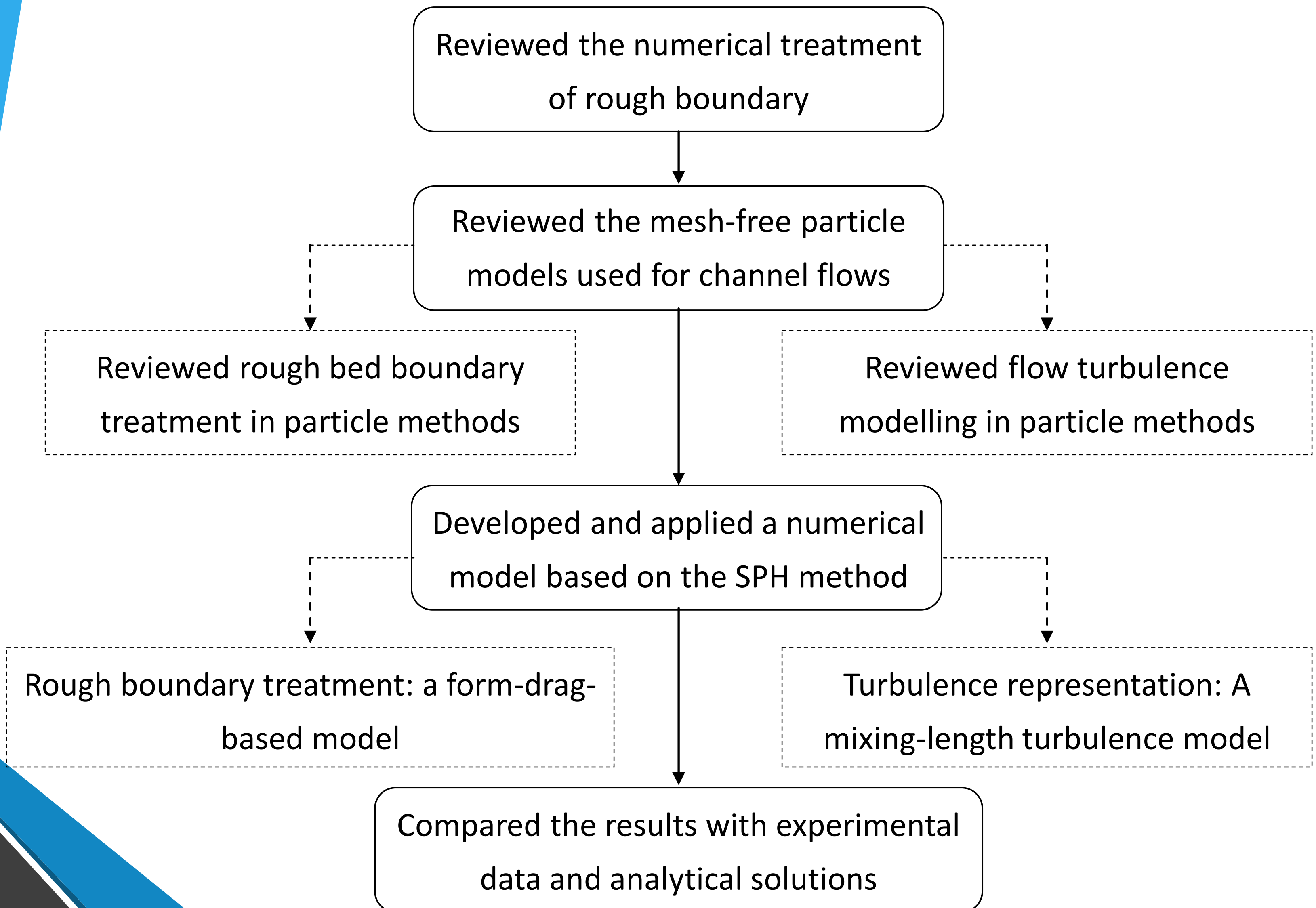
Potential application of mesh-free SPH method in turbulent river flows

Ehsan Kazemi, Simon Tait, Songdong Shao, Andrew Nichols

Summary


Reviewed the numerical treatment of rough boundary
Reviewed the mesh-free particle models used for channel flows
Reviewed rough bed boundary treatment in particle methods
Reviewed flow turbulence modelling in particle methods
Developed and applied a numerical model based on the SPH method
Rough boundary treatment: a form-drag-based model
Turbulence representation: A mixing-length turbulence model
Compared the results with experimental data and analytical solutions

Literature review — Numerical treatment of rough boundary

Boundary treatment method	Turbulence model	Characteristics	Examples
Wall function model	k-ε model	Suitable for smooth and small-scale boundary roughness, but not efficient for large-scale one as the velocity distribution is not logarithmic near rough wall	Hsu et al. (1998), Nicholas and Smith (1999), Zeng and Li (2012)
Modified turbulence model	Mixing length model	Simple but applicable only for shear flows where the distribution of the mixing length is known	van Driest (1956), Rotta (1962), Granville (1985, 1988), Krogstad (1991)
Drag-force model	Any turbulence model	Suitable for rough boundaries with large discrete roughness elements, also reflects the effects of rough wall based on shape and geometry of the roughness element	Christoph and Pletcher (1983), Taylor et al. (1985), Wiberg and Smith (1991), Cui et al. (2003), Zeng and Li (2012)

Literature review — Bed boundary treatment in particle methods

Particle model (SPH or MPS)	Channel bed treatment	Characteristics
Violeau et al. (2002), Violeau and Issa (2002)	Wall function approach	To impose the logarithmic velocity near the wall. Not applicable for large-scale roughness
Lopez et al. (2010), Chern and Syamsuri (2013)	Repulsive force on the bed	This numerical resistance force does not reflect the physical conditions of the bed roughness
Fu and Jin (2013)	Wall imaginary particles with artificial velocity in opposite direction of the flow	Numerical adjustment of velocity at the bed, which is not based on an actual physical mechanism

Literature review — Turbulence modelling in particle methods

Particle model (SPH or MPS)	Turbulence model	Characteristics
Sahebari et al. (2011), Chern and Syamsuri (2013), Fu and Jin (2013)	SPS model	Smagorinsky constant was chosen in the range of 0.12 to 0.15
Violeau and Issa (2007)	k-ε, Explicit algebraic Reynolds stress model (EARSM), 3D SPS	despite of its simplicity, LES-SPS model needs more computational costs compared to the traditional RANS turbulence closures
Lopez et al. (2010)	Variable artificial viscosity, k-ε	Relating the variable artificial viscosity to the flow vorticity

Objective

- ❑ Ideally, the production of near-wall velocity gradient can be modelled by an appropriate drag force model and the transportation of shear to upper layers can be modelled by a suitable turbulence model. Therefore both turbulence and rough ness effect are necessary to be considered.

<u>Approach:</u>

- Applying a drag force model to include the effects of the bed roughness in the model
- Using a turbulence model based on the mixing length approach to to address the flow turbulence effect. The mixing length approach is preferred because of its effectiveness in modelling shear flows.

SPH model — Governing equations

Mass conservation equation

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u}$$

Momentum conservation equation

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla P + \mathbf{g} + \nu_0 \nabla^2 \mathbf{u} + \frac{1}{\rho}\nabla \cdot \boldsymbol{\tau} + \frac{1}{\rho}\boldsymbol{\tau}_d$$

Pressure gradient

Viscosity

Turbulent shear

Drag shear

$\rho =$ Density

$\mathbf{u} =$ Velocity

$P =$ Pressure

$\bar{\tau} =$ Turbulent shear stress

$\mathbf{g} =$ Gravitational acceleration

$t =$ Time

SPH model — Roughness and turbulence modeling

⇨ Form-drag shear:

$$\boldsymbol{\tau}_d = \frac{\mathbf{F}_d}{A_\tau} \qquad \mathbf{F}_d = -\frac{1}{2} C_d \rho A_d \mathbf{u}|\mathbf{u}|$$

$$C_d = 0.5 \qquad A_d = d_p \qquad A_\tau = d_s d_p$$

⇨ Turbulent shear stress:

$$\frac{\tau_{xz}}{\rho} = \nu_t \left(\frac{\partial u}{\partial z} \right), \qquad \nu_t = l_m^{\ 2} \left| \frac{\partial u}{\partial z} \right|$$

$$l_m = \kappa z \left(1 - \frac{z}{H} \right)$$

$\boldsymbol{\tau}_d =$ Shear stress

$A_\tau =$ Area affected by drag

$\mathbf{F}_d =$ Drag force

$A_d =$ Cross-sectional area

$d_s =$ Bed grain diameter

$d_p =$ SPH particle spacing

$\nu_t =$ Eddy viscosity

$l_m =$ Mixing length

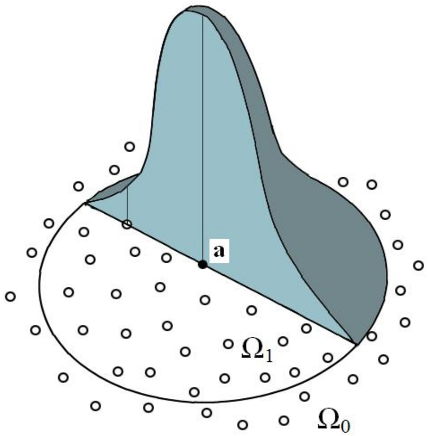
$\kappa = 0.41$ von-Karman constant

$H =$ Water depth

SPH model — Discretization of equations

Mass equation

$$\frac{D\rho_a}{Dt} = \rho_a \sum_b \frac{m_b}{\rho_b} \mathbf{u}_{ab} \cdot \nabla_a W_{ab}$$



Momentum equation

$$\frac{d\mathbf{u}_a}{dt} = -\sum_b m_b \left(\frac{P_a}{\rho_a{}^2} + \frac{P_b}{\rho_b{}^2} \right) \nabla_a W_{ab} + \mathbf{g}$$
$$+ \sum_b m_b \frac{4\nu_0}{(\rho_a + \rho_b)} \frac{\mathbf{r}_{ab} \cdot \nabla_a W_{ab}}{|\mathbf{r}_{ab}|^2 + \eta^2} \mathbf{u}_{ab}$$
$$+ \sum_b m_b \left(\frac{\boldsymbol{\tau}_a}{\rho_a{}^2} + \frac{\boldsymbol{\tau}_b}{\rho_b{}^2} \right) \cdot \nabla_a W_{ab} + \frac{1}{\rho_a} (\boldsymbol{\tau}_d)_a$$

$$f(\mathbf{r}_a) = \sum_b m_b \frac{f(\mathbf{r}_b)}{\rho_b} W(\mathbf{r}_a - \mathbf{r}_b, h)$$

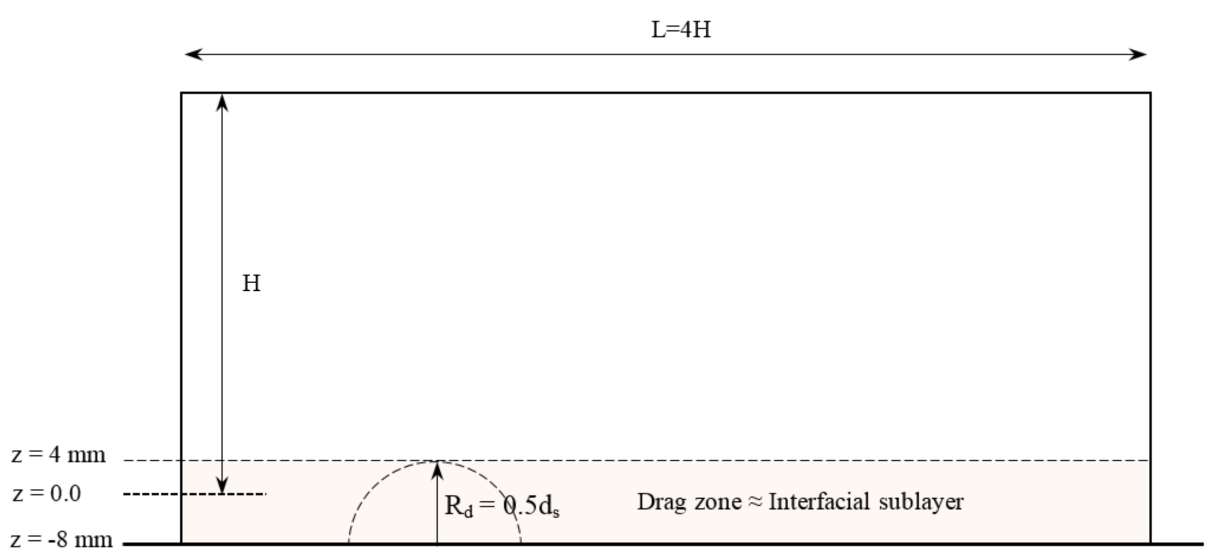
SPH model — Application

Experiments by Andrew Nichols:

- A steady uniform flow with water depth H = 50 mm in a 12 m long laboratory flume with a gradient of S_0 = 0.004.
- Hexagonally packed spheres with a diameter d_s = 24 mm on the bed.
- Two-dimensional PIV technique to measure the time-dependent flow field

Computational parameters

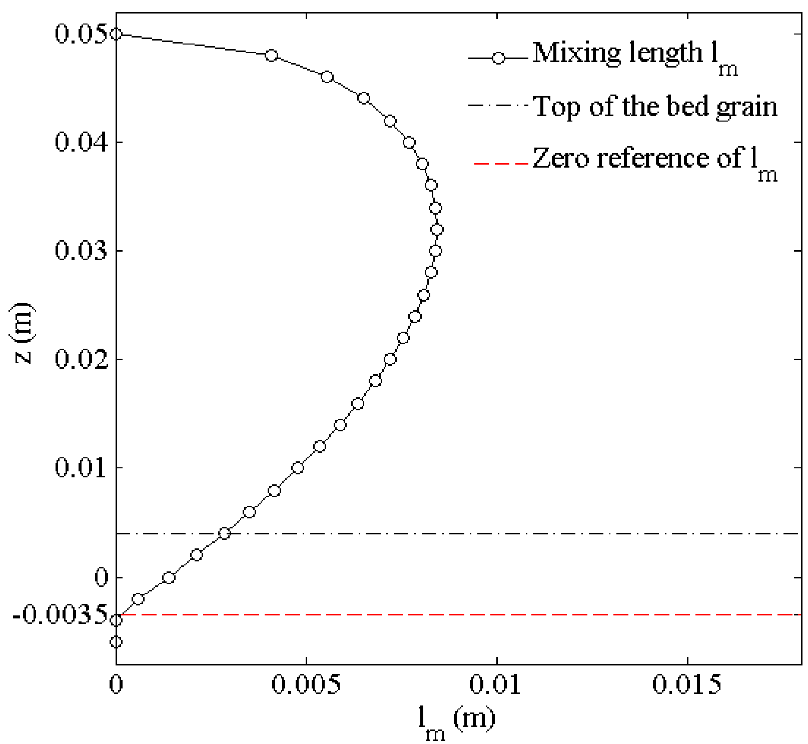
H (mm)	S_0	d_s (mm)	C_d	R_d (mm)	d_p (mm)
50	0.004	24	0.5	12	2



SPH model — Results

Nezu and Rodi (1986) mixing length

According to Nezu and Rodi (1986), the decrease in the l_m near the free surface is due to the fact that the water surface restricts the size of turbulence

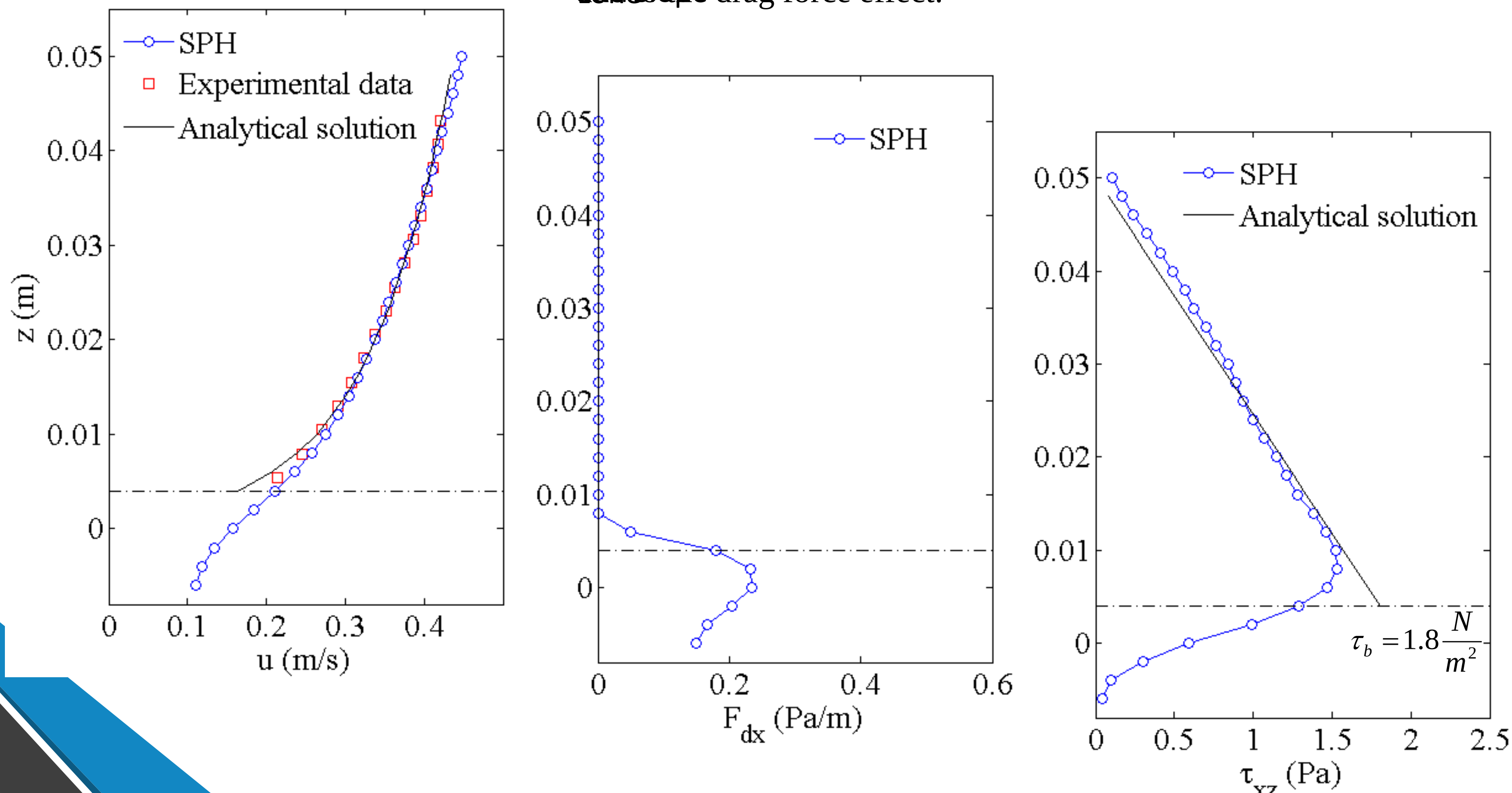


SPH model — Results

Results obtained by the SPH model with the drag force model and the mixing length model:

The SPH velocity profile is in good agreement with the experimental and analytical ones

… due to the drag force effect.



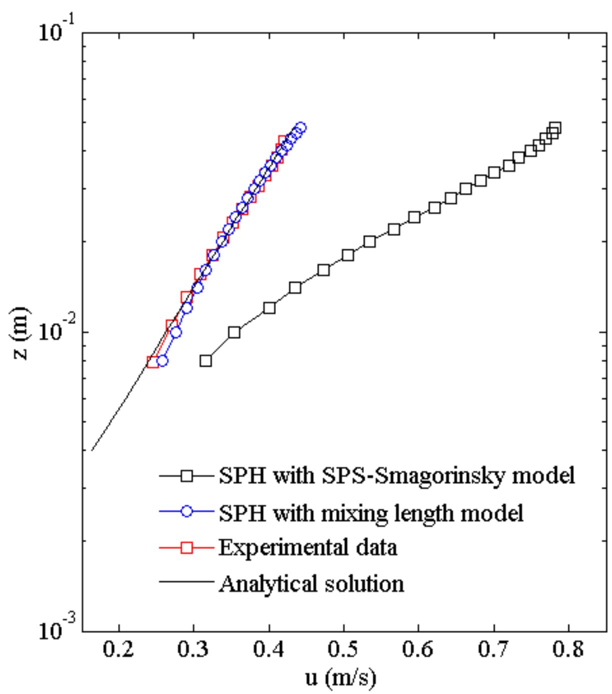
SPH model — Results

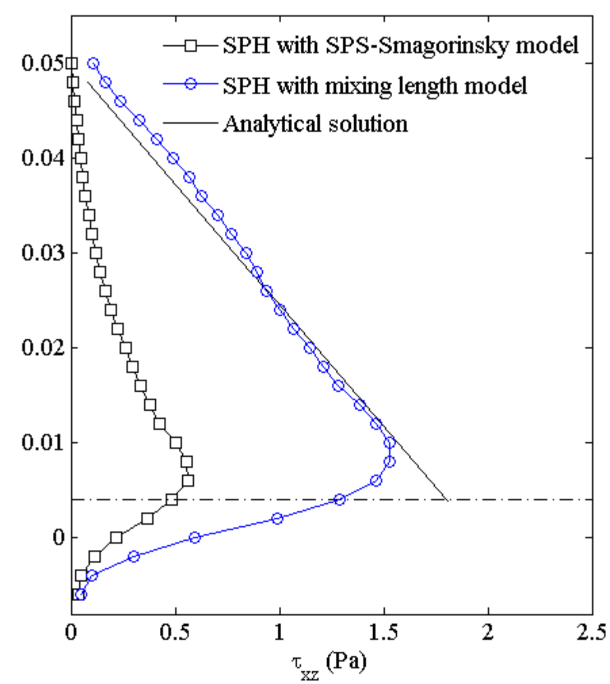
Results obtained by the SPS-Smagorinsky model

To further investigate the importance of the turbulence model, the calculations have been repeated by applying the SPS model with the Smagorinsky constant Cs = 0.15 and a filter size (Δ) equal to the SPH particle spacing

$$\frac{\tau_{ij}}{\rho} = 2\nu_t S_{ij} - \frac{2}{3} k\delta_{ij} \quad , \qquad \nu_t = (C_s \Delta)^2 |S|$$

Mixing length theory: $\nu_t = l_m{}^2 \left| \frac{\partial u}{\partial z} \right|$





Conclusions

- the drag force model successfully reproduced the mechanism of velocity reduction in the shear boundary layer and the mixing length model correctly transferred this effect to the upper flow.
- the SPS-Smagorinsky model with C_s = 0.15 was unable to reproduce the correct turbulent shear stress in uniform open channel flows over rough bed.
- In modelling turbulent open channel flows over rough walls by SPH:
  - drag-force model is suggested for treating boundaries with large roughness
  - SPS model is proposed for calculation of turbulence but with a mixing length approach to determine the eddy viscosity, instead of using the fixed Smagorinsky constant
- This model will be further developed to simulate turbulent flows over porous beds

Thank you

SPH model — Results

The magnitude and distribution of the mixing length is not known due to the difficulty in measuring velocity and shear stress in the interfacial zone

the zero-reference of the mixing length has been found by using numerical trials so as to achieve the best fit of mean velocity profile to the measured data.