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Lattice Boltzmann method for the numerical simulations of the melting and floating of ice

By: Ayurzana Badarch (Ph.D. student)
Hosoyamada Tokuzo (Professor)

長岡技術科学大学
Nagaoka University of Technology
Japan

Motivations and methods

An ice in an open channel flow

↓

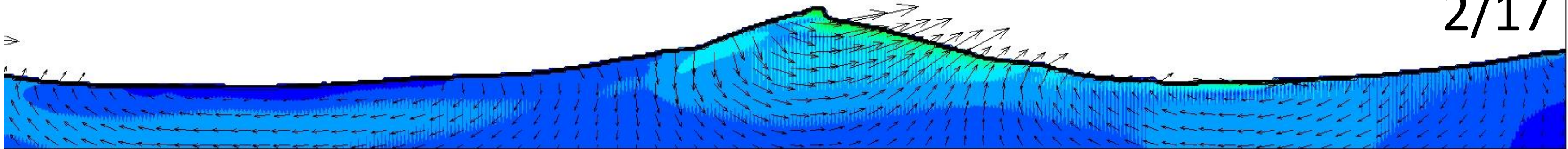
- A free surface flow
- A heat transport (transfer)
- Liquid-solid phase transitions
- A moving body

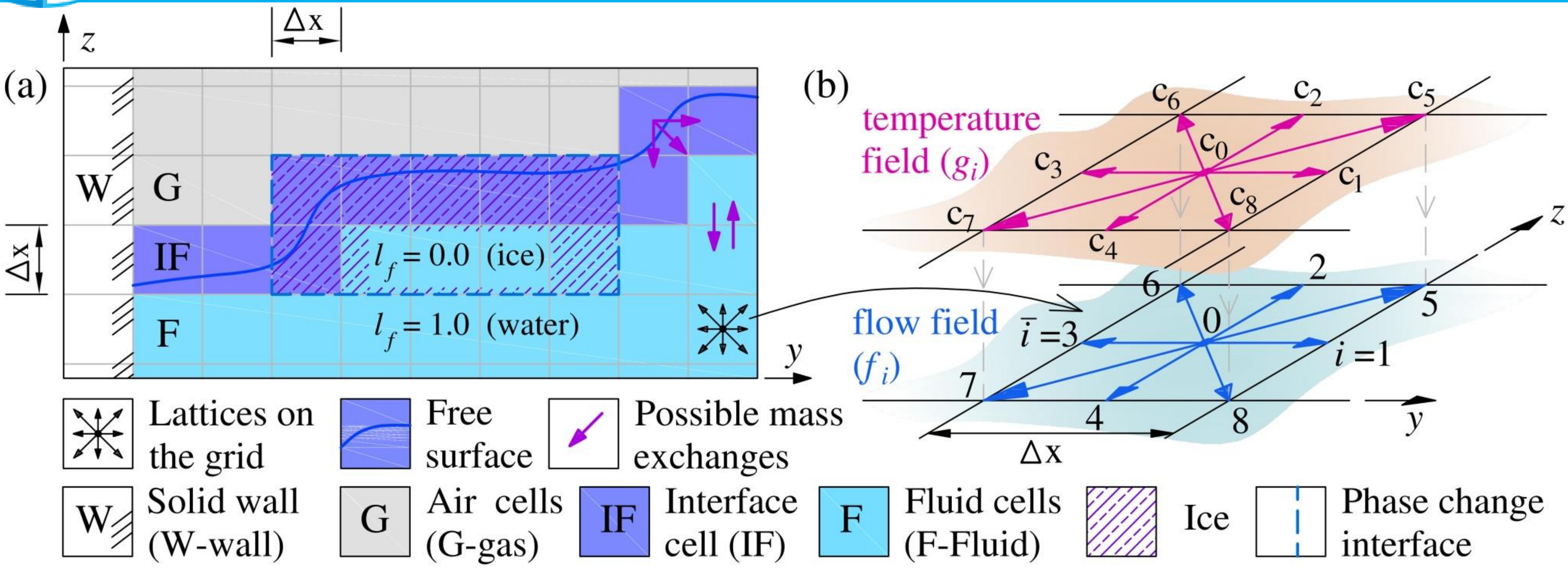
↓

Thermal – Free surface – immersed boundary – **lattice Boltzmann method**

Free surface flow modeling → ← Heat transfer and phase change modeling

Notes:





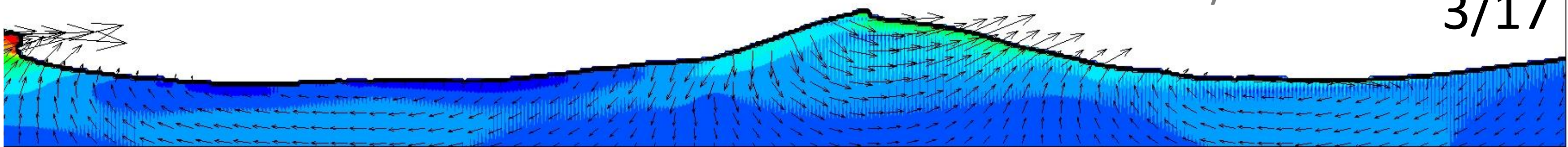
The discretized LBE for fluid flow:

$$f_i(\mathbf{x}+\mathbf{c}_i\Delta t, t+\Delta t) - f_i(\mathbf{x},t) = -\frac{\Delta t(1-\beta)}{\tau_{tot}} \times (f_i(\mathbf{x},t) - f_i^{eq}(\mathbf{x},t)) + \beta f_i^{m}(\mathbf{x},t) + \Delta t F_i,$$

The discretized LBE for scalar field:

$$g_i(\mathbf{x}+\mathbf{c}_i\Delta t, t+\Delta t) - g_i(\mathbf{x},t) = -\frac{g_i(\mathbf{x},t) - g_i^{eq}(\mathbf{x},t)}{\tau_h} - w_i \frac{L_h}{c_p}(l_f(\mathbf{x},t-\Delta t) - l_f(\mathbf{x},t)),$$

Notes: T-FS-IB-LBM is Thermal – Free surface – Immersed Boundary – LBM



2D Numerical model (T-FS-IB-LBM)

Macroscopic variables for fluid flow:

$$\rho=\sum_{i=0}^{8} f_i \quad \text{and} \quad \rho u=\sum_{i=0}^{8} c_i f_i+\frac{F\Delta t}{2}$$

Macroscopic variables for heat transport:

$$\vartheta=\sum_{i=0}^{8} g_i \;,\; T=\frac{T_{max}-T_{melt}}{\vartheta_{max}-\vartheta_{melt}}\left(\vartheta-\vartheta_{melt}\right)+T_{melt}$$

Liquid fraction:

$$l_f(\boldsymbol{x})=\begin{cases} 1 & \text{for } En>En_s+L_h=En_l \\ 0 & \text{for } En<En_s=c_p\vartheta_{melt} \\ \dfrac{En-En_s}{En_l-En_s} & \text{for } En_s\leq En\leq En_s+L_h \end{cases}$$

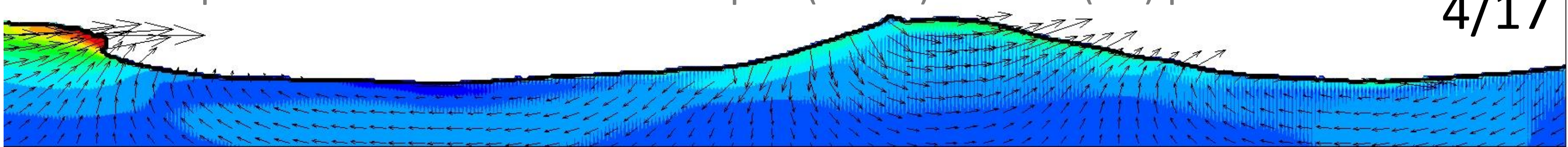
Some relations:

$$\alpha^{water}=\nu/Pr$$

$$F_R=F_L=\frac{u_0}{\sqrt{\boldsymbol{g}L}}$$

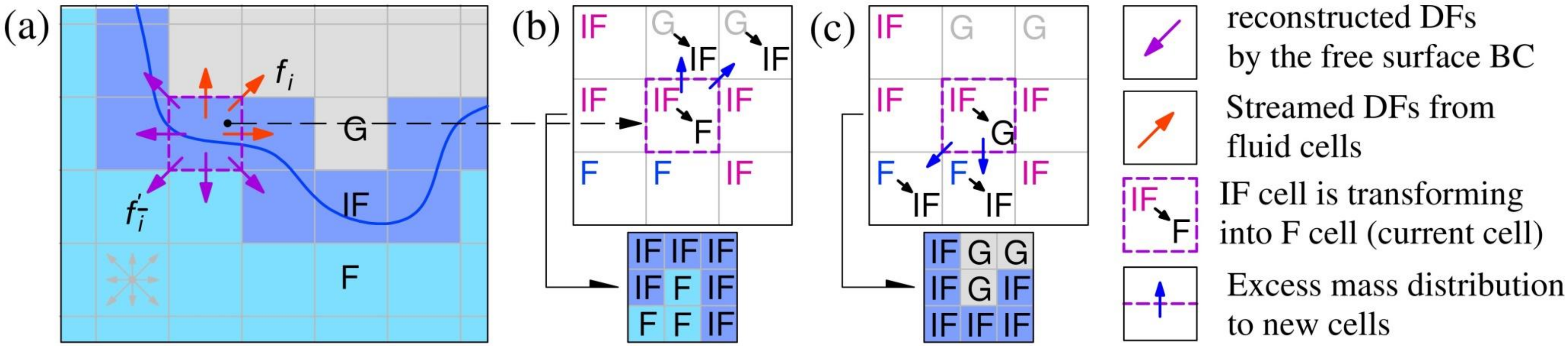
$$\boldsymbol{g}_R=\boldsymbol{g}\frac{\Delta x}{\Delta t_f^2} \quad \text{and} \quad \alpha_R^{ice}=\alpha^{ice}\frac{\Delta x^2}{\Delta t_h^2}$$

Notes: Liquid fraction defines if cell is liquid (water) or solid (ice) phases.



2D Numerical model (T-FS-IB-LBM)

Example of free surface transformation and free-surface boundary condition



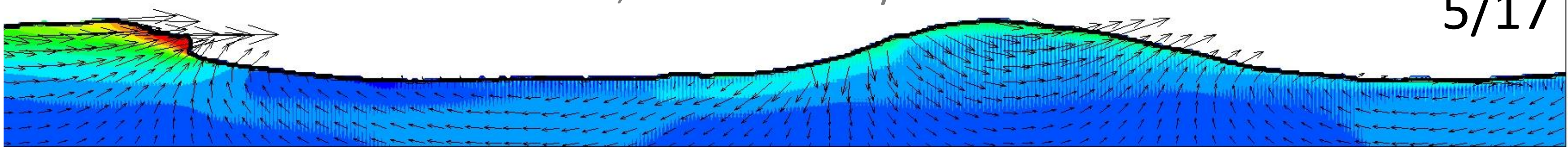
$$\frac{\partial m}{\partial t} + \mathbf{u} \cdot \nabla m = 0 \quad \rightarrow\rightarrow \quad m(\mathbf{x}, t + \Delta t) = m(\mathbf{x}, t) + \sum_{i=1}^{8} \Delta m_i(\mathbf{x}, t + \Delta t)$$

Mass evaluation:
$$\Delta m_i(\mathbf{x}, t + \Delta t) = \left(f_{\bar{i}}(\mathbf{x} + \mathbf{c}_i, t) - f_i(\mathbf{x})\right) \frac{\epsilon(\mathbf{x} + \Delta t \mathbf{c}_i, t) + \epsilon(\mathbf{x}, t)}{2}$$

$$m(\mathbf{x} + \Delta t \mathbf{c}_i) = m(\mathbf{x} + \Delta t \mathbf{c}_i) + m^{ex}\left(\frac{\eta_i}{\eta_{total}}\right)$$

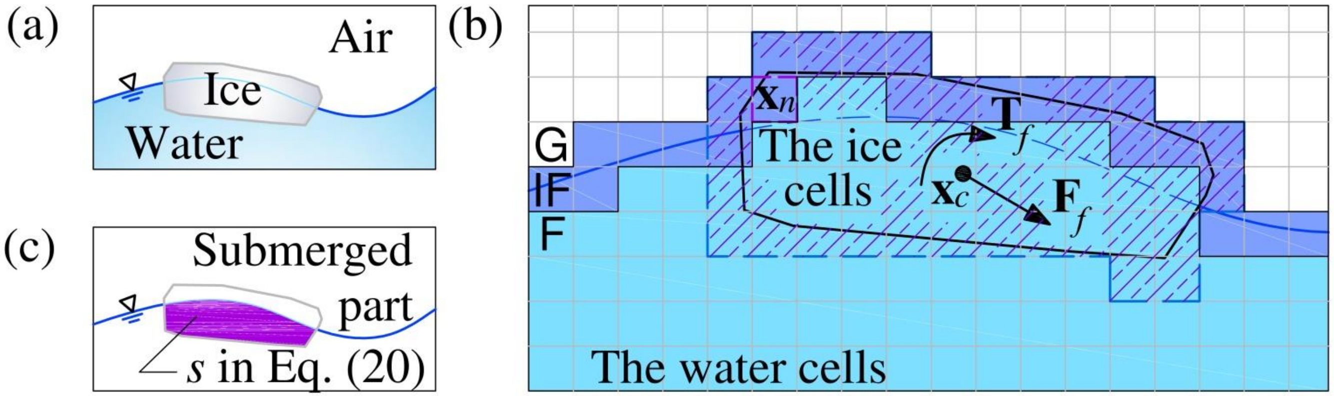
Boundary condition:
$$f_{\bar{i}}{}'(\mathbf{x}, t + \Delta t) = f_i{}^{eq}(\rho_A, \mathbf{u}) + f_{\bar{i}}{}^{eq}(\rho_A, \mathbf{u}) - f_i(\mathbf{x}, t)$$

Notes: DF – Distribution Function, BC – Boundary Condition



2D Numerical model (T-FS-IB-LBM)

Time-dependent arbitrary shaped floating body



- A shape is arbitrary
- A shape changes over time
- A body has physical properties

$$\mathbf{F}_f = s \cdot \frac{\Delta x^2}{\Delta t} \sum_n \beta \sum_{i=0}^{8} f_i^m \mathbf{c}_i \text{ and}$$

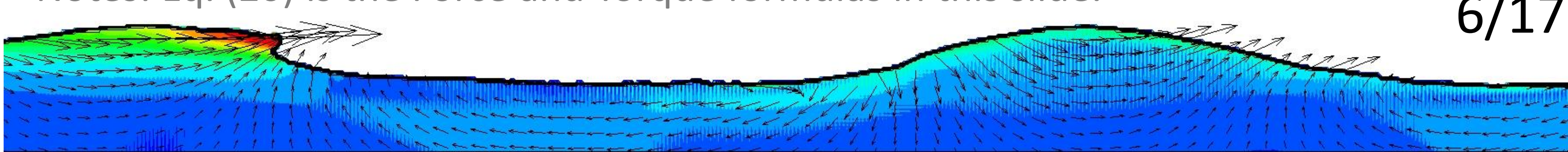
$$\mathbf{T}_f = s \cdot \frac{\Delta x^2}{\Delta t} \sum_n (\mathbf{x}_n - \mathbf{x}_c) \times \left(\beta \sum_{i=0}^{8} f_i^m \mathbf{c}_i \right),$$

$$\longrightarrow \quad m \frac{d^2\mathbf{x}}{dt^2} = \mathbf{F}_f + \mathbf{F}_b \text{ and } I \frac{d\omega}{dt} = \mathbf{T}_f$$

Submerged volumetric percent

Integer center of gravity

Notes: Eq. (20) is the Force and Torque formulas in this slide.



2D Numerical code (T-FS-IB-LBM)

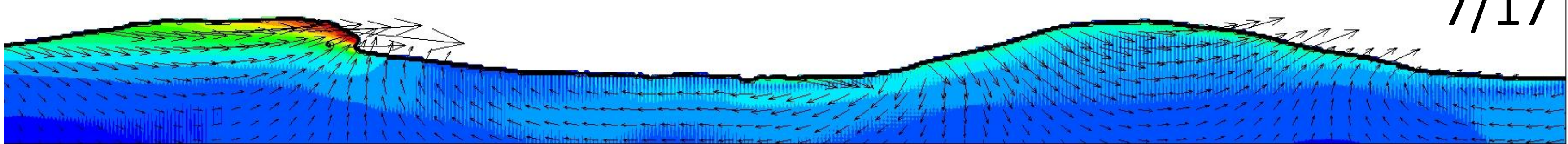
```
    program phase_free_surface
    include 'paramc.h'
    open(*,file='****.dat')  ! new files for result printing
! Initialazation
    compute dimensionless number
            for fluid flow and scalar field
    compute viscosity or relaxation parameter
            for fluid flow and scalar field
    call initial !where initilize the variables and distribution functions
                 !as well as flagging for each cell
    define time cycling !if code uses different time steps
! main computational loop
    DO kk=1,mtotal  ! discrete time step (lattice time step)
       time=kk*dt  ! dt is physical time step in second
    call massev    ! mass evaluations on cells
    call streaming ! streaming for fluid flow
    call fluidbound! boundary conditions for fluid flow
    call densvel   ! evaluation of density and velo
    call subgrid   ! subgrid scale model for turbu
    call collision ! collision for fluid flow
```

Free surface module

Heat transport module

```
    call coll_scal ! collision for scalar field
    call scalbound ! Boundary conditions for scalar field
    call streaming ! streaming for scalar field
    call move      ! ice motion is computed here
    call scalcalcu ! evaluation of scalar variable
    call result    ! printing results
    call change    ! updating of cell information for the free surface
       check the time criteria
    END DO
! end of the main computational loop
    stop
    end
! end of the main program
```

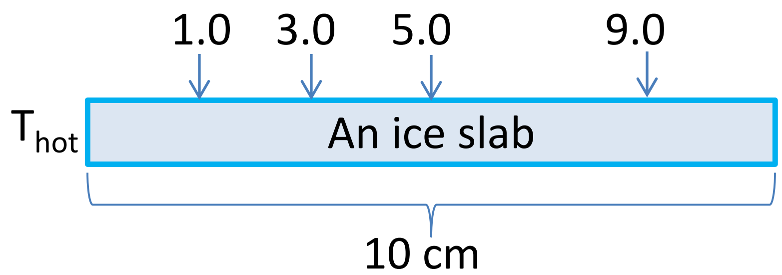
Notes: Not so efficient Fortran 77 code.

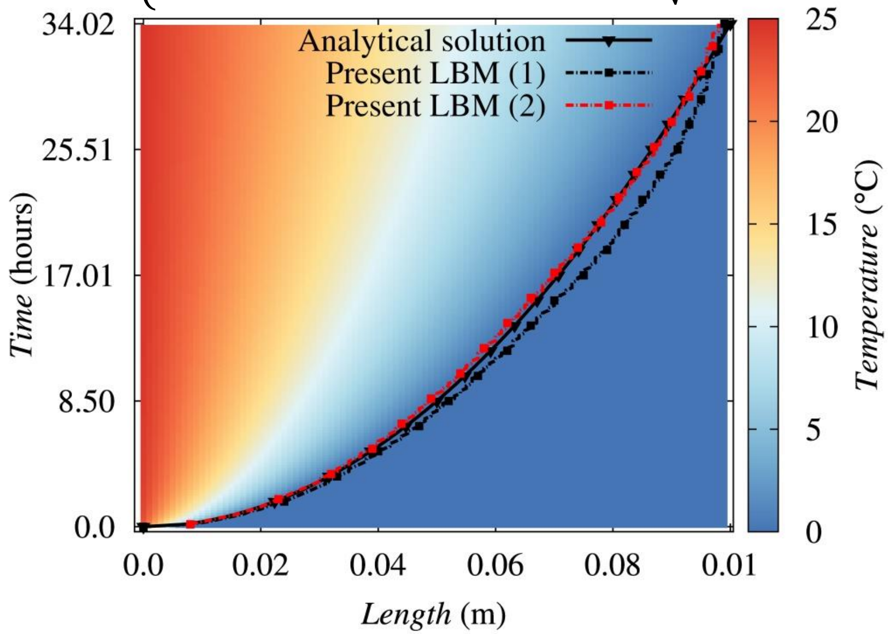


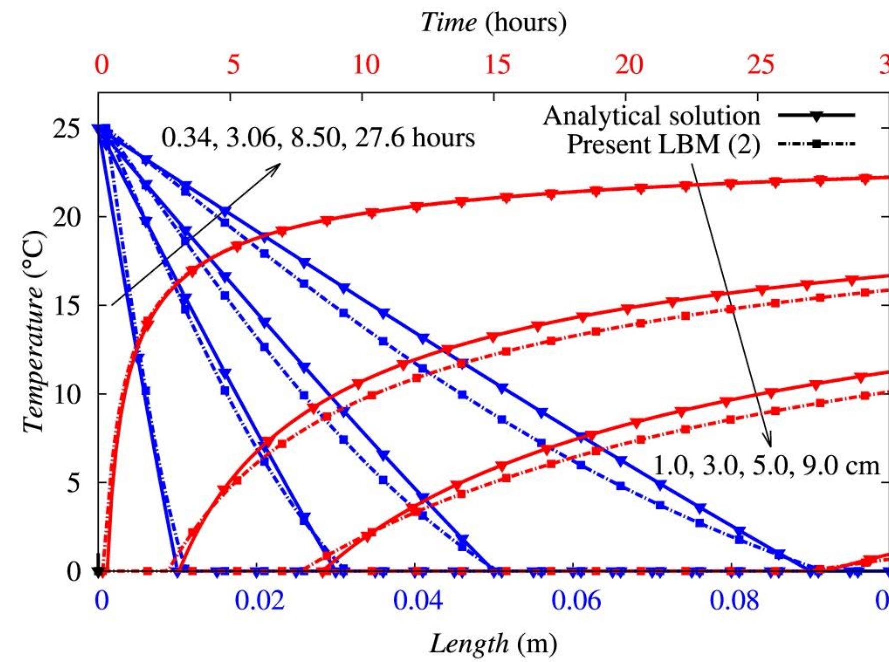
Validation for phase change

1D Stefan problem

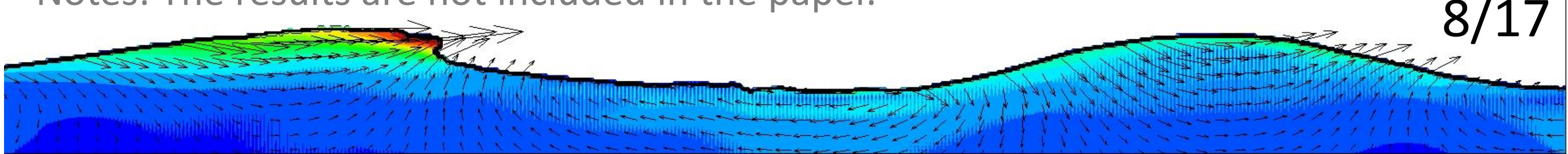
$$
\begin{cases}
X(t) = 2\chi\sqrt{\alpha_R^{water} t} \\
T(x,t) = T_{max} - (T_{max} - T_{melt}) \dfrac{erf\left(x/2\sqrt{\alpha_R^{water} t}\right)}{erf(\chi)} \\
\chi e^{\chi^2} erf(\chi) = \dfrac{St}{\sqrt{\pi}}
\end{cases}
$$





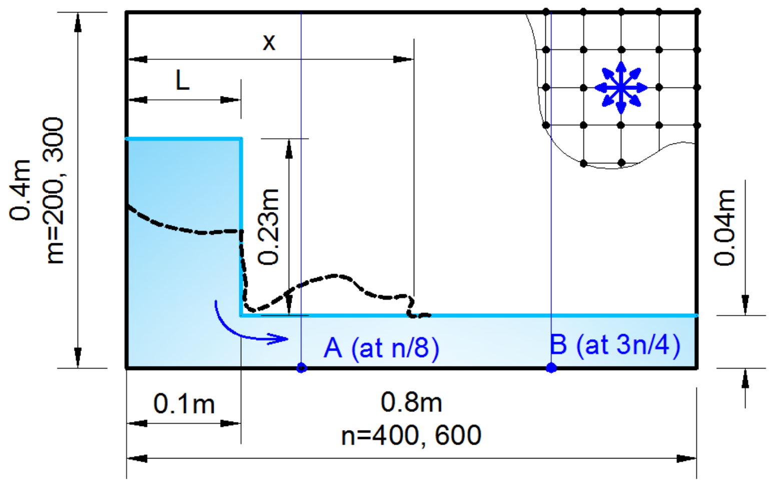


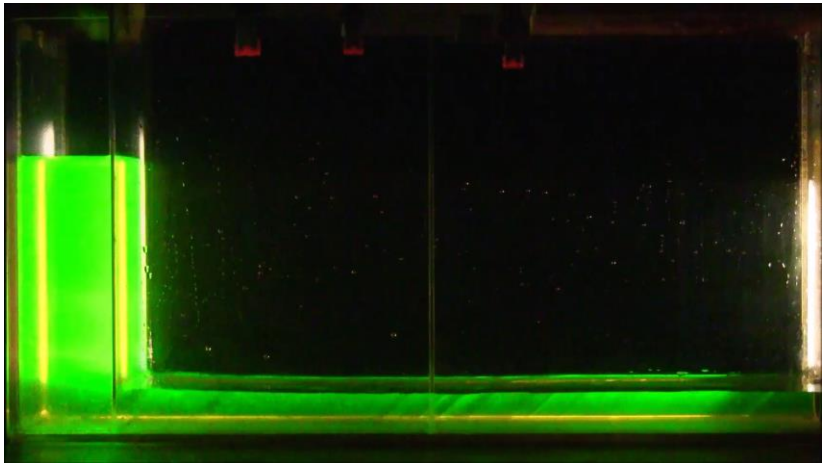
Notes: The results are not included in the paper.

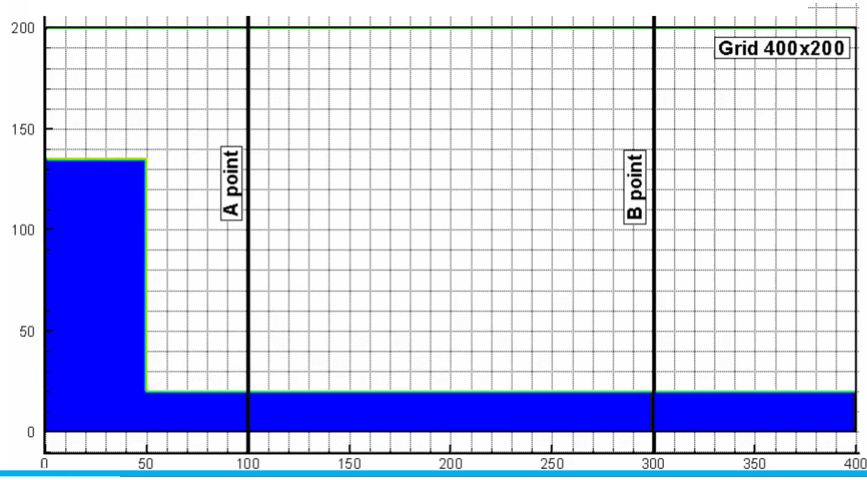


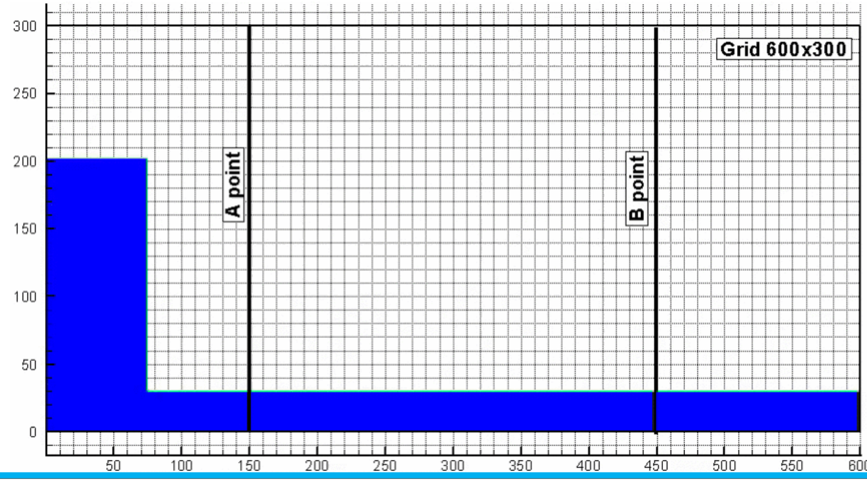
Validation for free surface flows

Dam break problem with wet bed









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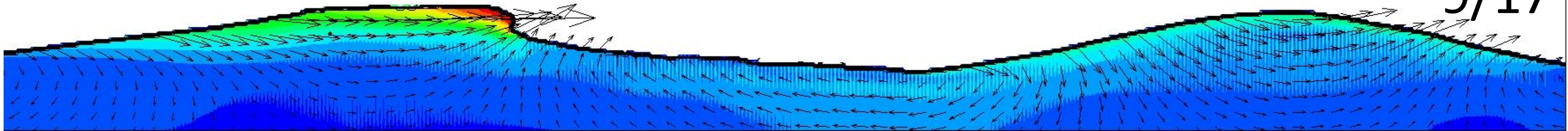
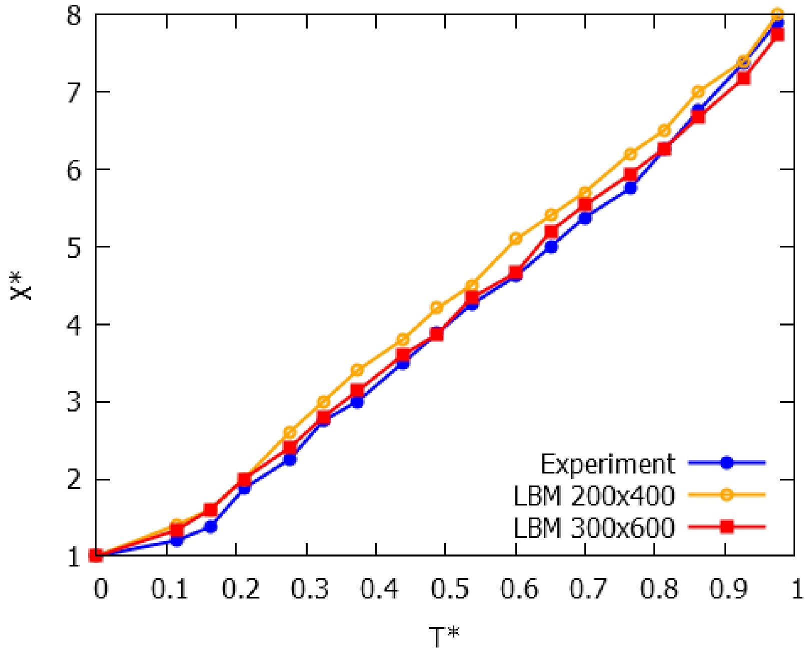
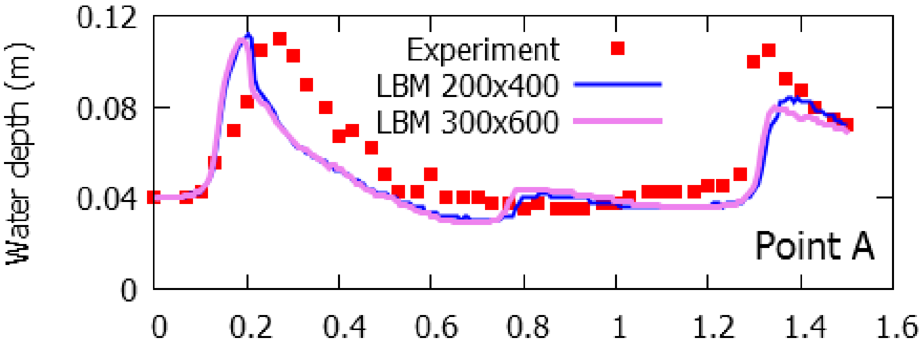
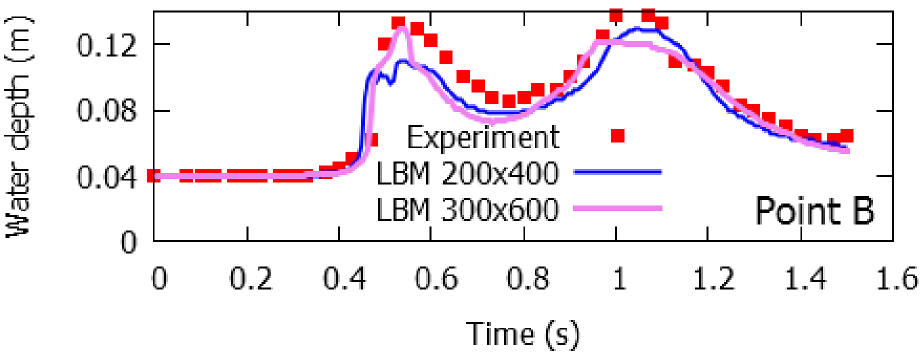


Dam break problem with wet bed



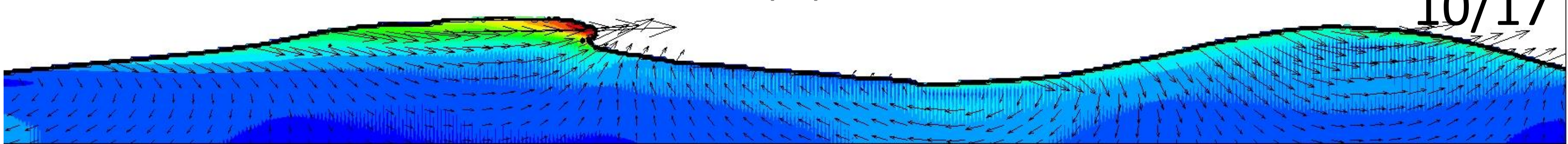
Time evolution of water-front position (X*) in experiment and numerical tests at dimensionless time T*





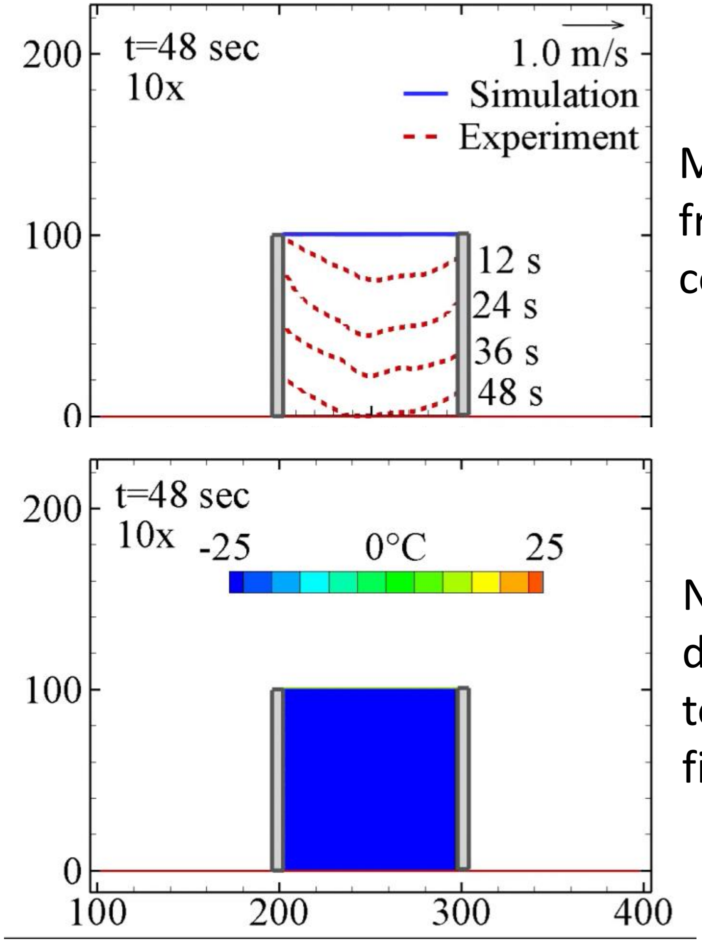
Time evolution of water level at control point A and B in experimental and numerical tests

Notes: The results are not included in the paper.



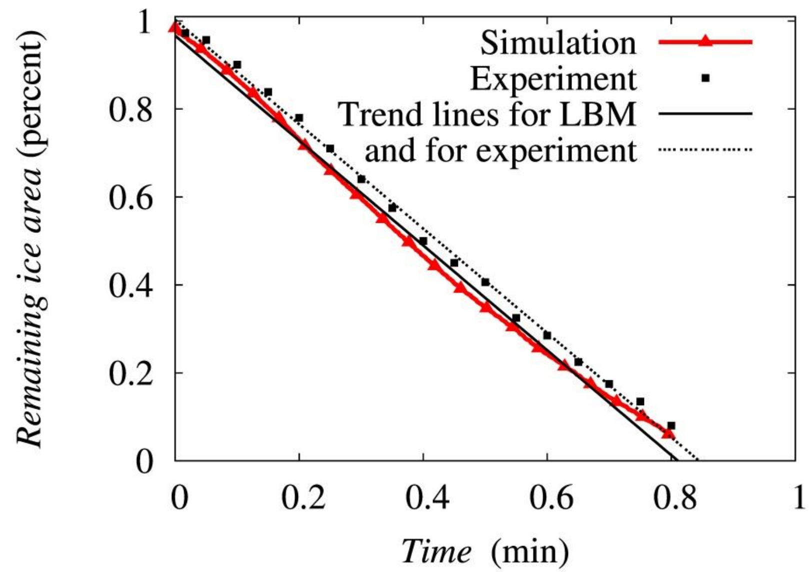
Validation for ice melting in FS flows

Ice cube melting by poured water



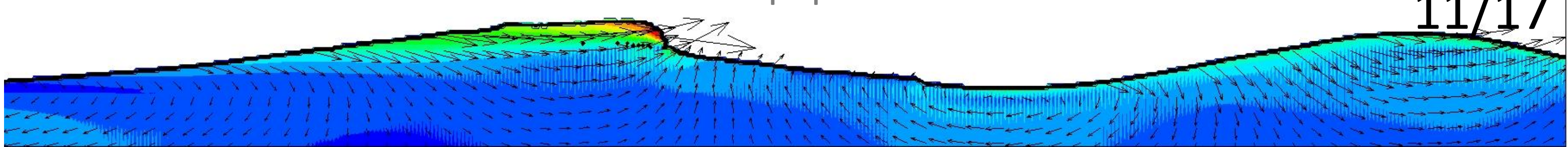
Melting front comparison

Numerically defined temperature field

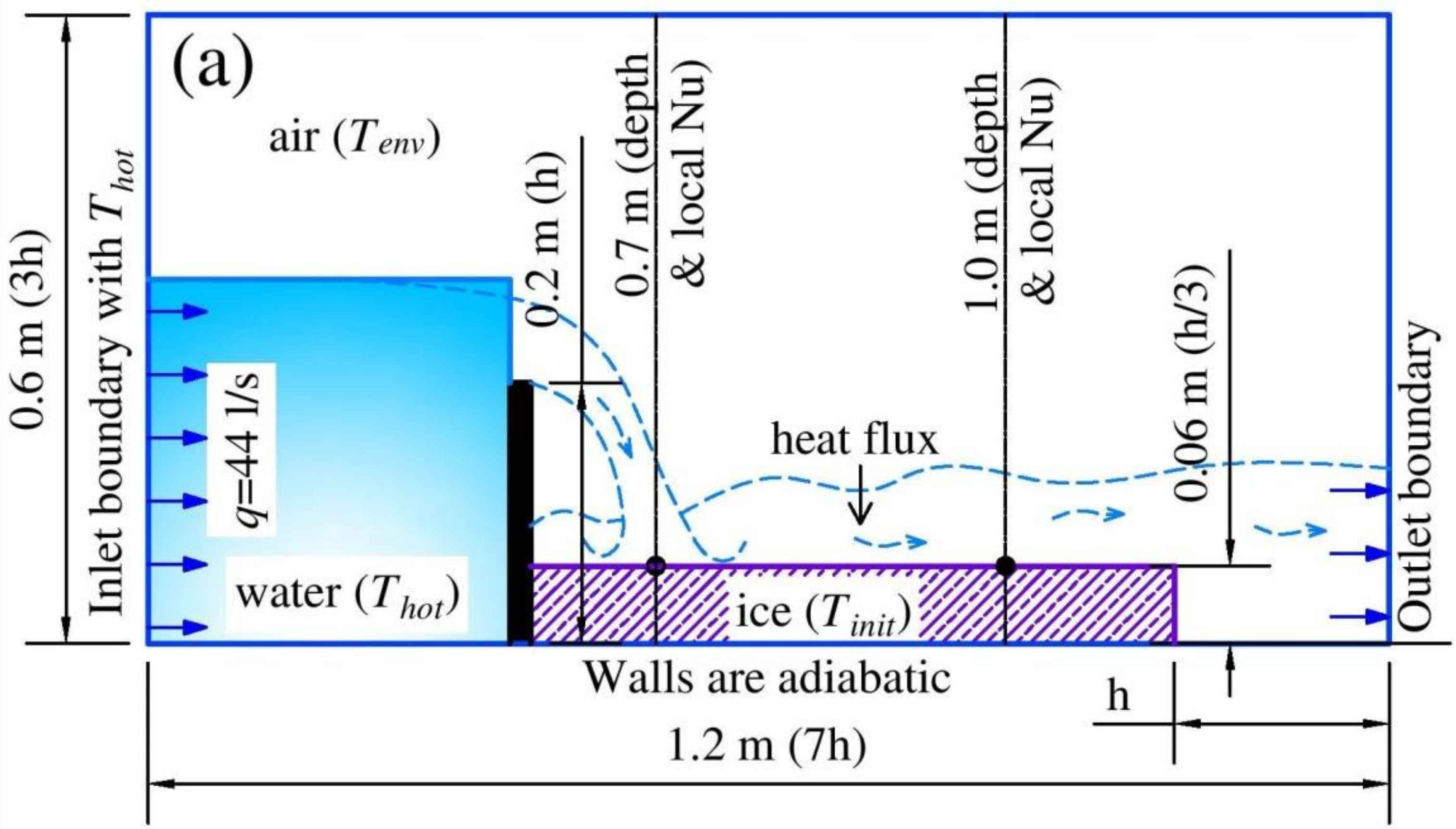


Data: 4.5 cm cube ice with -30°C , 30°C pouring water, 25°C ambient air,

Notes: The results are not included in the paper. FS – free surface



Melting of motionless ice in FS flow



Notes:

Melting of motionless ice in FS flow

Initial temperatures

$T_{ice} = -30°C$

$T_{wat} = 30°C$

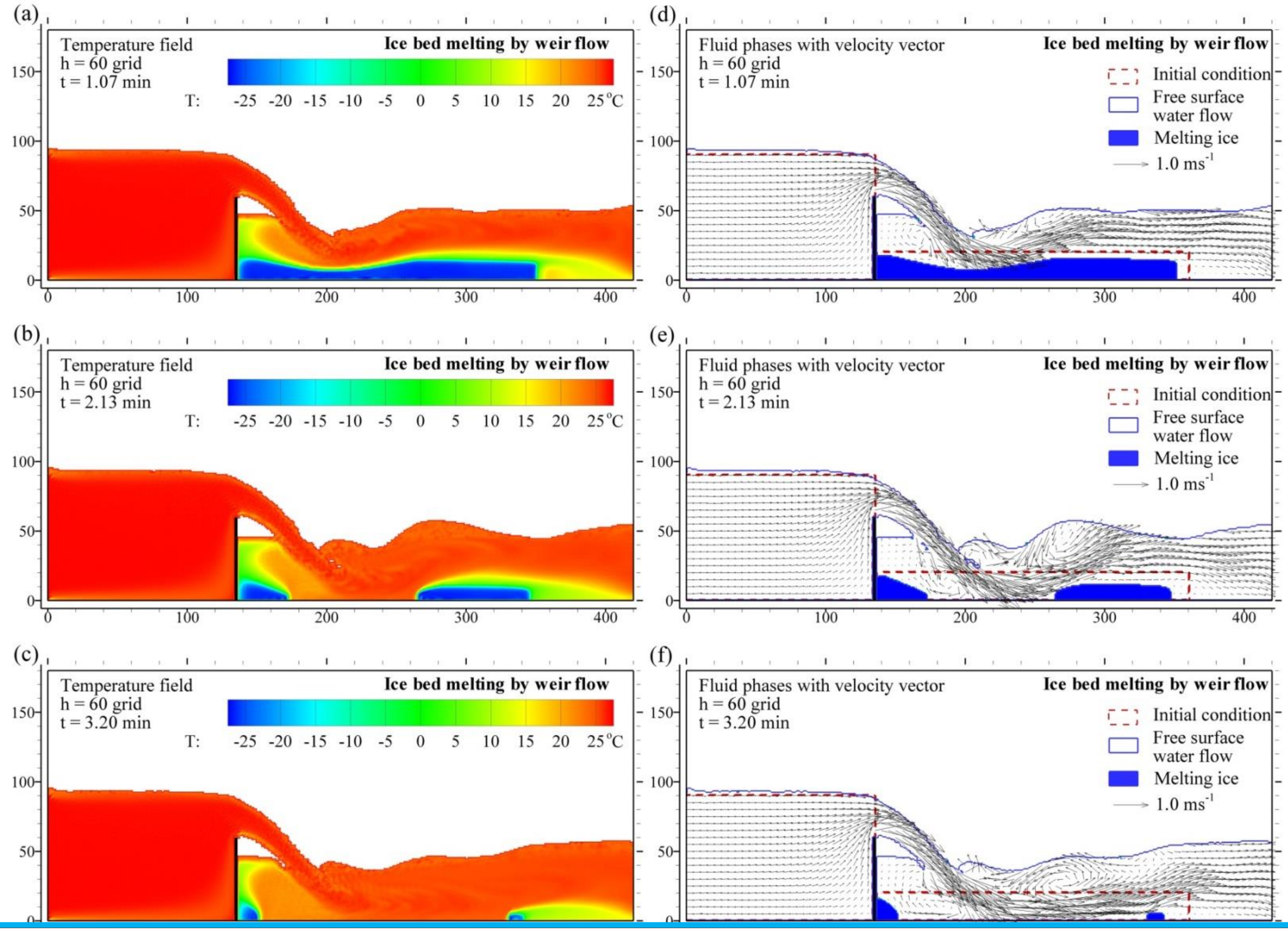
$T_{air} = 20°C$

Some facts:

$h_c = 648 - 1188$

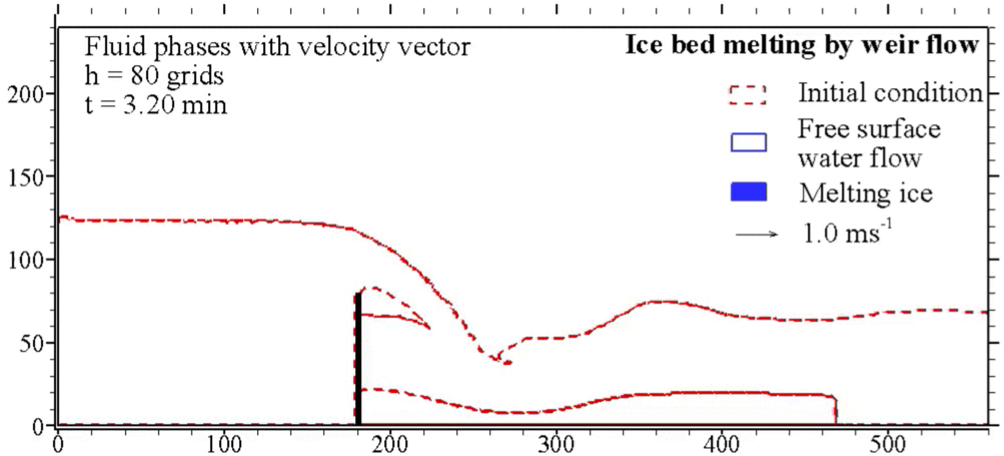
$WK^{-1}m^{-2}$

$M_{rate} \approx 42\ kg/min$



Notes:

Melting of motionless ice in FS flow



Some parameters:

Δx = 2.5e−3 m

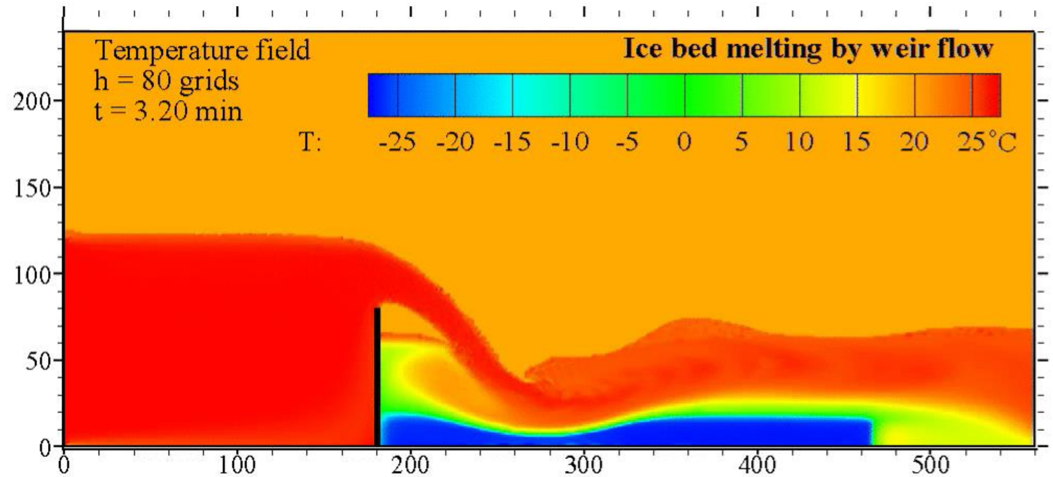
Δt = 5.41e−3 s

Pr = 11.59

Ra = 10^5

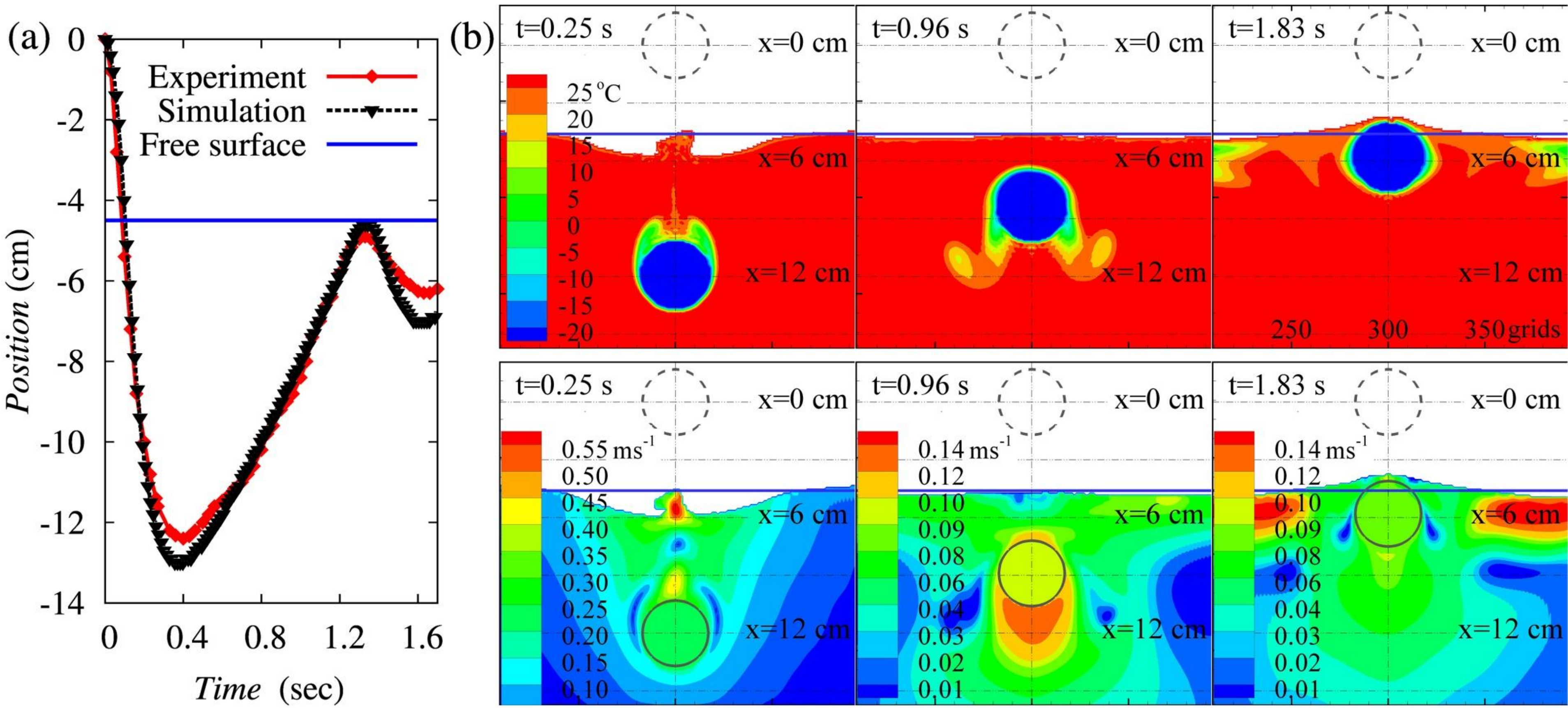
Fr = 0.13

Nappe flow plays an important role in melting of ice bed. Therefore heat transfer and phase changes are strongly induced by flow velocity in open channel flow.



Notes:

Floating of an ice cylinder

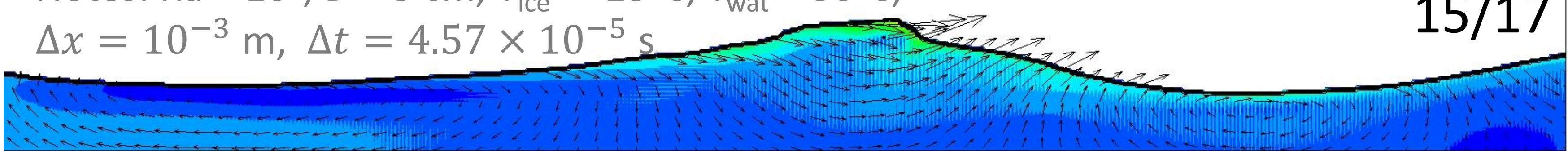


Displacement

Temperature field and Flow field

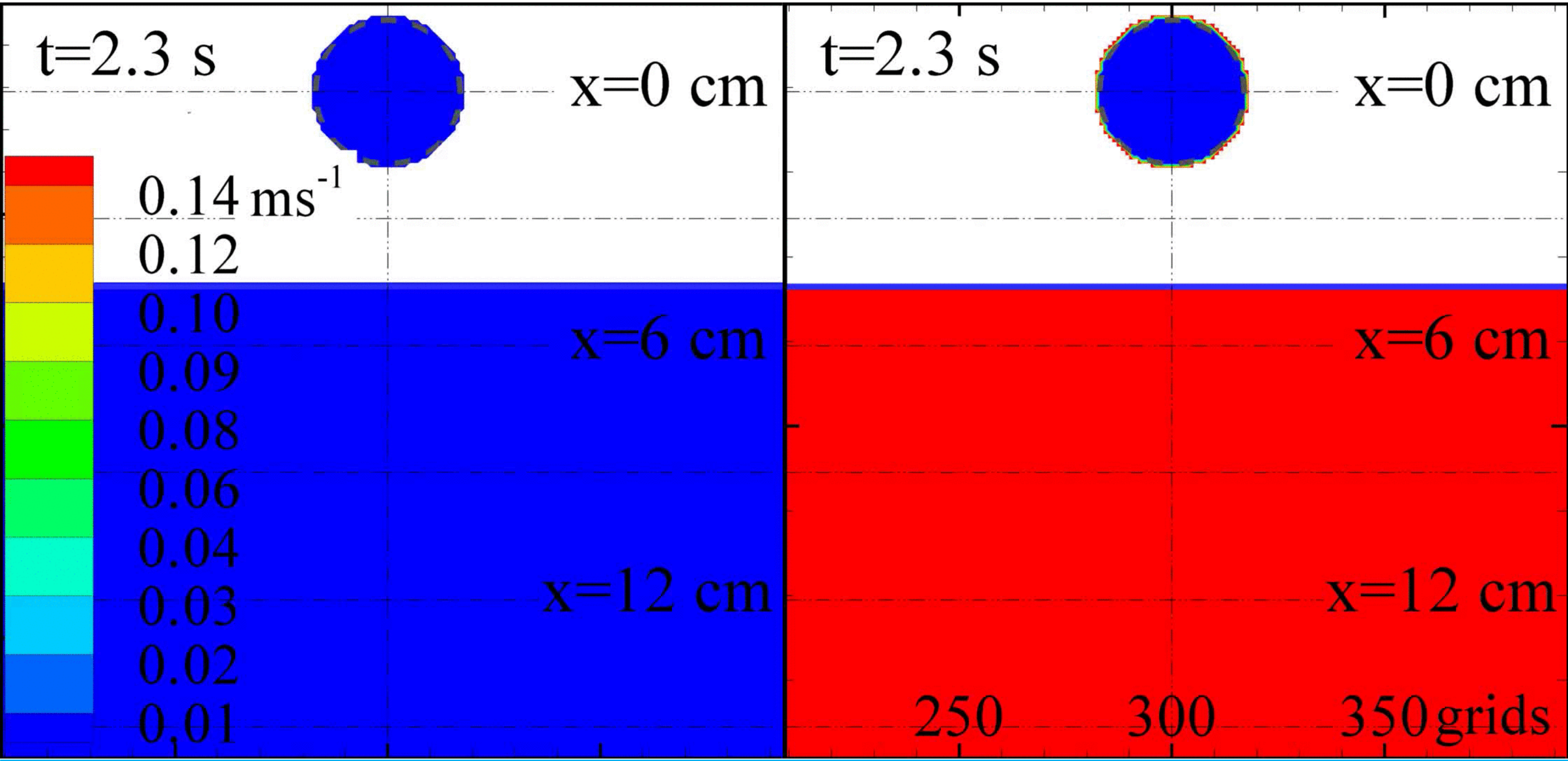
Notes: Ra = 10^4, D = 3 cm, T_{ice} = -25°C, T_{wat} = 30°C,

$\Delta x = 10^{-3}$ m, $\Delta t = 4.57 \times 10^{-5}$ s



Floating of an ice cylinder

Velocity field | Temperature field

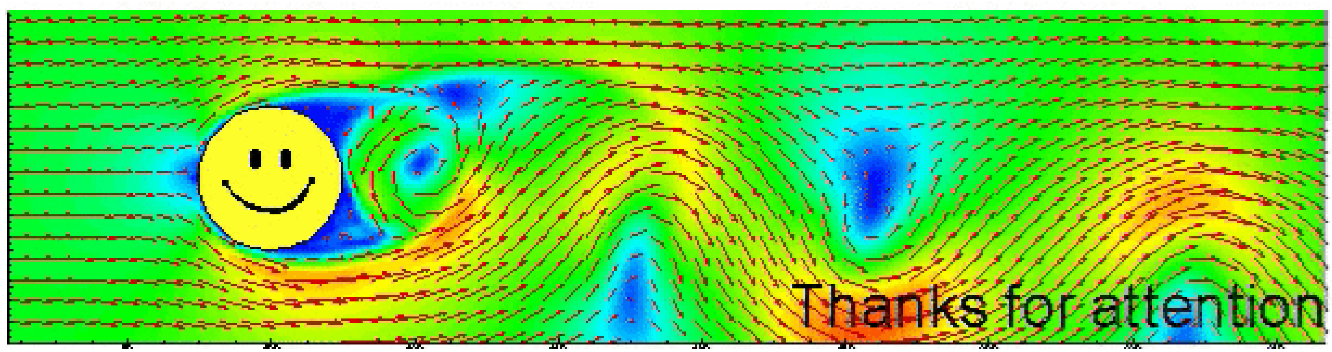


Notes: Ra = 10^4, D = 3 cm, T_{ice} = -25°C, T_{wat} = 30°C,
$\Delta x = 10^{-3}$ m, $\Delta t = 4.57 \times 10^{-5}$ s

Conclusions and Future works

- Thermal – Free surface – Immersed boundary – lattice Boltzmann method
- Liquid-solid phase transitions
- Time-dependent arbitrary shaped floating or immersed bodies
- LBM is applicable for hydraulics problem (All in 1 LBM concept)

- 3D numerical model and parallelization
- Multi-body interactions
- More validations



Notes: In logo, ice, sediment, free surface are depicted. Contact: ayur_426@yahoo.com