



**Russian Academy of Sciences  
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# Mathematical modelling of sand-gravel bed evolution in one dimension

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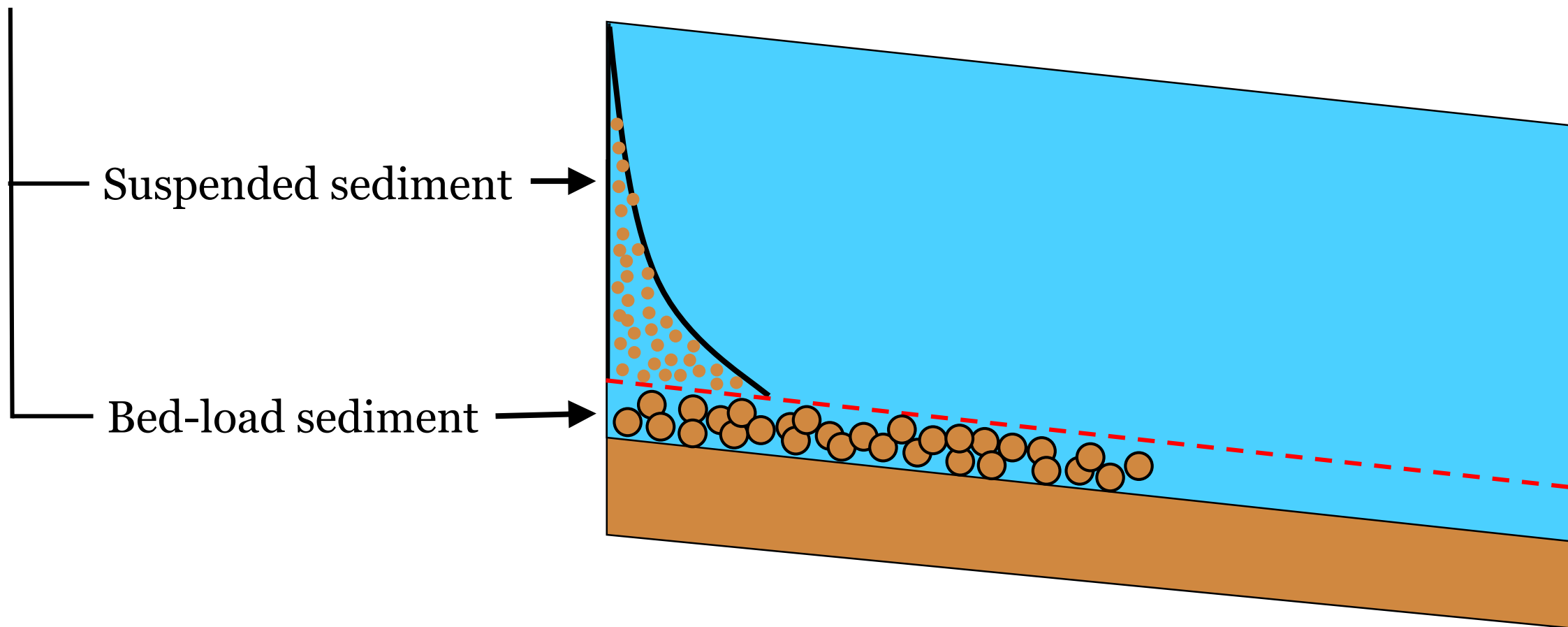
# Presentation outline

1. Introduction
2. The purpose of the research
3. Mathematical formulation
4. The model verification
5. Conclusion



# Introduction

Sediment



Bed-load sediment mostly forms local-scale bed shape



# Introduction

Bed-load transport models

[Shamov, 1959]

$$G = k \left( \frac{v_m}{v_s} \right)^3 (v_m - v_s) \left( \frac{d_{50}}{H} \right)^{1/4}$$

[Bagnold, 1966]

$$G = \frac{e_b}{\tan \alpha_*} \psi G$$

Phenomenological parameters

[van Rijn, 1986]

$$G = 0.1 H d_{50} u'_{*,c} \frac{\tau^{3/2}}{D_*^{0.3}}$$

[Wu, Wang and Jia, 2000]

$$\frac{G_k}{p_k \sqrt{\gamma_s (\gamma - 1) g d_k^3}} = 0.0053 \left[ \left( \frac{n'}{n} \right)^{3/2} \frac{\tau_b}{\tau_{ck}} - 1 \right]^{2.2}$$

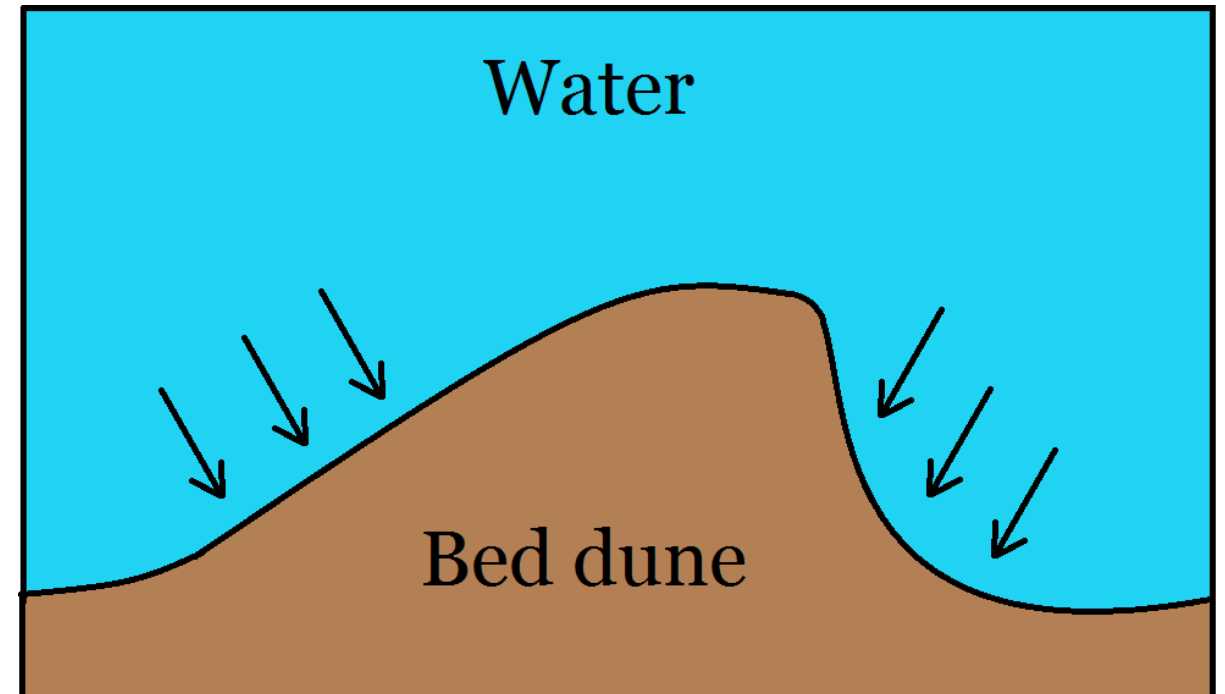


# The purpose of the research

To model accurately **local-scale** bed evolution, e.g.:

- degradation behind the dam;
- reservoir siltation;
- dunes development.

Analytical bed-load transport model





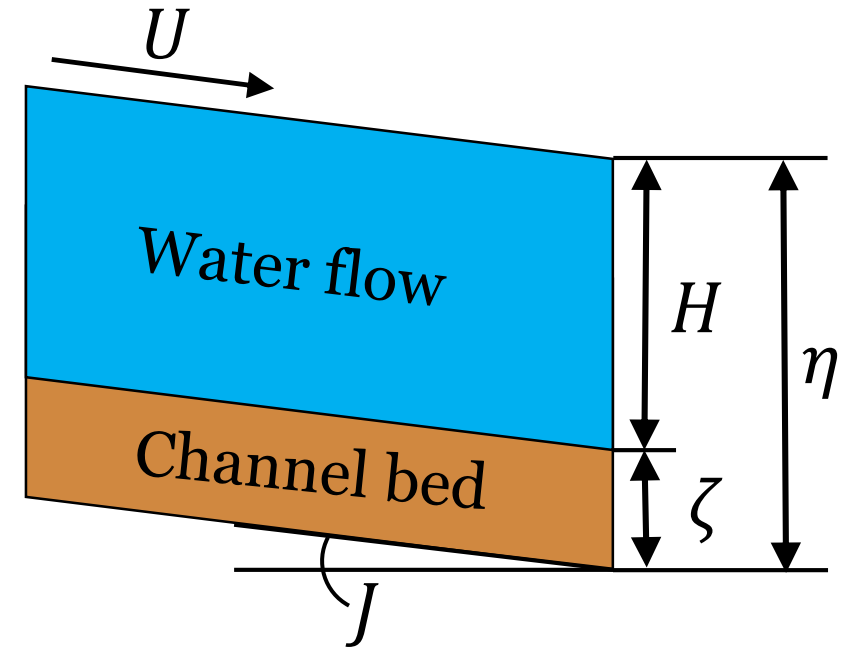
# Mathematical formulation

The stationary hydrodynamic equations in shallow water approach [Grishanin, K.V., 1974]

$$(1) \quad \frac{\partial}{\partial x} \left( \frac{U^2}{2g} + \eta \right) + \frac{\tau}{gH\rho_w} = 0, \quad Q = UH,$$

The Chezy-Manning formula for bed shear stress  
[Grishanin, K.V., 1974]

$$(2) \quad \tau = \rho_w g \frac{U^2}{C^2}, \quad C = \frac{H^{1/6}}{n_s}, \quad n_s = \frac{H^{2/3} \sqrt{J}}{U}.$$

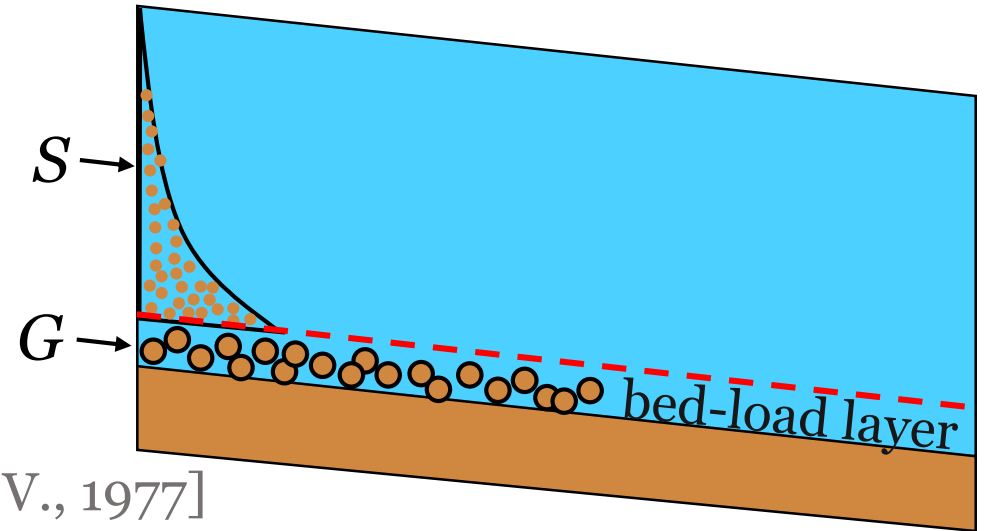


Where  $Q$  - the water rate,  $\tau$  - the bed shear stress,  $n_s$  - the Manning roughness coefficient.

# Mathematical formulation

The Exner equation [Exner, F.M., 1925]

$$(3) \quad (1 - \varepsilon)\rho_s \frac{\partial \zeta}{\partial t} + \frac{\partial G}{\partial x} = -\alpha \frac{W}{H} (S_* - S),$$



The suspended transport equations [Karashev, A.V., 1977]

$$(4) \quad \frac{\partial S}{\partial t} + \frac{\partial SU}{\partial x} = \alpha \frac{W}{H} (S_* - S), \quad S_* = \begin{cases} \beta \frac{U^3}{WH}, & W < u_*, \\ 0, & W \geq u_*; \end{cases} \quad u_* = \sqrt{\frac{\tau}{\rho_w}}.$$

Phenomenological parameters

Where  $\rho_s$  - the water density,  $\varepsilon$  - the bed material porosity,  $W$  - the particle fall velocity,  $\alpha \in (0; 1)$  - the adaptation coefficient,  $\beta = 0.2$ ,  $S_*$  - the suspended transport capacity



# Mathematical formulation

The analytical bed-load rate formula [Petrov, P.G., 1991]

Granulometrical and physical-mechanical parameters of the bed material

$$(5) \quad G = G_0 \tau^{3/2} \left[ (1 - \chi) - \left(1 - \frac{\chi}{2}\right) \frac{1}{\tan \varphi \cos \gamma} \frac{\partial \zeta}{\partial x} \right],$$

Local bed slope

$$(6) \quad G_0 = \frac{4}{3} \frac{\rho_s m}{\kappa \sqrt{\rho_w} (\rho_s - \rho_w) g \tan \varphi \cos \gamma}, \quad m = \begin{cases} 1, & \chi < 1, \\ 0, & \chi \geq 1; \end{cases}$$

Bed-shear stress

$$(7) \quad \chi = \sqrt{\frac{\tau_*}{\tau}}, \quad \tau_* = \frac{3}{8} \frac{\kappa^2 d (\rho_s - \rho_w) g \tan \varphi \cos \gamma}{c_x}.$$

Where  $\tau_*$  - the critical bed-shear stress,  $\varphi$  - the internal friction angle of particles,  $c_x$  - the frontal particle drag coefficient,  $\kappa$  - the Karmans constant.





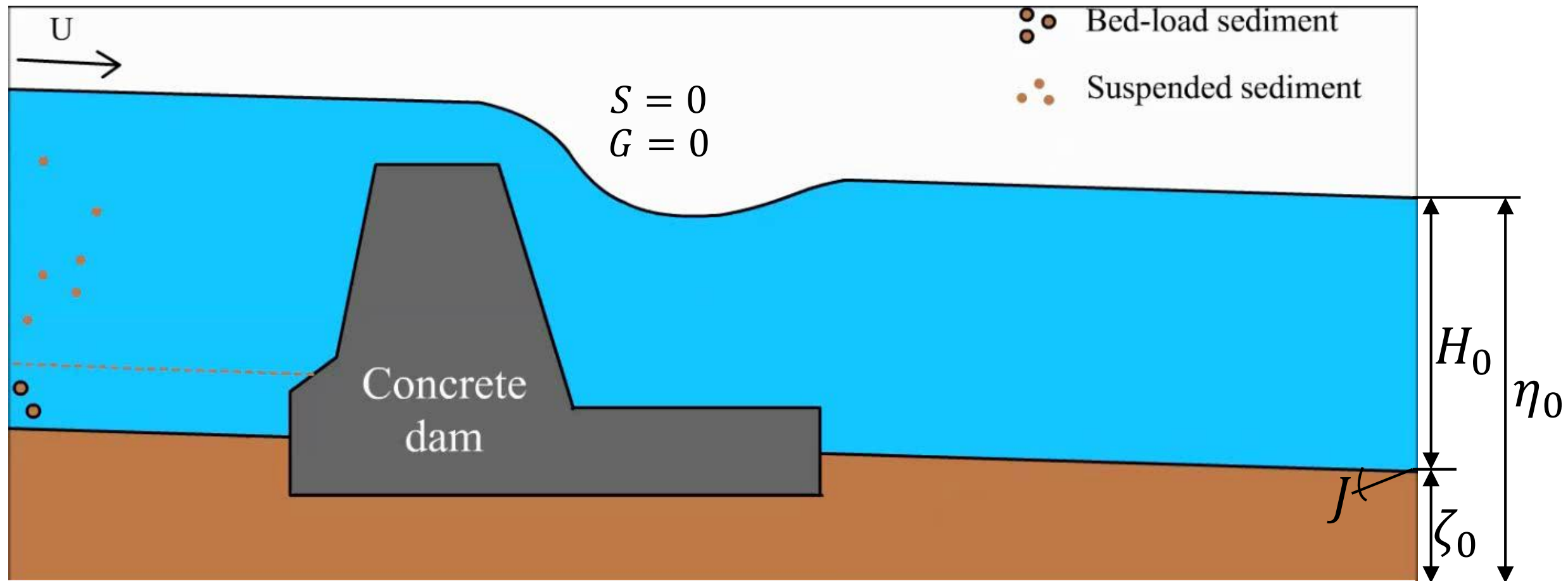
# The model verification

In order to estimate the ability of the model to describe local-scale bed evolution the next problems are solved:

- local bed degradation behind the dam,
- local bed aggregation,
- trench evolution.

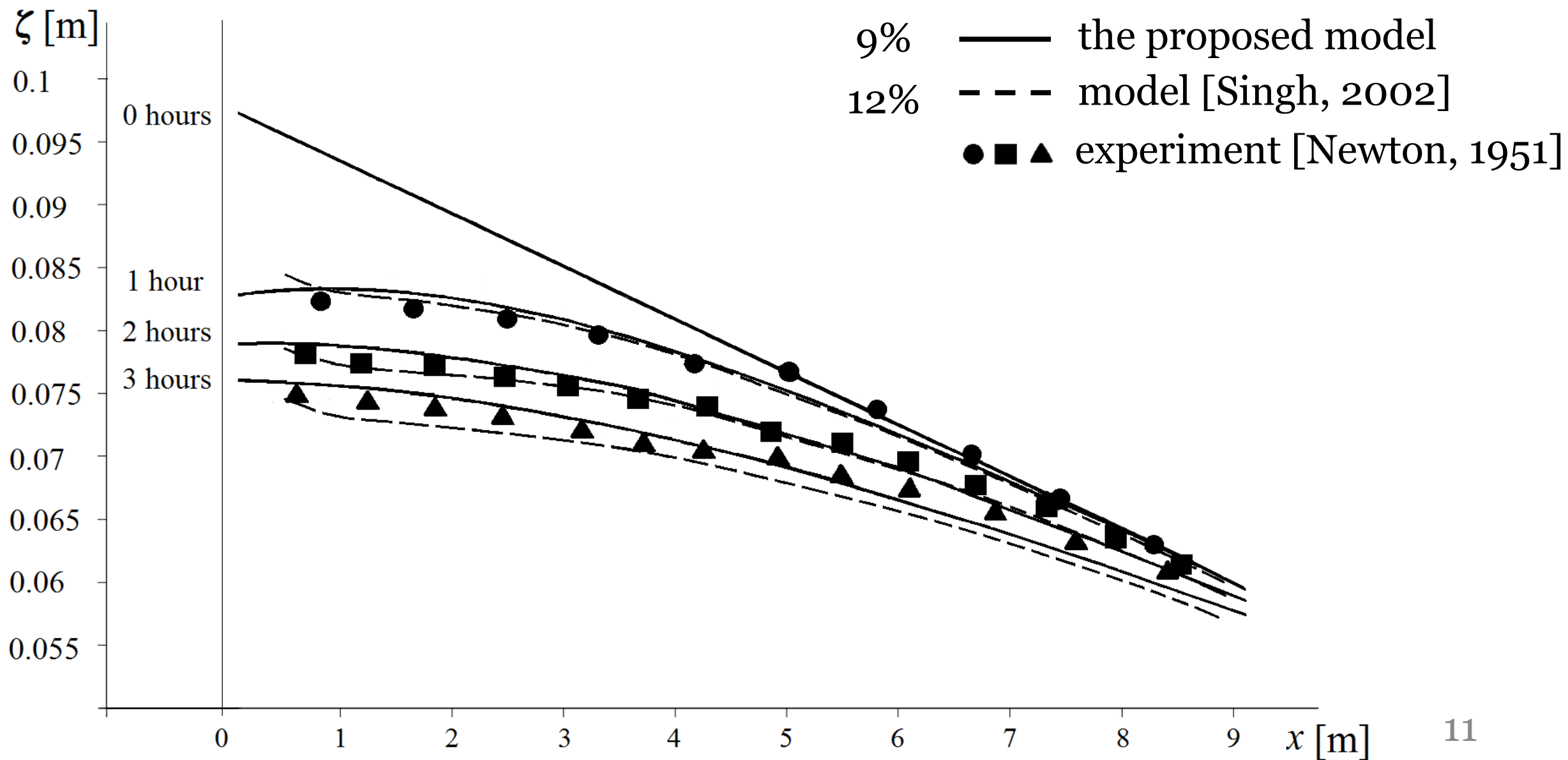


# Bed degradation



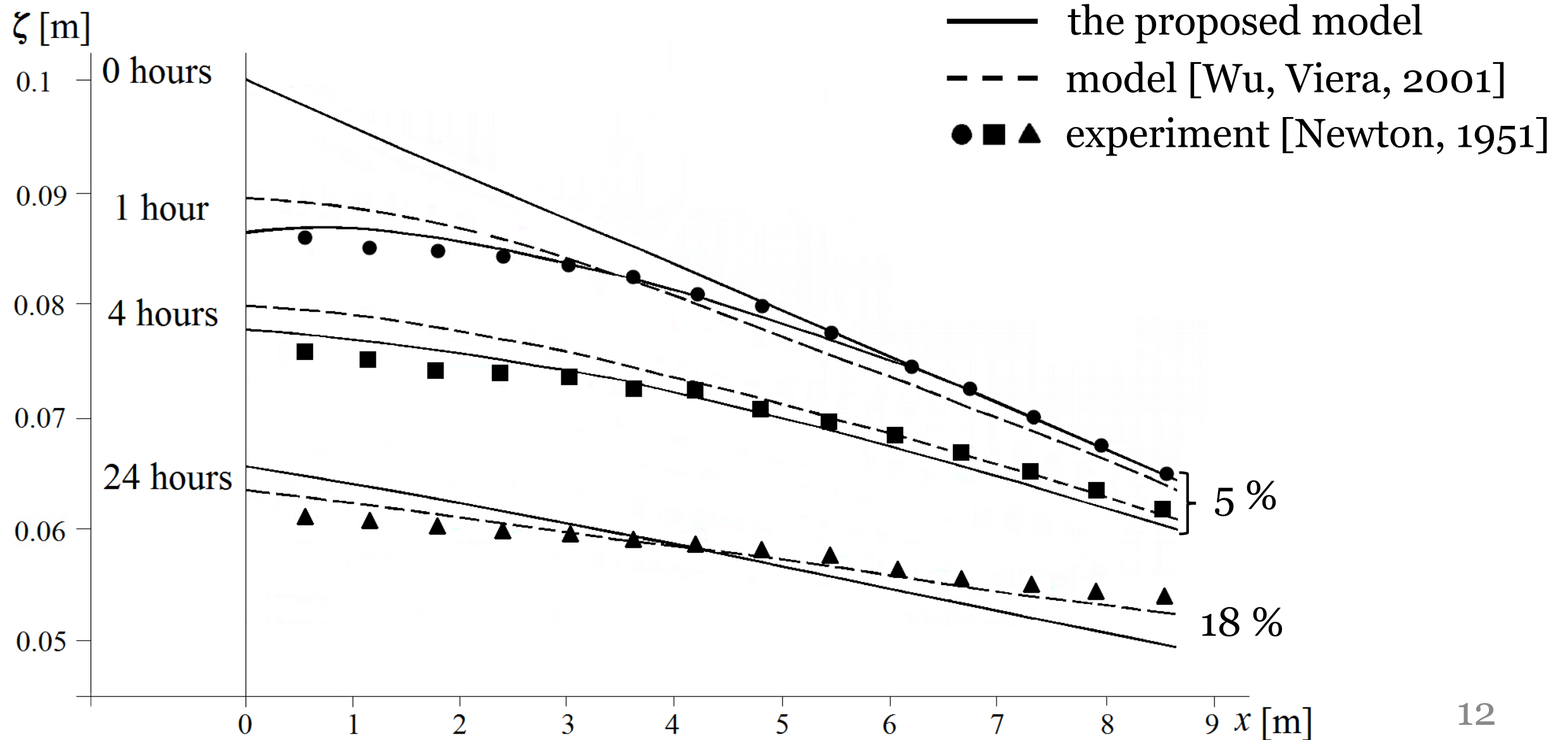


# Bed degradation



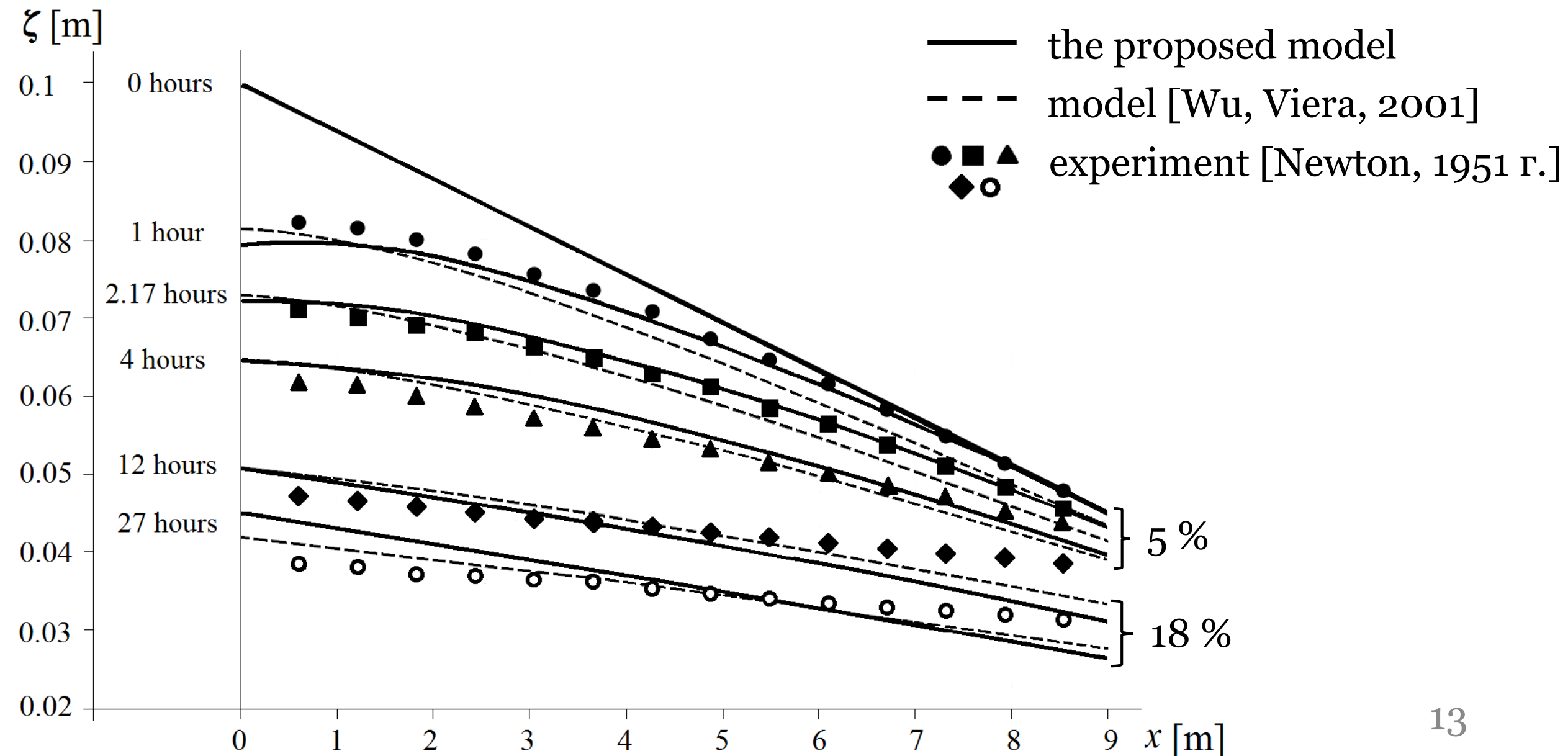


# Bed degradation



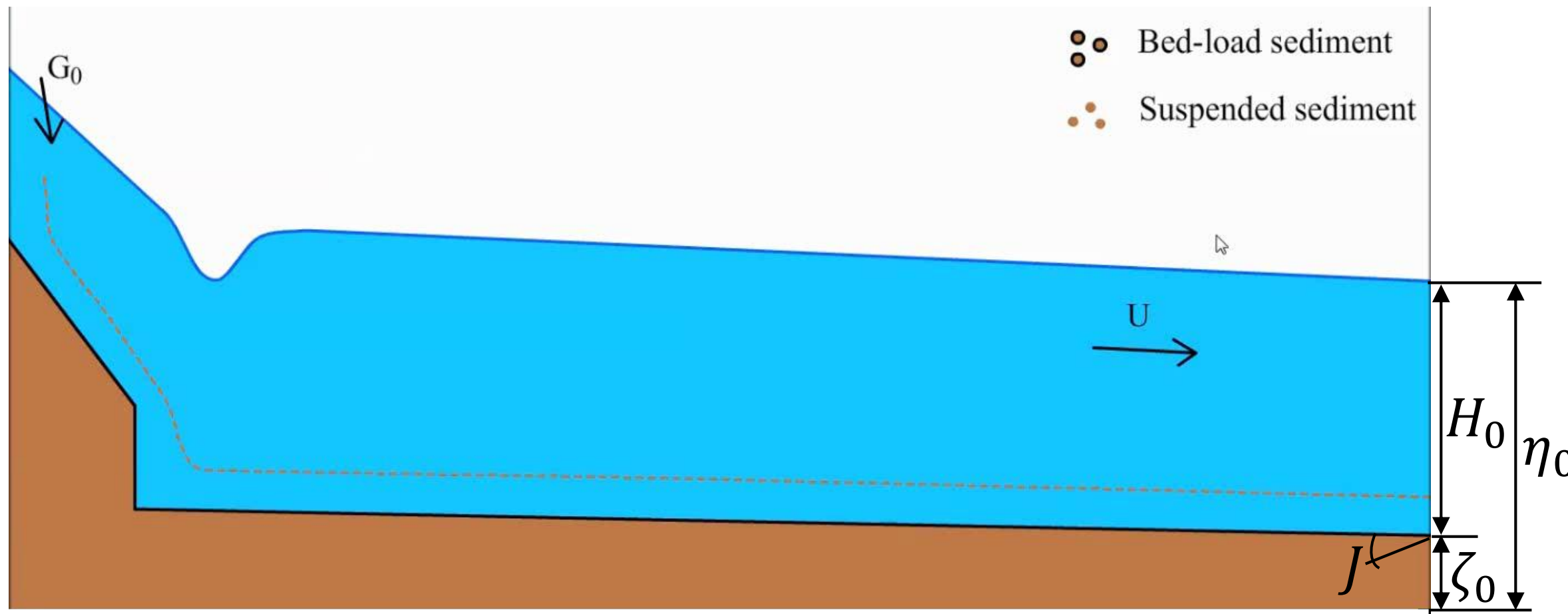


# Bed degradation



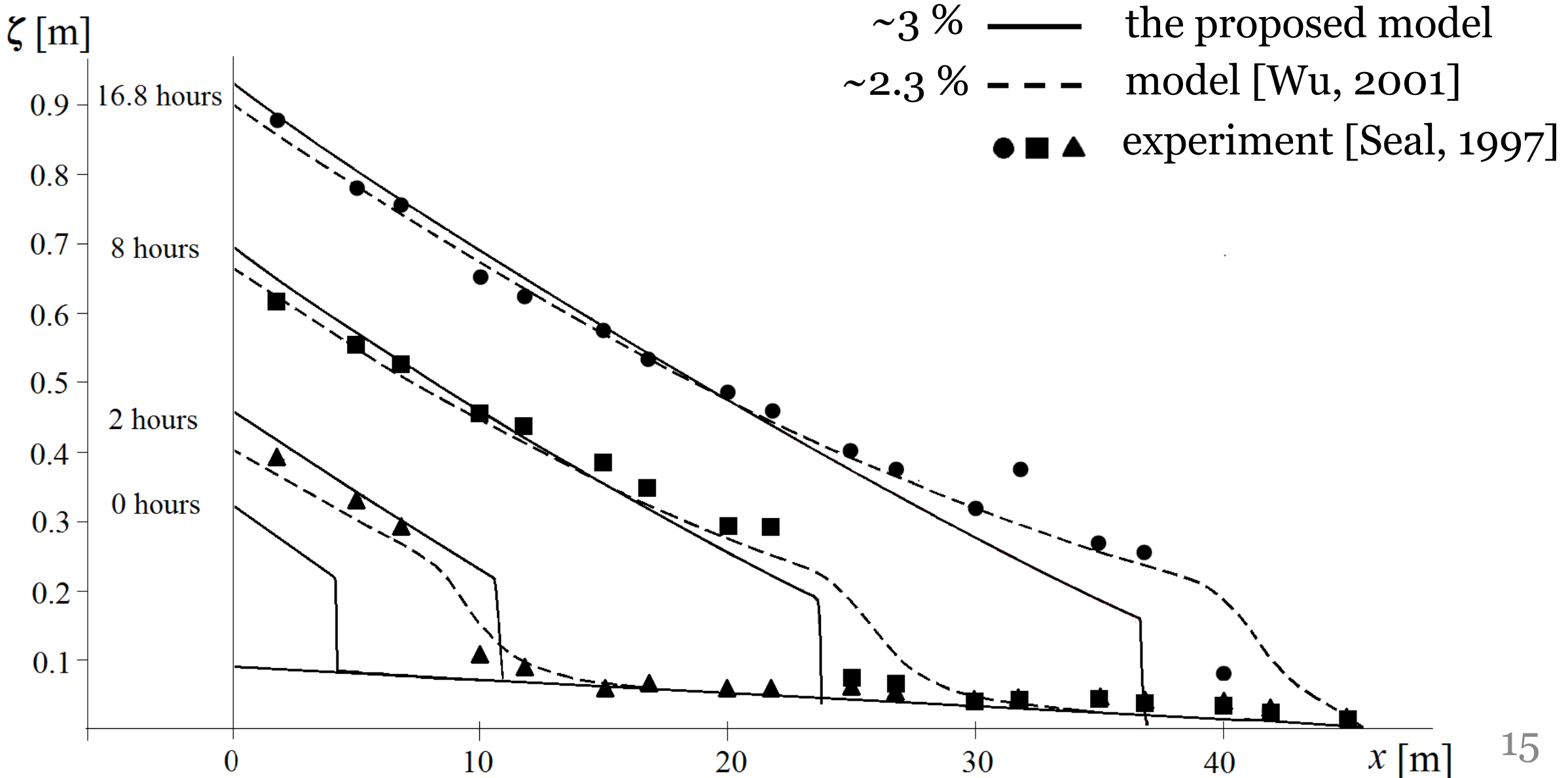


# Bed aggregation



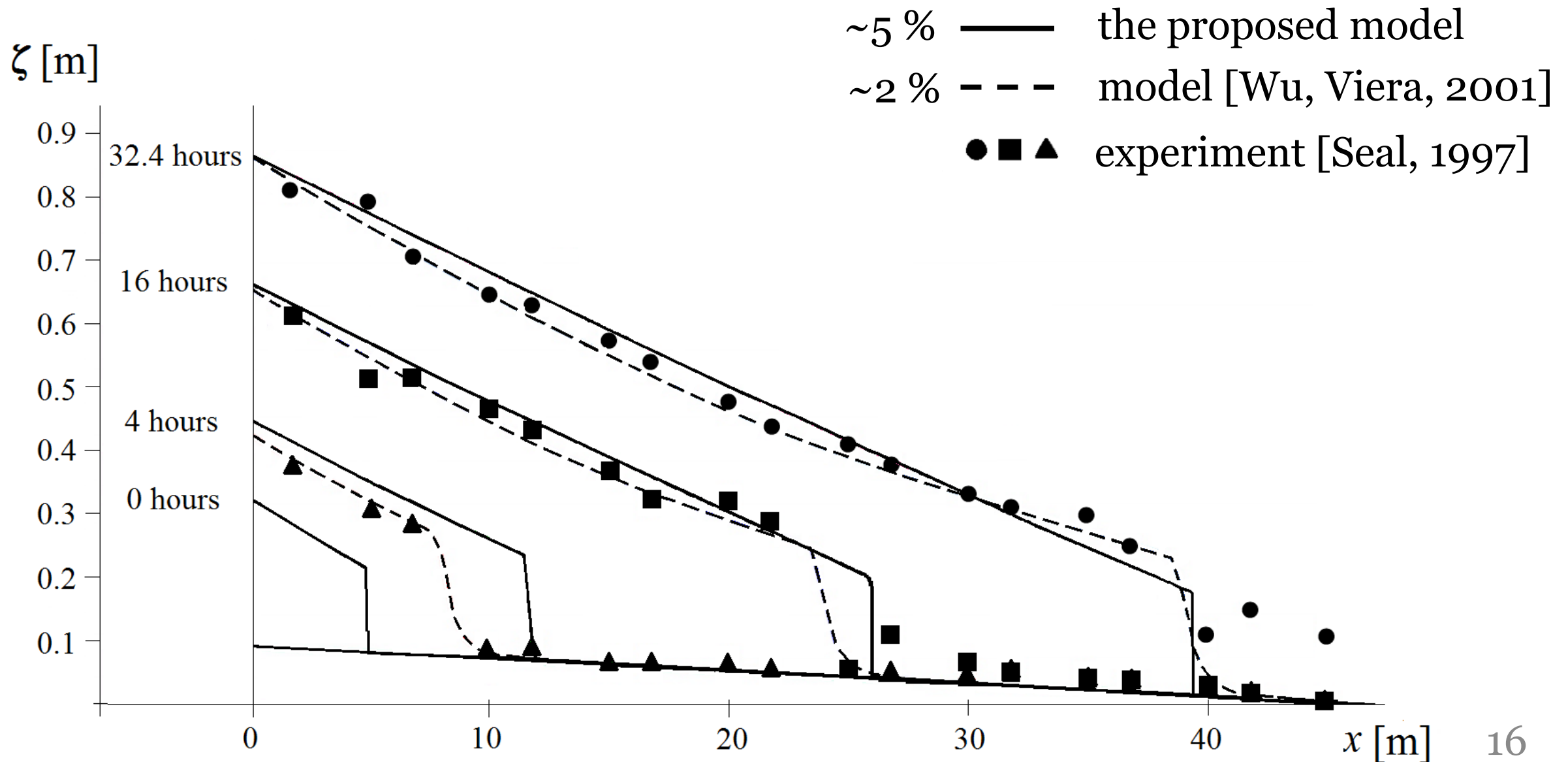


# Bed aggregation





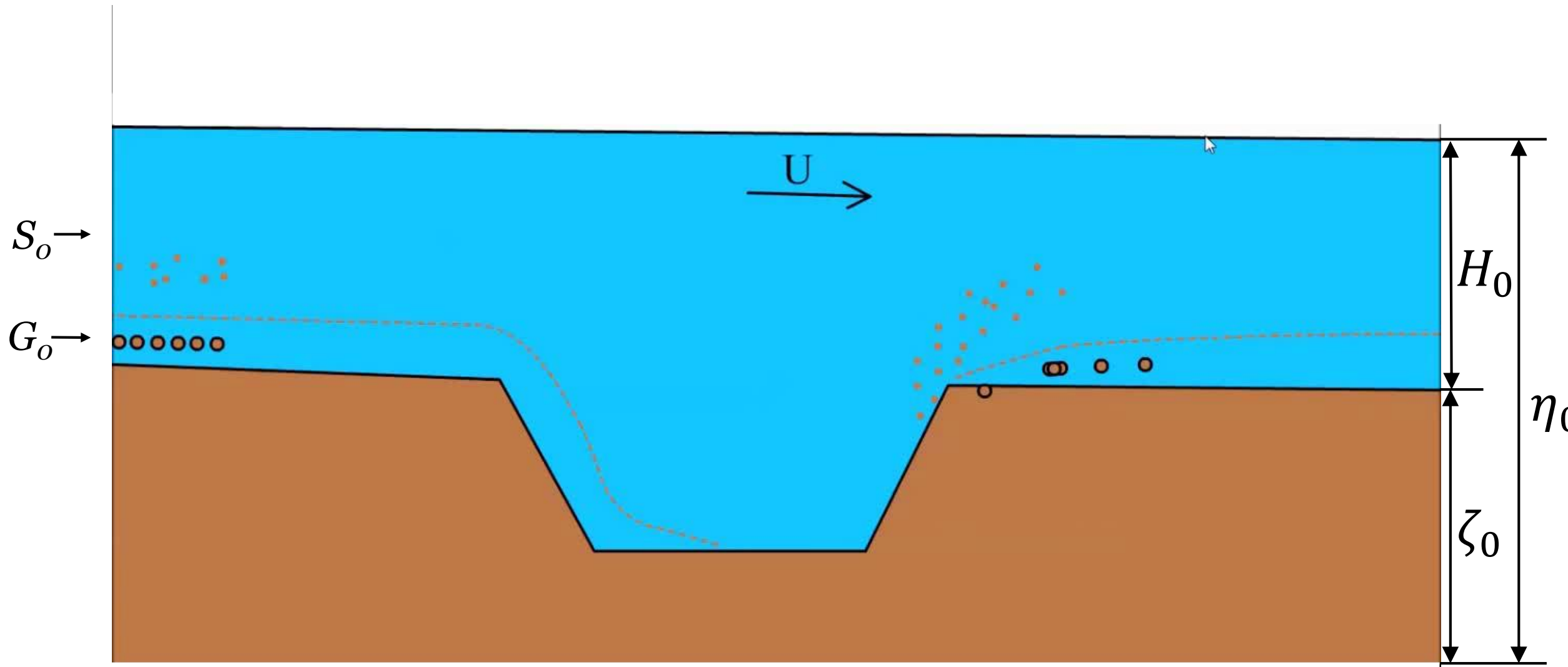
# Bed aggregation





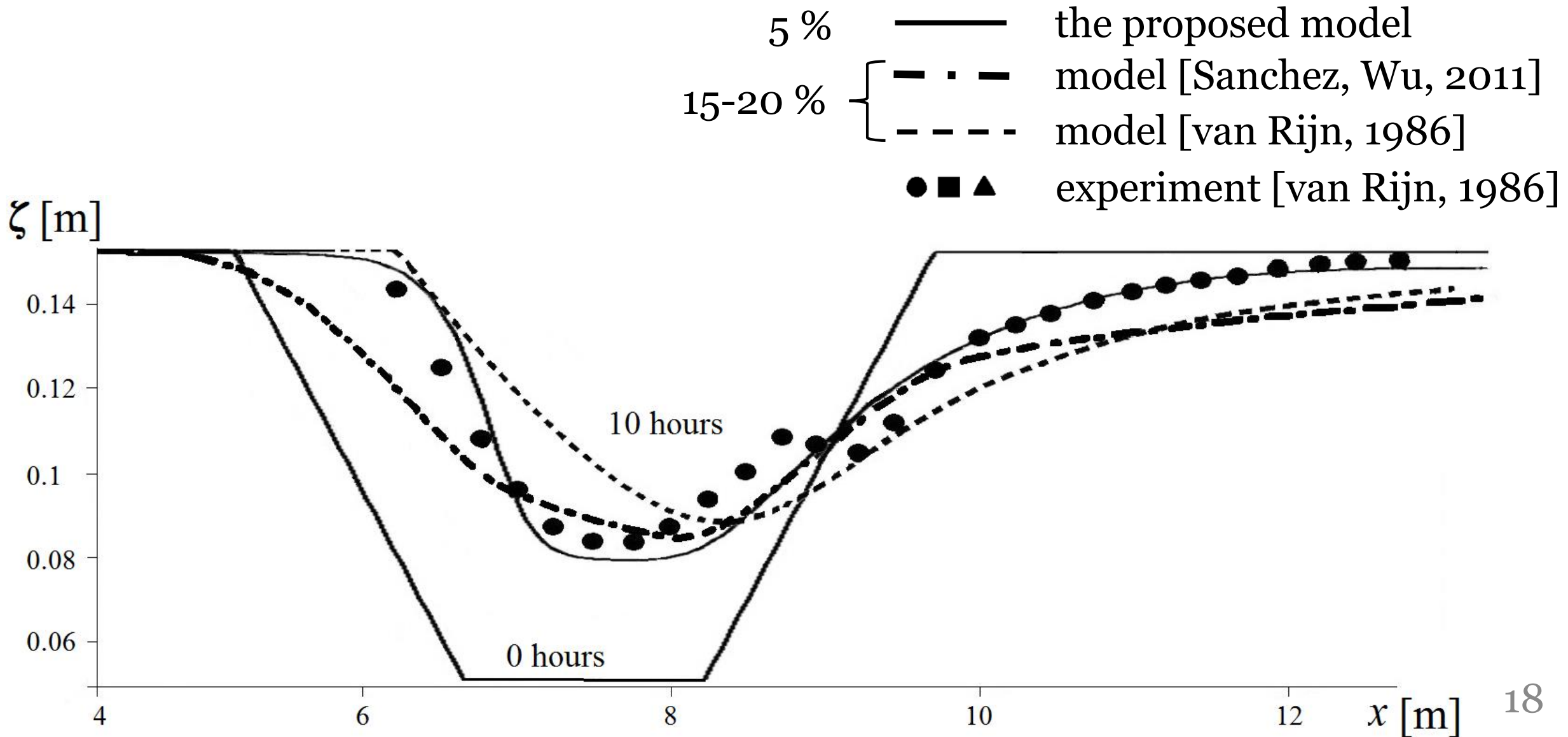


# Trench evolution





# Trench evolution





# Conclusion

- The proposed model included the analytical bed-load rate formula is verified on three practice problems.
- The proposed model is adequately describes local-bed evolution.
- It has almost the same accuracy as models [Sanchez, A., Wu, W., 2011; Singh, V., 2002; van Rijn, L. C. 1986; Wu, W, 2001; Wu, W., Vieira, D.A., 2002].

## Next research

- By using the analytical bed-load rate formula with 2D hydrodynamic and 2D suspended sediment models to research the bed microforms evolution and estimate their influence on the sediment rate and hydraulic resistance.



# Thank you!



Photo from <http://www.gazprom.ru/press/news/2013/april/article161128/>