajectory and flow intensity



$\sigma_r = k\ \mu_r$

k = 0.4 → 0.8



u^*_{NEW} 10% greater

Influence on:

- step length statistics
- entrainment rate

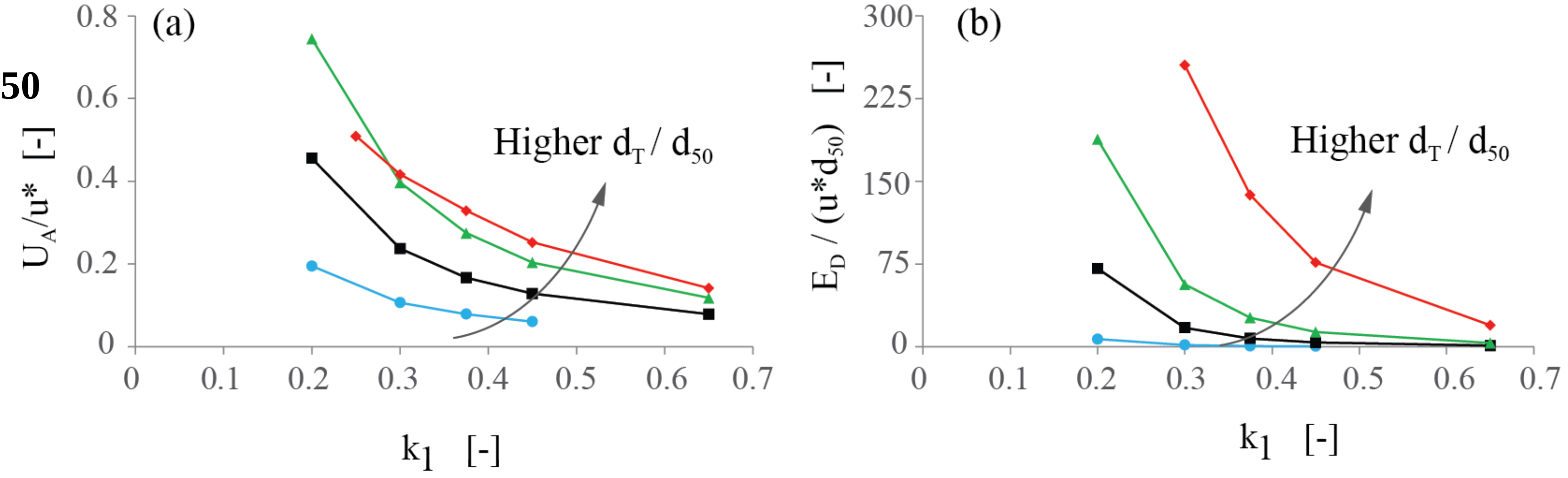
Enhanced mobility

	d_{50}[mm]	d_T [mm]	u^*/u^*_c [-]	E [m^3/m^2s]	v_p [m/s]	k [-]	U_A/u^* [-]	E_D/u^*d_{50} [-]
Original (Fig. 2)	1	0.8	2.15	$3*10^{-5}$	0.17	0.4	0.17	75
k case (Fig. 3a)	1	0.8	2.15	$3*10^{-5}$	0.17	0.8	0.17	110
u* case (Fig. 3b)	1	0.8	2.40	$4.5*10^{-5}$	0.19	0.4	0.22	119

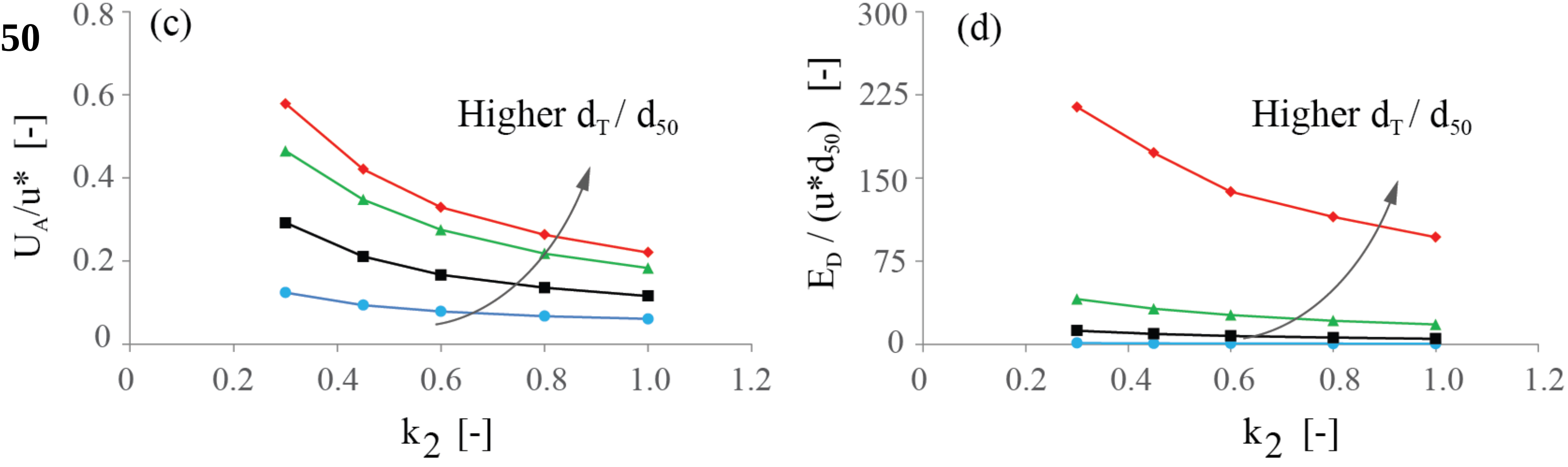
Influence of bed configuration

Advection and diffusion studied for d_T/d_{50} = 0.4; 0.8; 1.2; 2.5

$\sigma_b = k_1 d_{50}$



$L_a = k_2 d_{50}$



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- Model (considering step length) performs the expected advection and diffusion motion of bedload particles

- Flow intensity, $u*$, greatly affects mobility

- bed configuration = driving factor on sediment transport
  - Bed roughness, σ_b
  - Active layer, L_a

→ Negative correlation on particles' mobility, more significant for $d_T \geq d_{50}$

- Active layer acts as storage → diffusion for longer time scales

Future investigations: bed roughness role in diffusive processes under longer time scales

Acknowledgement to Project “HYTECH: Hydrodynamic Transport in Ecologically Critical Heterogeneous Interfaces”

Thanks for your attention