Time Variation of Scour at Downstream Pier for Two Piers in Tandem Arrangement

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Introduction

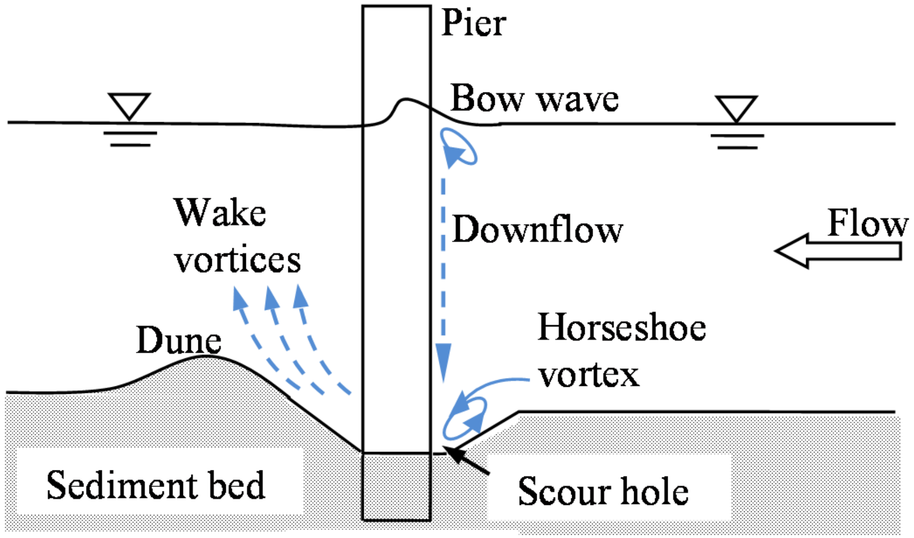

Figure: Definition sketch of pier scour showing the flow field.

Local scour can be defined as erosion of bed elevation in the vicinity of an obstacle owing to the removal of bed material by the erosive action of flowing water.

Types of local scour: clear-water scour and live bed scour.

- **Two Piers in Tandem Arrangement:**
  - * The existence of parallel railway-bridge
  - * Road-bridge
  - * A newly constructed bridge by the side of an old bridge gives rise to a situation.

- Failure of bridges due to local scour at pier foundations is a common occurrence.

- Undermining of structures because of scour around them, is still a challenging problem to civil, hydraulic, bridge engineers, and etc.

- Mode of sediment transport: as suspended load, bed load, saltation load, and wash load.

The State-of-the-Art

Author	year	Key parameters	Measurement technique	Quantities measured
Chabert and Engeldinger	1956	single pier	-	Studied the time-variation of scour at a single bridge pier.
Dey S	1999	Conservation of mass of sediment	ADV system	developed a theoretical model for the time variation of scour depth in an evolving scour hole at circular piers.
Dey et al.	2005	$l/h \leq 1$	Scale fitted to the inside of the abutment and ADV	developed a numerical model for the time variation of scour depth at short abutments.
Lu et al.	2011	$U/U_c \leq 0.9$	ADV system	proposed a semi-empirical model to compute the temporal variation of scour depth.

Author	year	Key parameters	Measurement technique	Quantities measured
Kothyari et al.	2012	$U/U_c \leq 0.92$	electronic bed profile indicator MKV and ADV	developed a mathematical model to compute the temporal variation of scour depth.
Khaple et al.	2017	$U/U_c \leq 0.9$	Point gage	Studied the time-variation of scour at two piers in tandem arrangement experimentally.

Comprehensive survey on the topic of time-variation of scour at a single pier was done by Raudkivi and Ettema (1983), Yanmaz and Altinbilek (1991), Dey (1999), and Dey and Raikar (2007).

Research Gap

Previous research works were just focused on obtaining an analytical solution for time variation of scour at an isolated pier.

However, time variation of scour caused by two piers in tandem arrangement is seems to be inadequate.

Mathematical Model

Model Background

The semiempirical model of the temporal variation of scour at two piers in tandem arrangements is derived based on the following assumptions:

(a) The primary horseshoe vortex at upstream face of the pier base is the principal agent of scouring,

(b) Sediment particles are removed from the upstream flat semi-circular zone where the maximum equilibrium scour depth occurs,

(c) The scour profiles are geometrically similar with the time as scouring progresses layer by layer,

(d) Rate of sediment deposition in the scour hole is equals to the difference between the sediment mass outflow from the scour hole and the sediment mass inflow rate into the scour hole by the approaching flow.

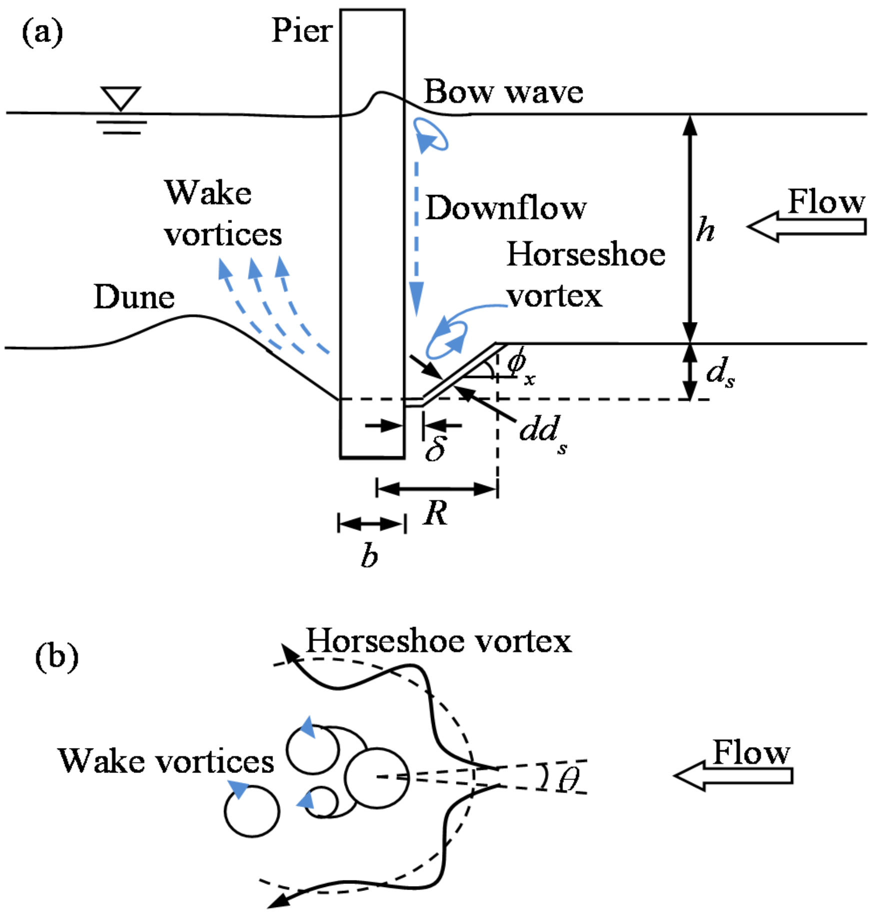

Figure: Definition sketch of scouring at a pier: (a) elevation view; (b) top view.

Formulation of Model

The mass rate of sediment picked up from the flat semicircular region during a small interval of time dt is

$$dm_1 = 0.5\theta\delta\left(\delta + b\right)E\mathrm{d}t \tag{1}$$

The width of the flat semicircular region δ can be expressed as

$$\delta = \varepsilon\left(R - 0.5b\right) \tag{2}$$

where ε is a geometric factor, and R is the radius of scour hole, thus R is given by

$$R = \left[\frac{d_s}{\left(1-\varepsilon\right)}\right]\cot\phi_x + 0.5b \tag{3}$$

substituting Eqs. (2) and (3) in Eq. (1), yields

$$dm_1 = 0.5\theta \frac{\varepsilon}{1-\varepsilon} d_s \cot\phi_x \left(\frac{\varepsilon}{1-\varepsilon} d_s \cot\phi_x + b \right) E\mathrm{d}t \qquad (4)$$

The sediment pick up rate E at the base of the pier due to scouring, determined using the equation suggested by van Rijn (1984), is given by

$$E = 0.00033\rho_s \left(\Delta g d_{50}\right)^{0.5} D_*^{0.3} T_s^{1.5} \qquad (5)$$

T_s is the transport-stage parameter due to scouring, that is $\left(\tau_b - \tau_{bc}\right)/\tau_{bc}$

The local bed shear stress at the pier base is calculated from the empirical formula given by Kothyari *et al*. (1992) as

$$\tau_b = 4\tau_0 \left[\left(\frac{2}{\pi}\right) \left(\frac{d_s}{d_v}\right)^2 \cot\phi_x + 1 \right]^{-0.57} \qquad (6)$$

d_v is the diameter of the horseshoe vortex at the beginning of scouring.

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d_v is determined from the empirical formula proposed by Kothyari *et al.* (1992) as

$$d_v = 0.28 h^{0.15} b^{0.85} \tag{7}$$

The bed shear stress of the approaching flow can be expressed as a function of dynamic pressure due to an average velocity U of flow as

$$\tau_0 = \frac{\lambda_D}{8} \rho U^2 \tag{8}$$

where λ_D is the Darcy-Weisbach friction factor.

The value of λ_D can be determined using an explicit form of the Colebrook-White equation was given by Haaland (1983),

$$\frac{1}{\lambda_D^{0.5}} = -0.782 \ln \left[\left(\frac{k_s P}{14.8 A} \right)^{1.1} + \frac{6.9}{R_e} \right] \tag{9}$$

Average velocity U of flow downstream of the upstream pier is determined, as a function of spacing between two piers, as

$$\frac{U}{U_a} = 0.2053 + 0.1491\left(\frac{S}{b}\right) - 0.0078\left(\frac{S}{b}\right)^2 \tag{10}$$

Applying the concept of geometrical similarity of the scour profiles with time, the reduction of the sediment mass is expressed as given below:

$$dm_2 = -0.5\left(1-\rho_0\right)\rho_s\theta \times$$

$$\left\{\delta\left(\delta+b\right) + \cot\phi_x \times \left[\left(R^2 - 0.25b^2\right) - \delta\left(\delta+b\right)\right]\right\} \mathrm{d}d_s \tag{11}$$

The right-hand side of the above equation is negative due to the reduction of sediment mass.

Substituting Eqs. (2) and (3) into Eq. (11), yields

$$dm_2 = -0.5\left(1-\rho_0\right)\rho_s\theta\frac{d_s}{1-\varepsilon}\times \left[\varepsilon\cot\phi_x\left(\frac{\varepsilon}{1-\varepsilon}d_s\cot\phi_x + b\right) + \frac{\left(1+\varepsilon\right)d_s\cot\phi_x + \left(1-\varepsilon\right)b}{\sin\phi_x}\right]\mathrm{d}d_s \qquad (12)$$

The fundamental equation to describe the scouring process can be obtained from the basic concept of conservation of the mass of sediment as

$$dm_1 + dm_2 = 0 \qquad (13)$$

Substituting Eqs. (4) and (12) in to Eq. (13) to obtain the following first-order differential equation of temporal variation of scour depth at downstream of pier when two piers are in tandem arrangements in nondimensional form as:

$$(1-\rho_0)\left[\varepsilon\cot\phi_x\left(\frac{\varepsilon}{1-\varepsilon}\hat{d}_s\cot\phi_x+1\right)+\frac{(1+\varepsilon)\hat{d}_s\cot\phi_x+(1-\varepsilon)}{\sin\phi_x}\right]\frac{\mathrm{d}\hat{d}_s}{\mathrm{d}\hat{t}}$$

$$=\varepsilon\cot\phi_x\left(\frac{\varepsilon}{1-\varepsilon}\hat{d}_s\cot\phi_x+1\right)\hat{b}\phi_p \qquad (14)$$

where $\hat{d}_s = d_s/b$, $\hat{t}$ = time parameter, that is , $td_{50}\sqrt{(\Delta g d_{50})}/b^2$,$\hat{b} = b/d_{50}$,
and ϕ_p = sediment pick up function due to scouring, that is
$E/\rho_s\sqrt{(\Delta g d_{50})}$

Equation (14) is a first-order differential equation, which can be solved by the fourth-order Runge-Kutta method to determine the variation of $\hat{d}_s$ with $\hat{t}$.

Results and Discussion

- In this study, the scour depth at the downstream pier is less than that of the upstream pier. As the distance between the piers increases, the scour depths at downstream pier decreases with time.

- Figures shows the temporal variation of $\hat{d}_s$ with $\hat{t}$ in uniform sediments under clear water scour condition for two piers arranged in tandem.

- The analytical results agree satisfactorily with the experimental data.

- It is observed that the computed value of sediment pick-up rate E from the scour hole area is inversely proportional to the scour depth d_s.

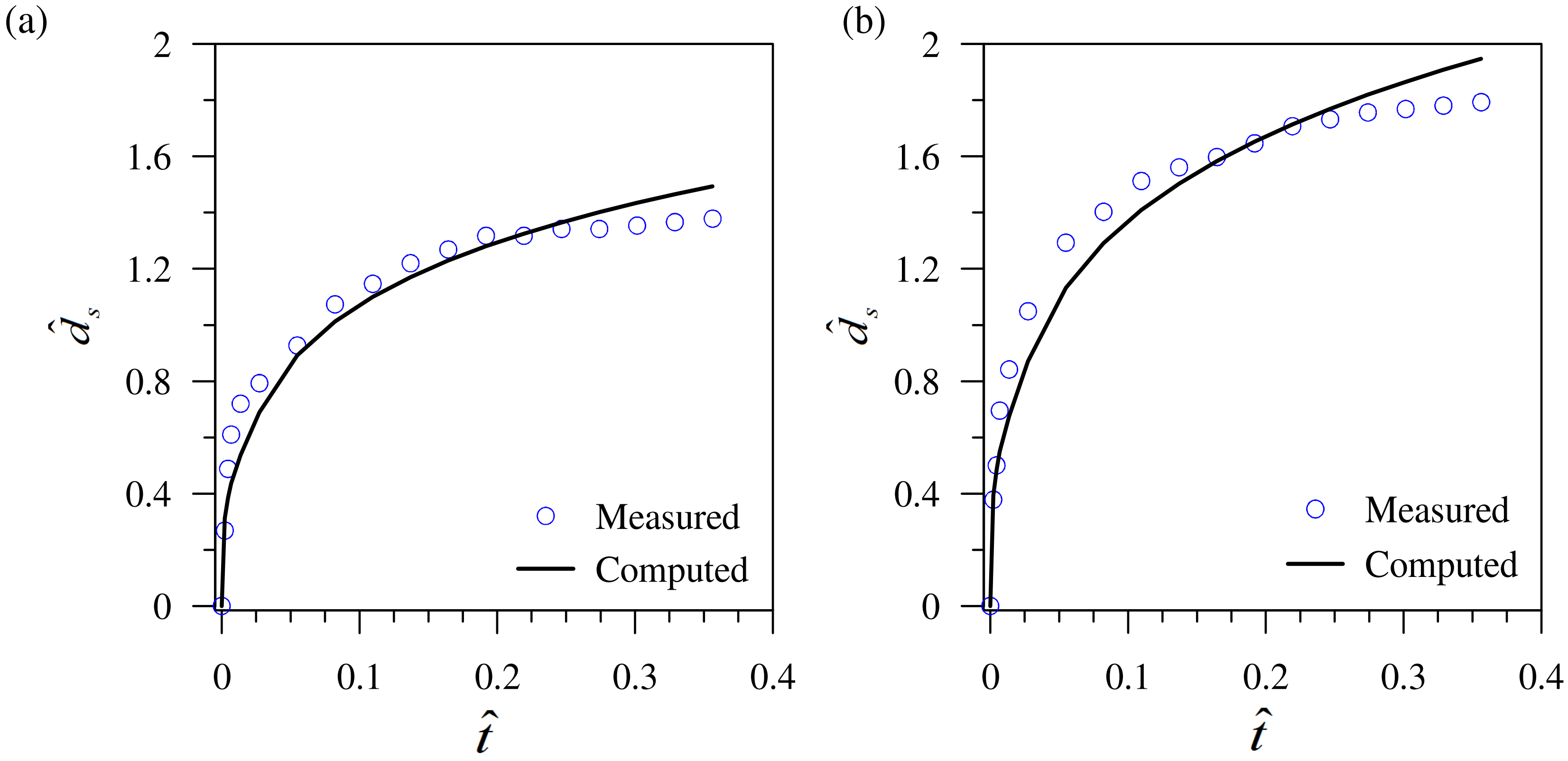

Figure: Temporal variation of scour depth at downstream of piers in tandem pier arrangements: (a) $S/b = 3$ and (b) $S/b = 5$

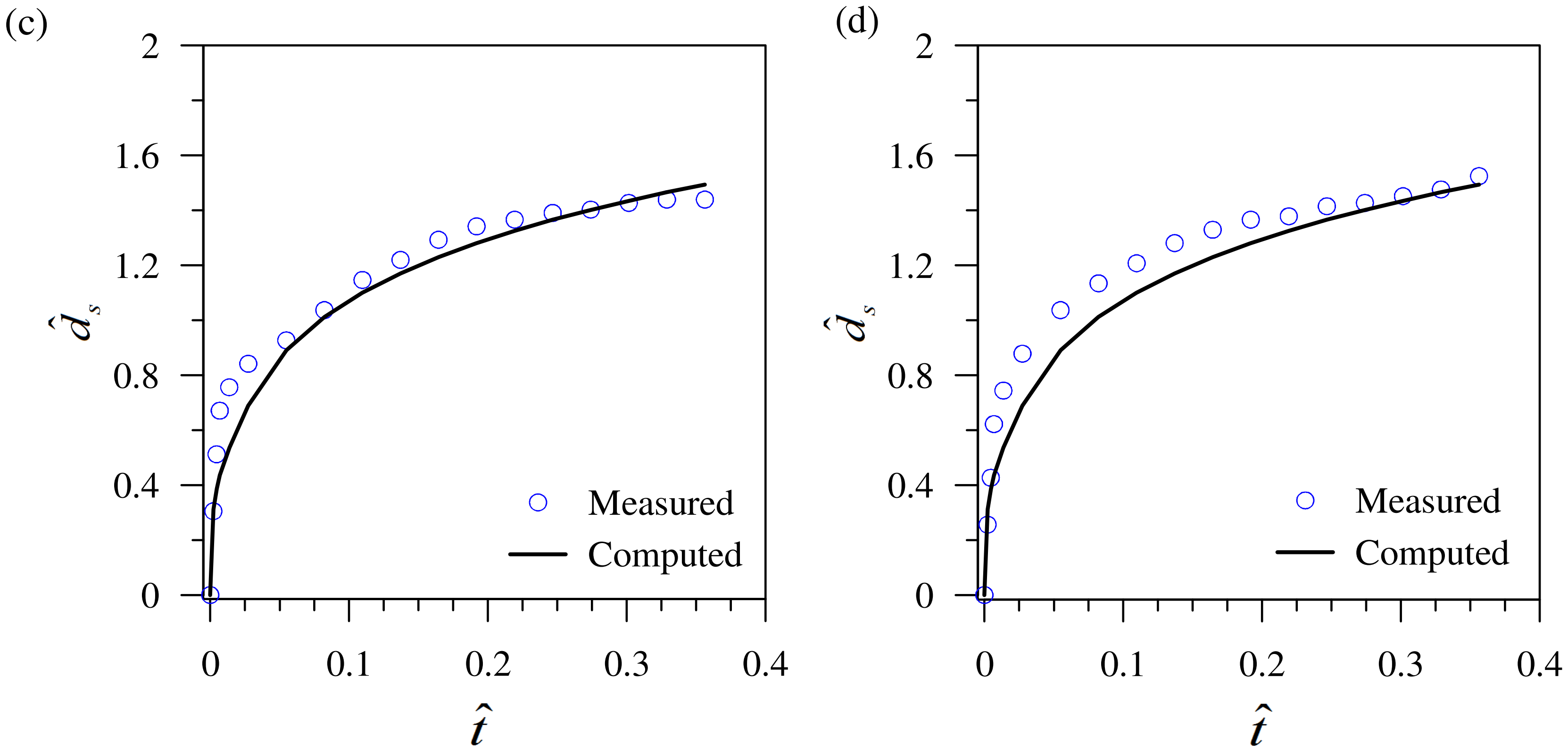

Figure: Temporal variation of scour depth at downstream of piers in tandem pier arrangements: (c) $S/b = 7$ and (d) $S/b = 9$

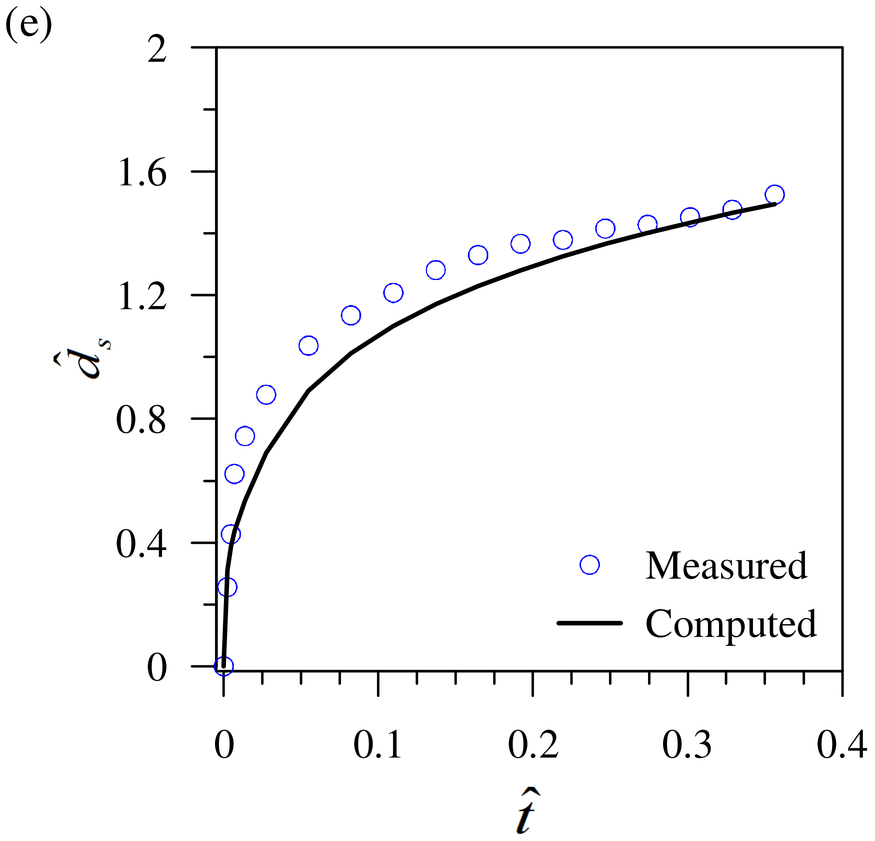

Figure: Temporal variation of scour depth at downstream of piers in tandem pier arrangements: (e) $S/b = 11$

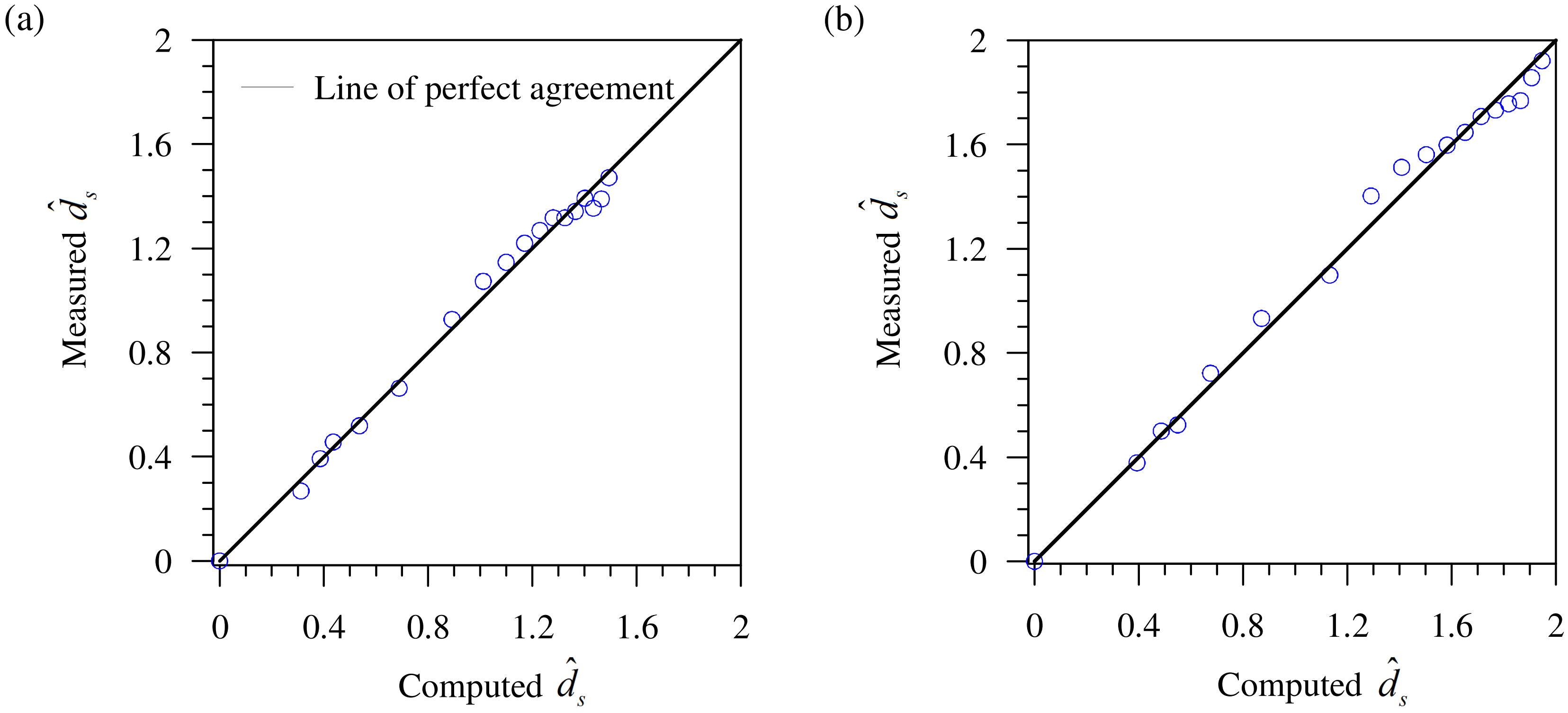

Figure: Comparison of computational data with measured data: (a) $S/b = 3$ and (b) $S/b = 5$

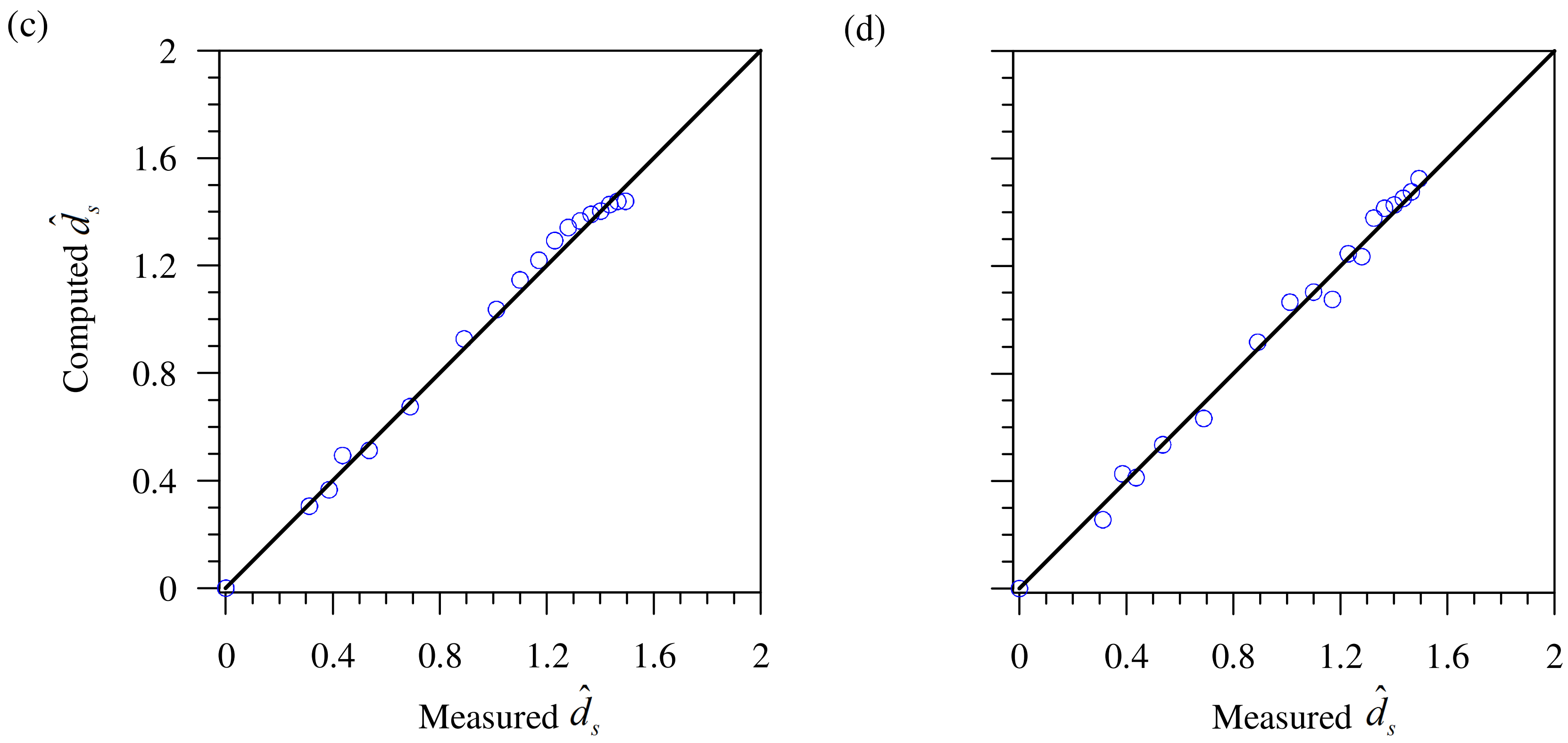

Figure: Comparison of computational data with measured data: (c) *S*/*b* = 7 and (d) *S*/*b* = 9

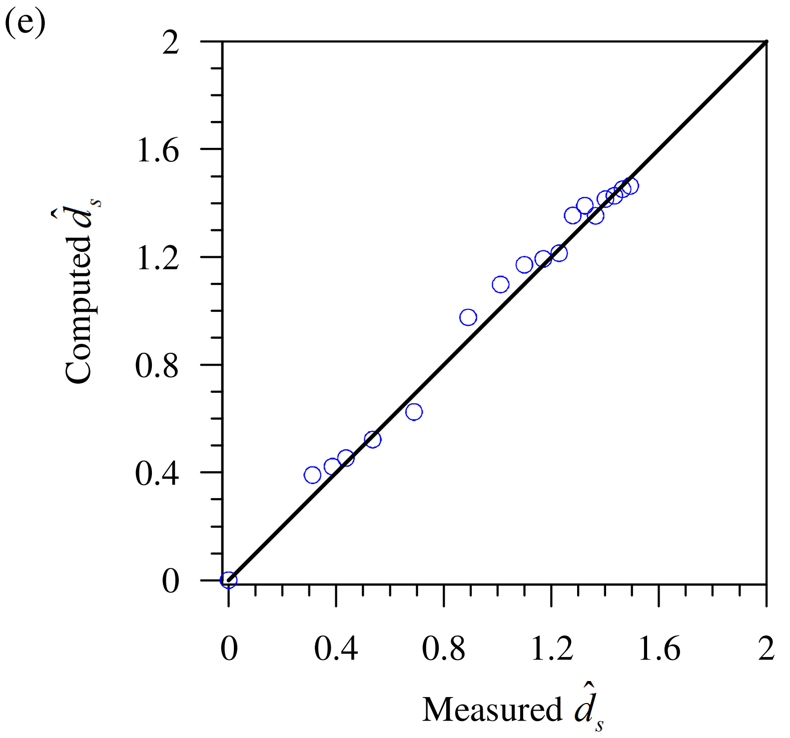

Figure: Comparison of computational data with measured data: (e) $S/b = 11$

Conclusions

The temporal variation of scour depth at piers in tandem arrangements under clear-water scour condition in uniform sediments has been analytically modeled. The primary findings are summarized as follows:

- The process of scouring at downstream pier has been defined using the concept of conservation of the mass of sediment that leads to a first-order differential equation, which has been solved numerically by the fourth-order Runge-Kutta method to compute the time-variation of scour depth.
- The temporal variation of scour depths computed using the present model is in reasonable agreement with the measured data.
- The analytical model solution can effectively be used by bridge engineers to design bridge pier foundations for the piers arranged in tandem arrangement.

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THANK YOU