



## MYTHS IN MEANDERING MORPHODYNAMICS



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# THE BEHAVIOUR OF MEANDERING RIVERS

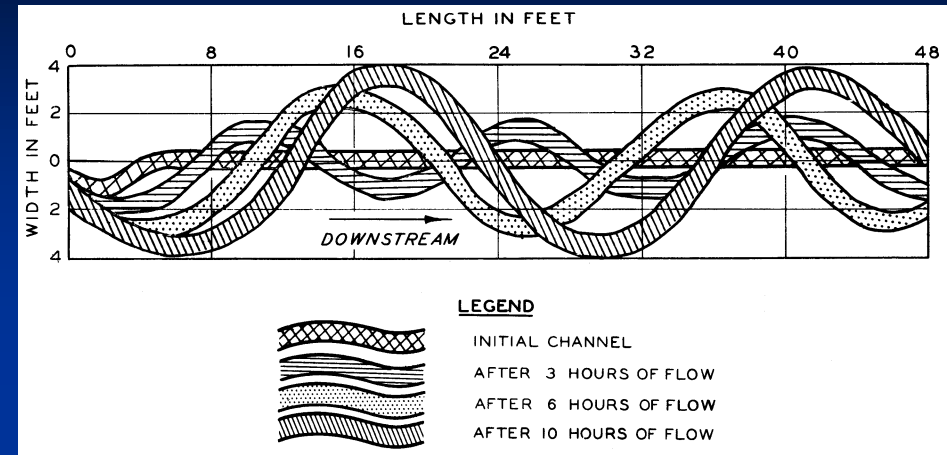
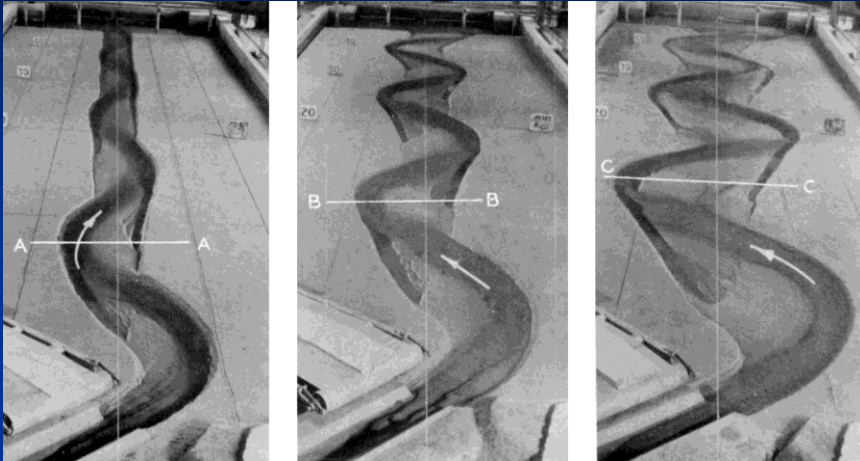
Bed deformation; Alva River, Portugal



Bank deformation; small stream in Germany  
(meander loop downstream migration  
and lateral expansion)

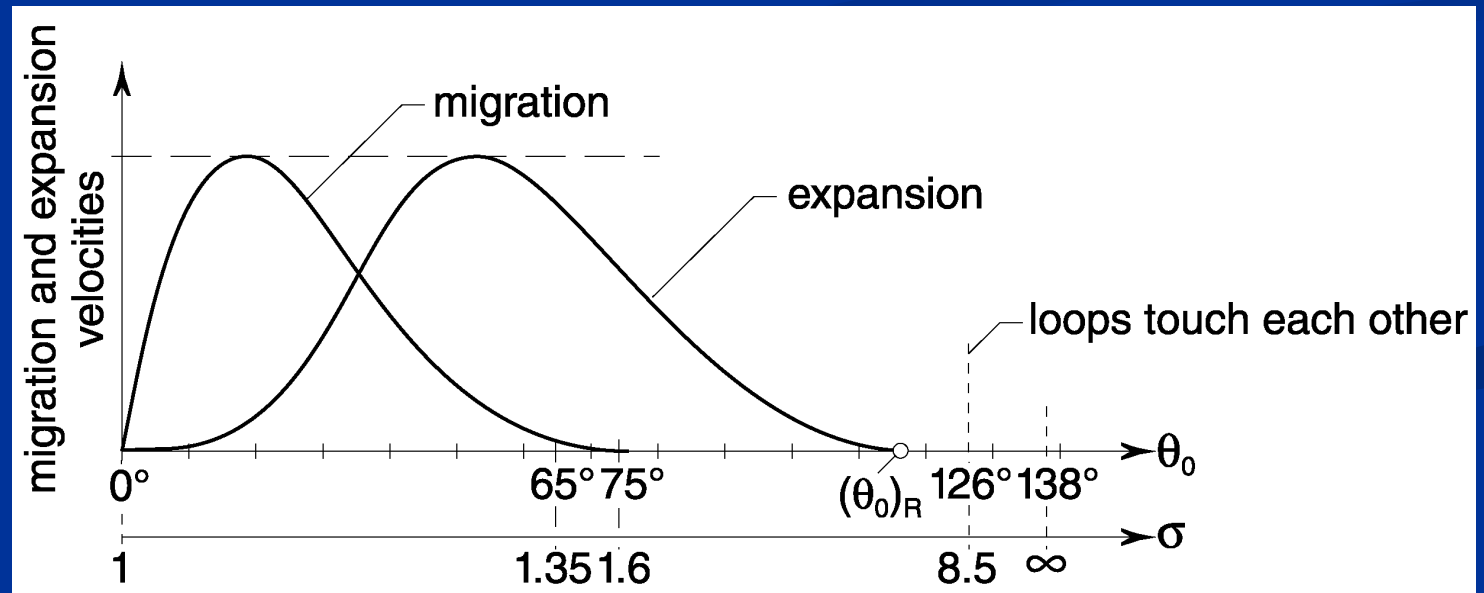


# Bank deformation: Expansion and migration of meander loops



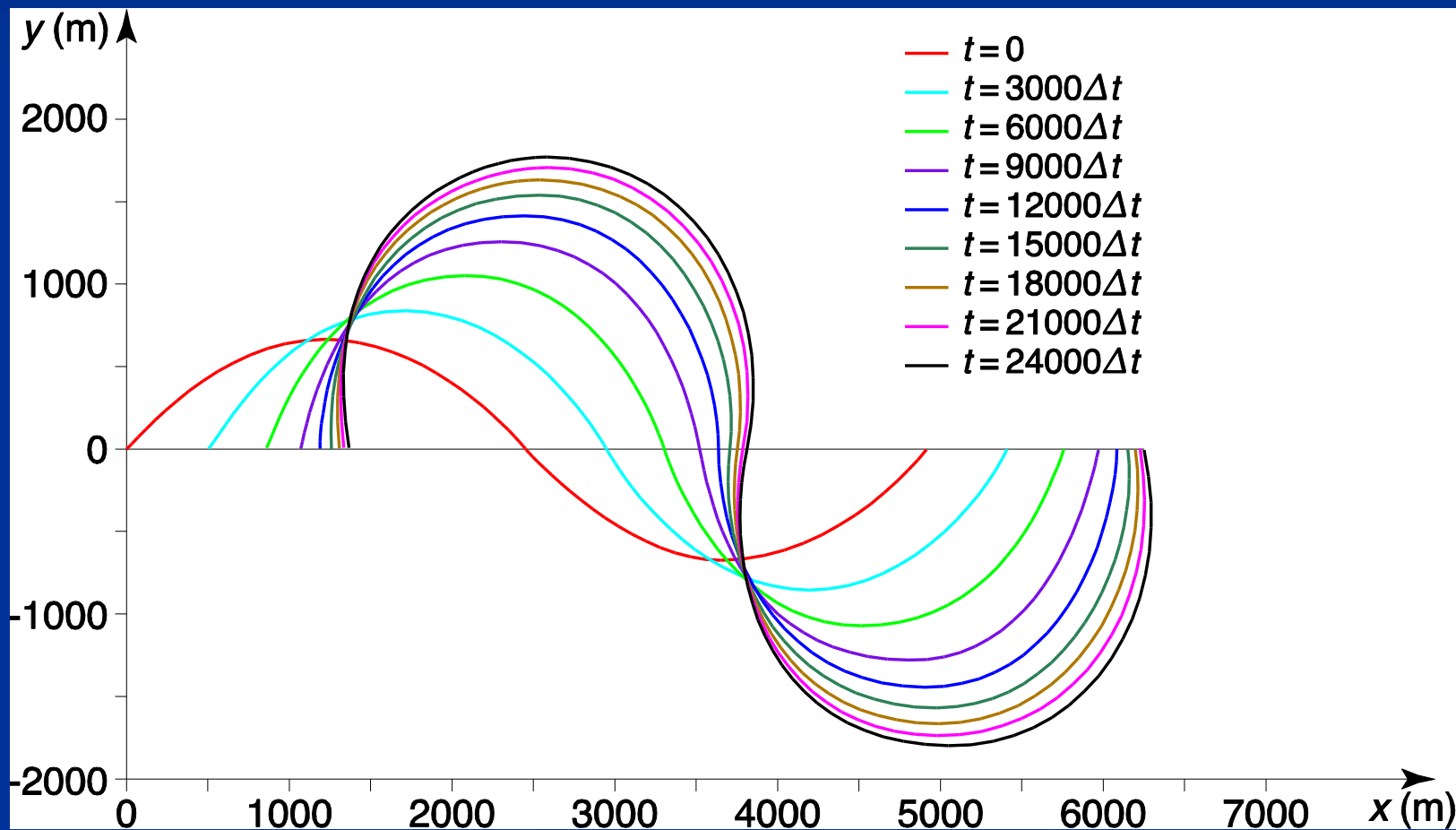
J. F. Friedkin, 1945, Waterways Experiment Station, Vicksburg, Mississippi

N. Kondratiev,  
I. Popov and  
B. Snishchenko 1982



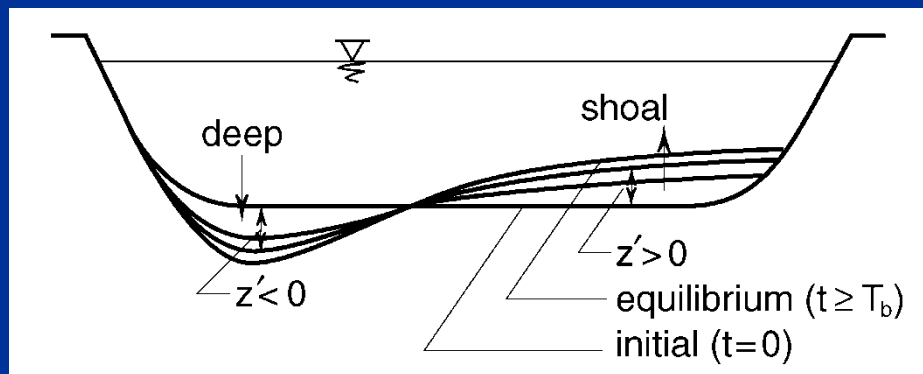
# THE BEHAVIOUR OF MEANDERING RIVERS

Simulation of evolution of stream centerline over time

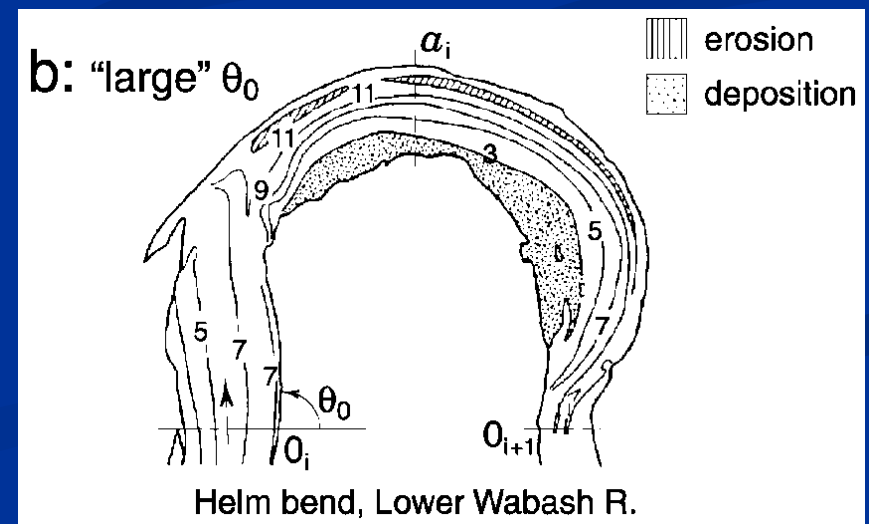
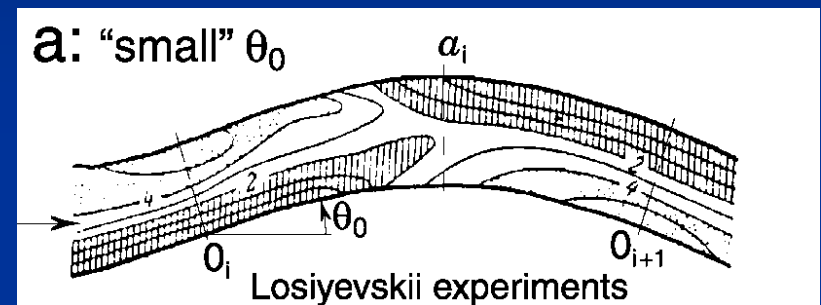


# Bed deformation in meandering streams

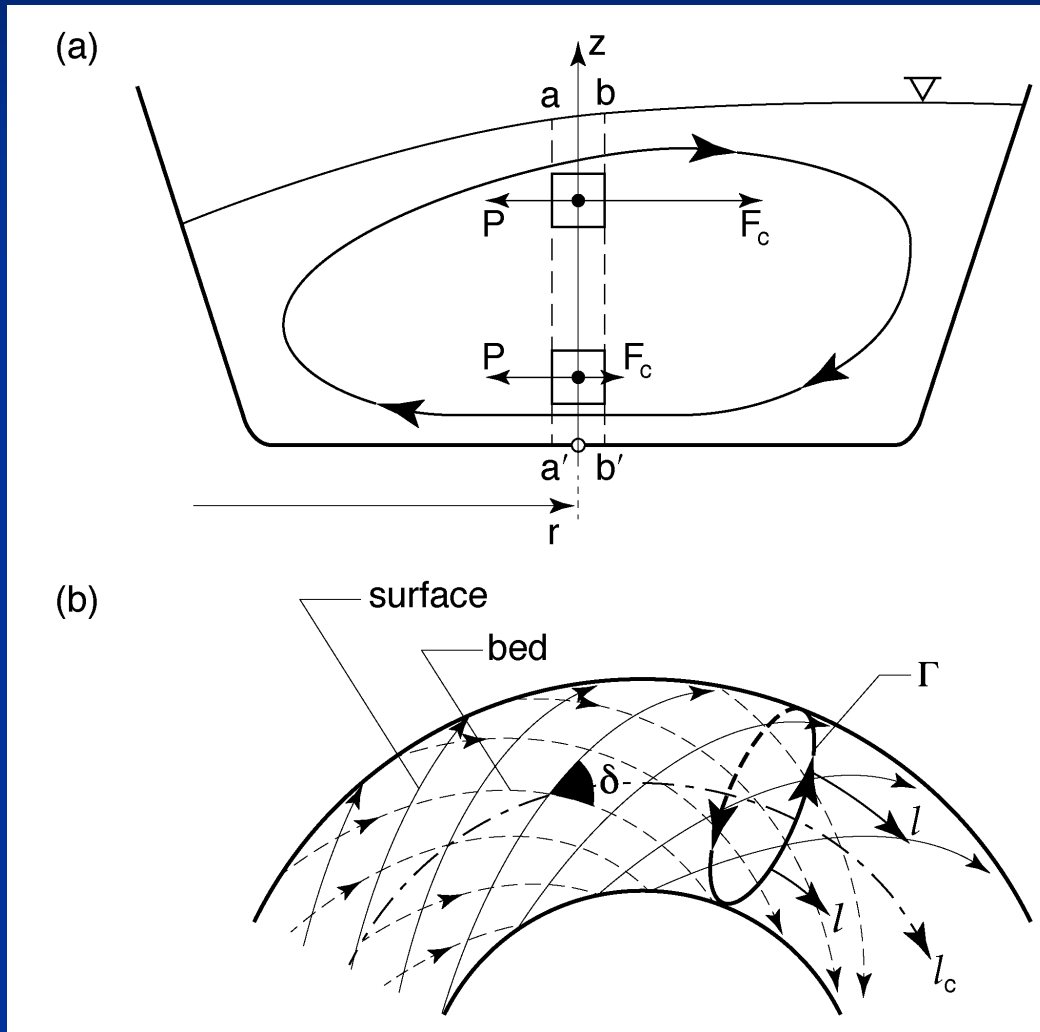
- At time  $t = 0 \rightarrow$  bed is **flat**
- At time  $t = T_b \rightarrow$  bed reaches its **equilibrium** (or **developed** state)

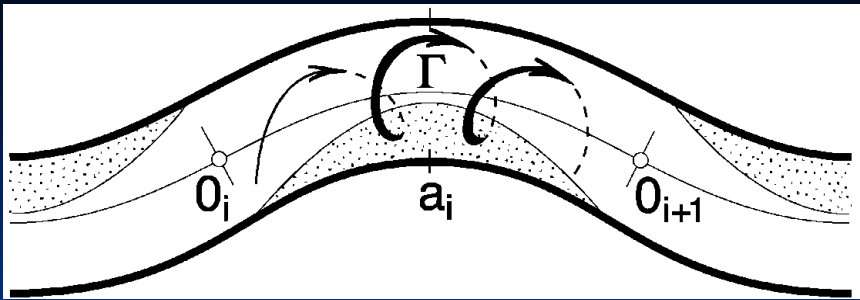


- laterally adjacent erosion "deeps" and deposition "hills"
- each deep+hill complex forms a  **$L/2$ - long erosion-deposition zone**

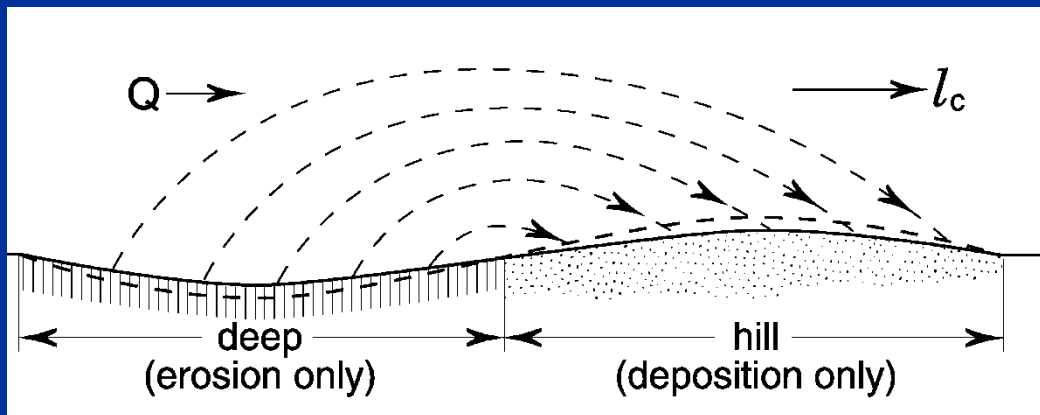
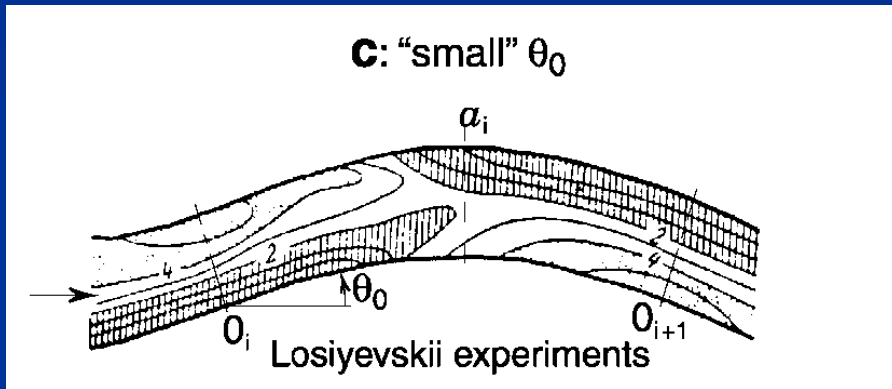


# Classical explanation for bed deformation: cross-circulatory motion





- If bed deformation is due to cross-circulation  $\Gamma$ , then it will initiate and develop predominantly at the apex (standard erosion-deposition pattern)



# Experiments carried out at Queen's University

a)  $\theta_0 = 70^\circ$

$B = 0.80\text{m}$



b)  $\theta_0 = 45^\circ$

$B = 0.30\text{m}$



c)  $\theta_0 = 95^\circ$

$B = 0.30\text{m}$



d)  $\theta_0 = 90^\circ$

$B = 0.80\text{m}$

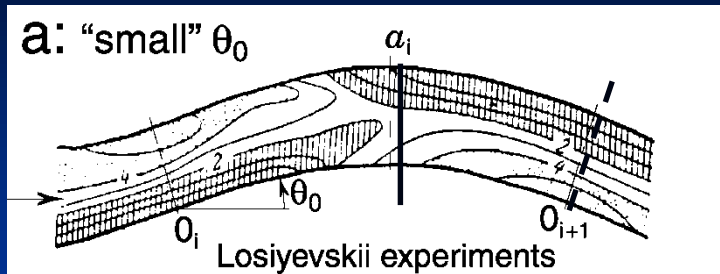




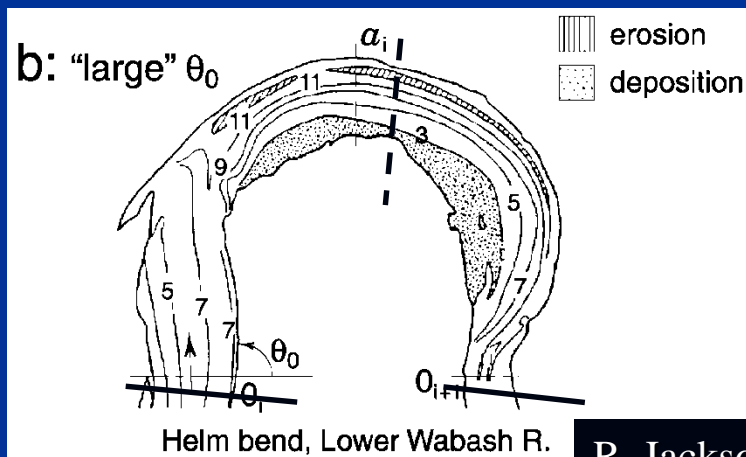
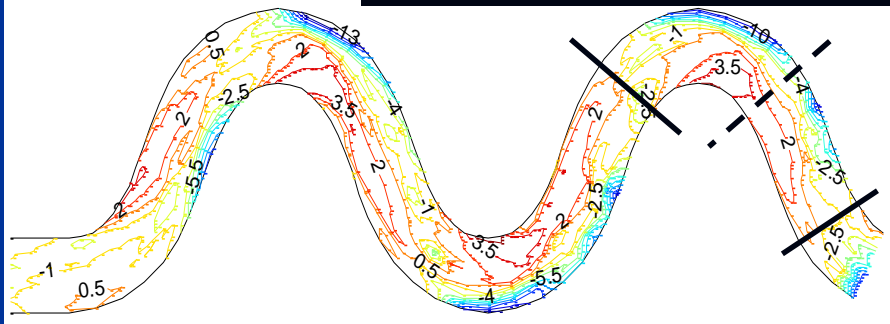
# Location in flow plan of erosion-deposition zones

is strongly dependent on  $\theta_0$

Losiyevskii 1974, Hooke 1974, Hasegawa 1983, Whiting and Dietrich 1993, Termini 1996, da Silva and El-Tahawy 2008, R.J. Jackson 1975, etc.

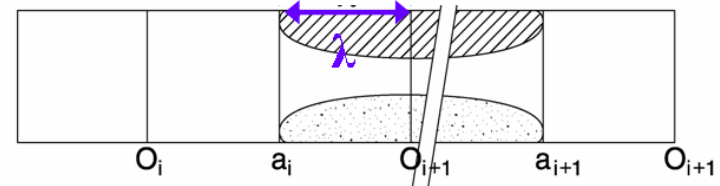


Experiments by A. Binns, 2006

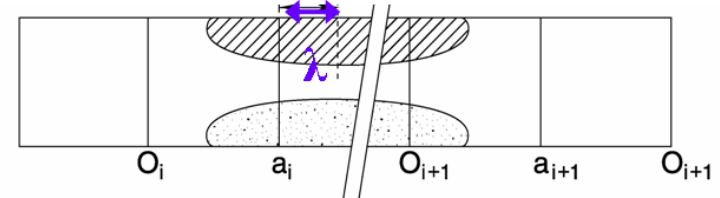


R. Jackson, 1975

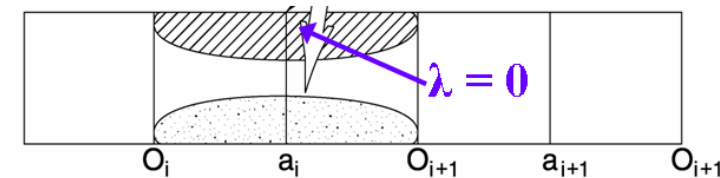
a) Small sinuosity channel: ( $\theta_0 \rightarrow 0$ )



b) Intermediate sinuosity channel:



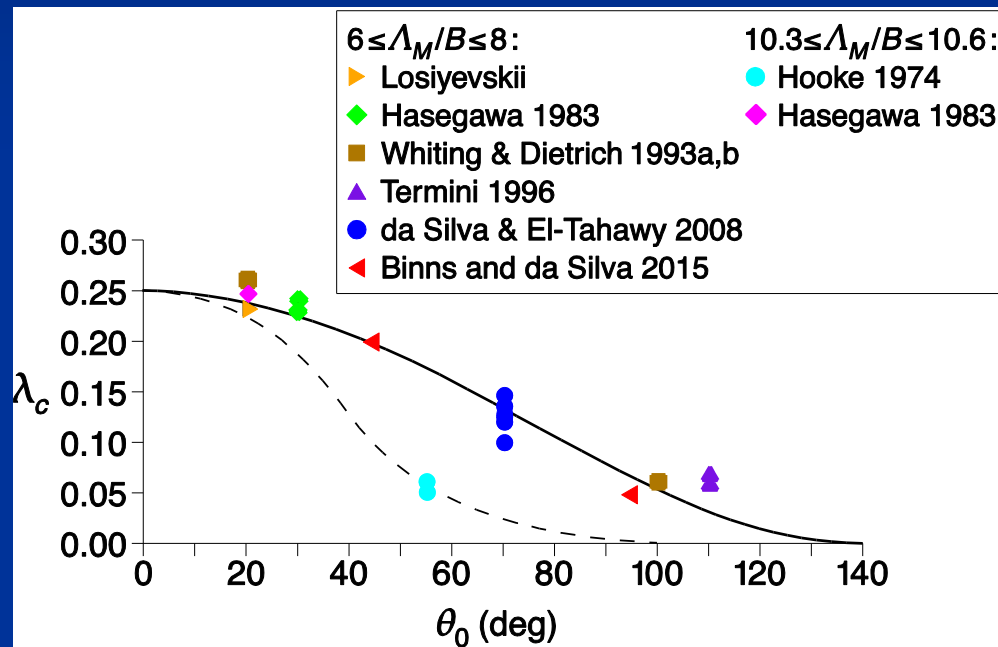
c) Large sinuosity channel: ( $\theta_0 \rightarrow 126^\circ$ )



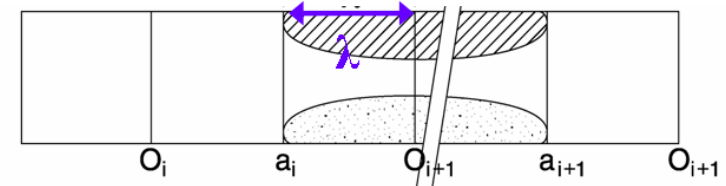
$\bar{\omega} > 0$  Erosion Deposition

# Location in flow plan of erosion-deposition zones is strongly dependent on $\theta_0$

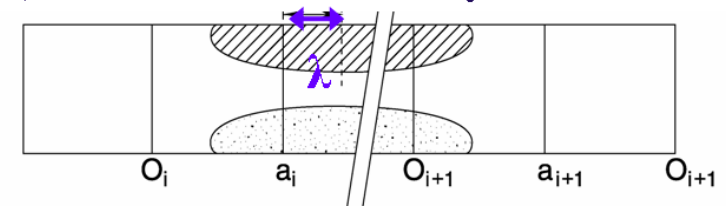
Losiyevskii 1974, Hooke 1974, Hasegawa 1983, Whiting and Dietrich 1993, Termini 1996, da Silva and El-Tahawy 2008, R.J. Jackson 1975, etc.



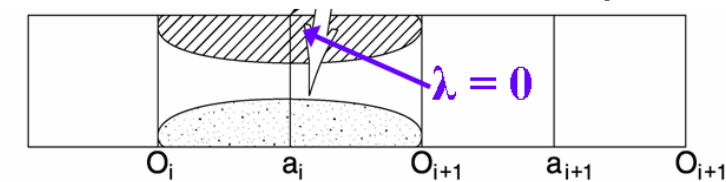
a) Small sinuosity channel: ( $\theta_0 \rightarrow 0$ )



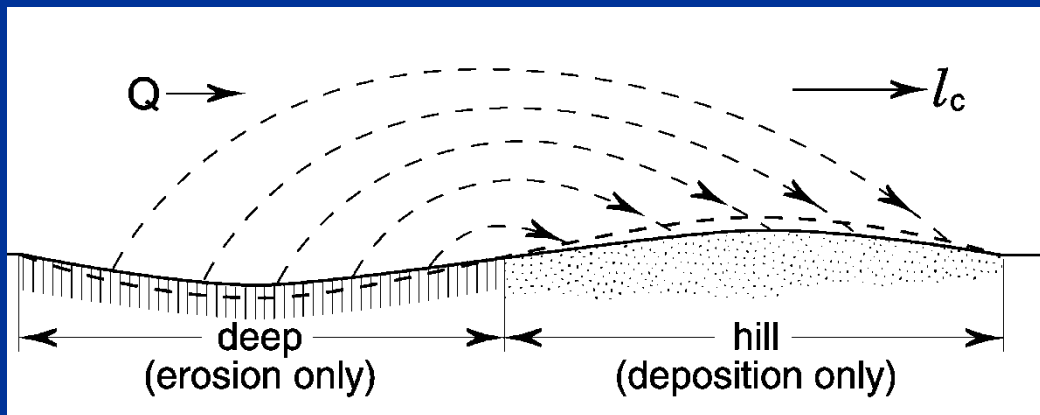
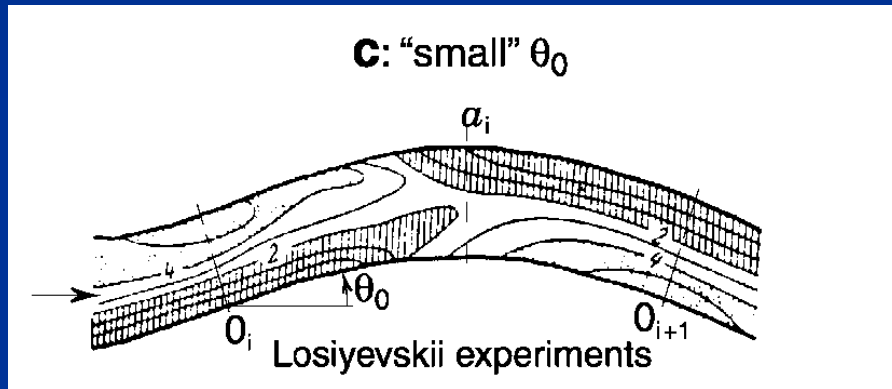
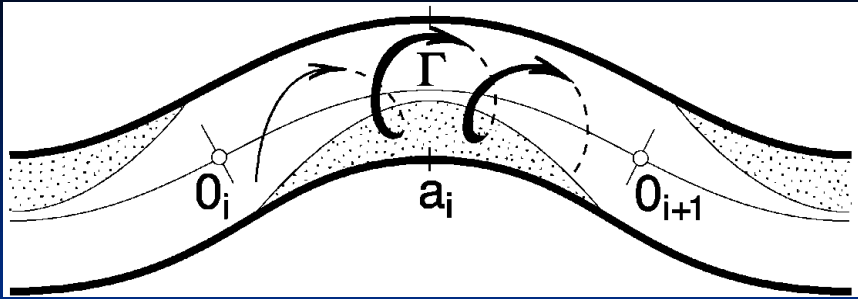
b) Intermediate sinuosity channel:



c) Large sinuosity channel: ( $\theta_0 \rightarrow 126^\circ$ )



■  $\bar{\omega} > 0$     ▨ Erosion    ▩ Deposition

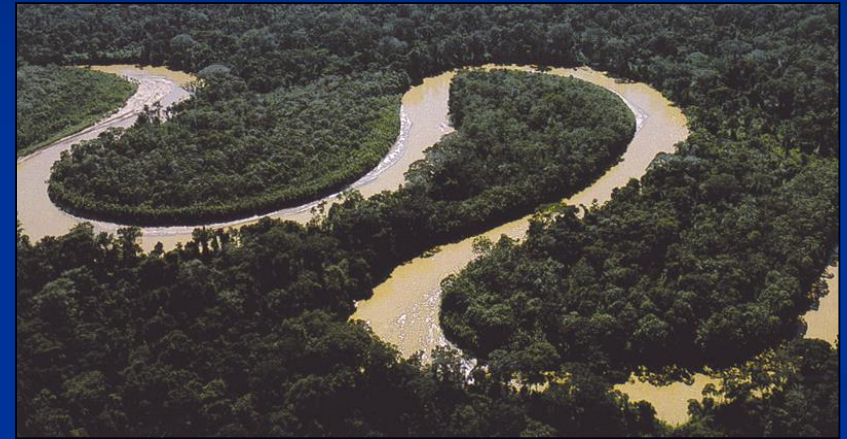


“The importance of cross-circulation in determining the geometry of river beds in meanders has been over-emphasized for many years, and it will take some time to bring the significance of such flow patterns into proper perspective”  
Hooke 1980

# Geometric characteristics of meandering streams:

## stream idealization

- In nature, different meandering streams exhibit different geometric characteristics, grain size distributions, flow regimes, etc., and in any given stream the conditions vary from one meander loop to another. Owing to this reason, scientific studies aimed at a general formulation of meandering processes are usually undertaken using abstract *idealized* meandering streams (or “stream models”).

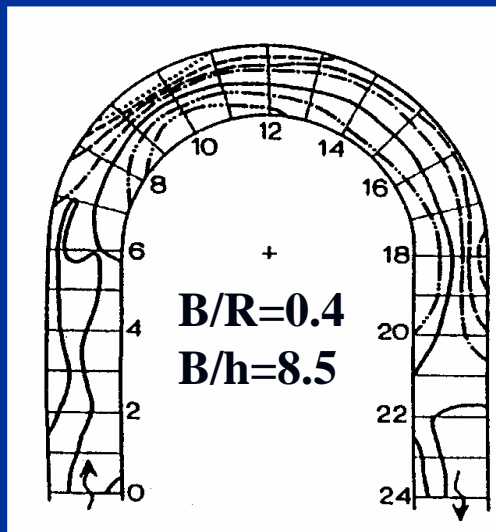


# Geometric characteristics of meandering streams: stream idealization

Definition of idealized meandering stream;

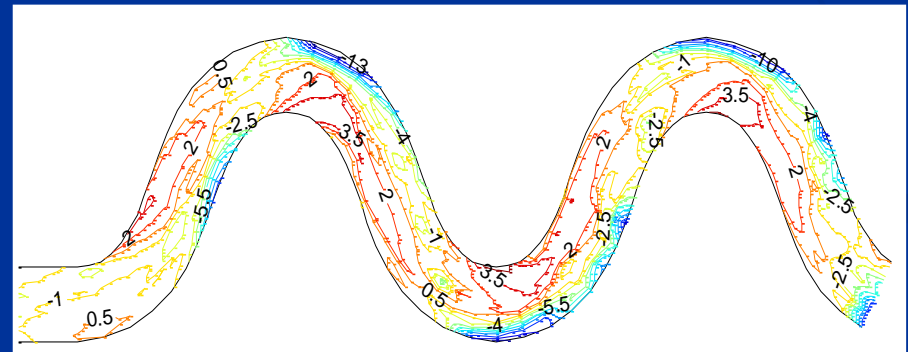
- Selection of a plan shape:

Circular bends



(Sutmuller & Glerum 1980)

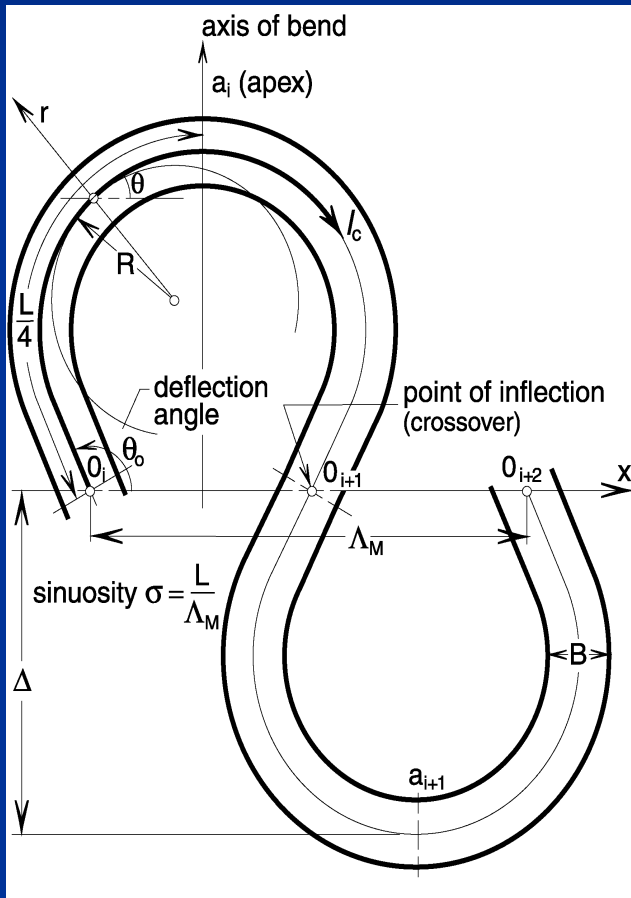
Sine-generated channels



- Selection of a width-to-depth ratio (with a few exceptions,  $B/h < 10$ )

# Geometric characteristics of meandering streams

## Meander path



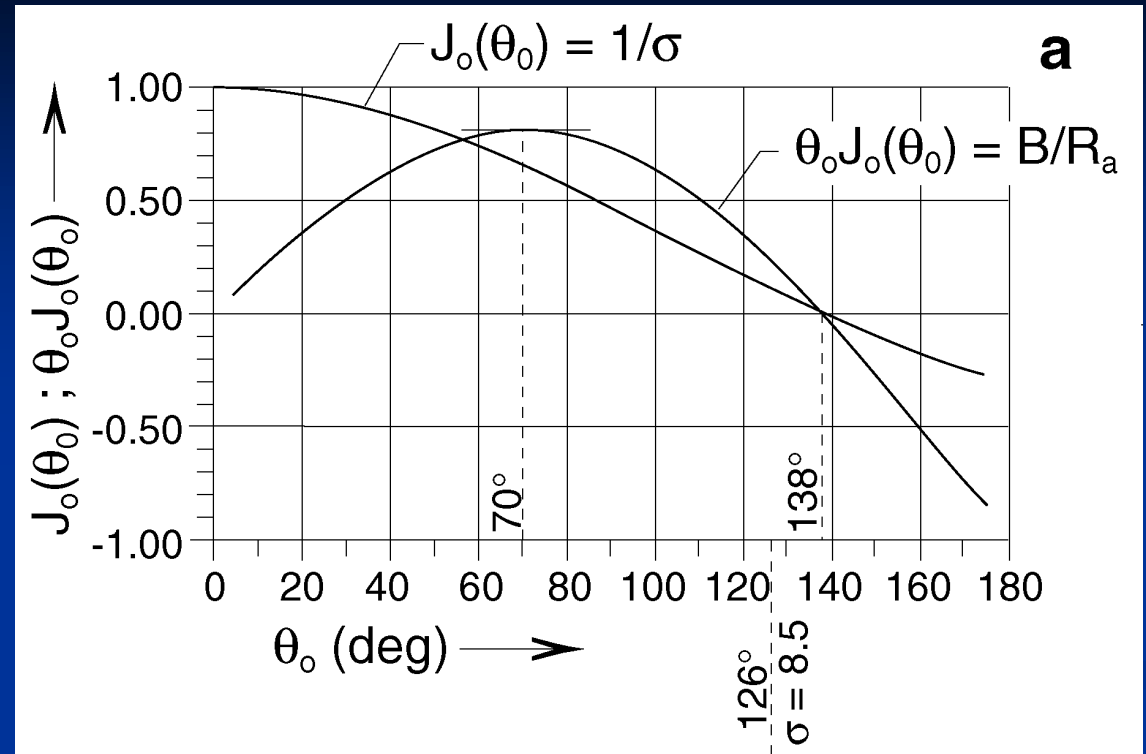
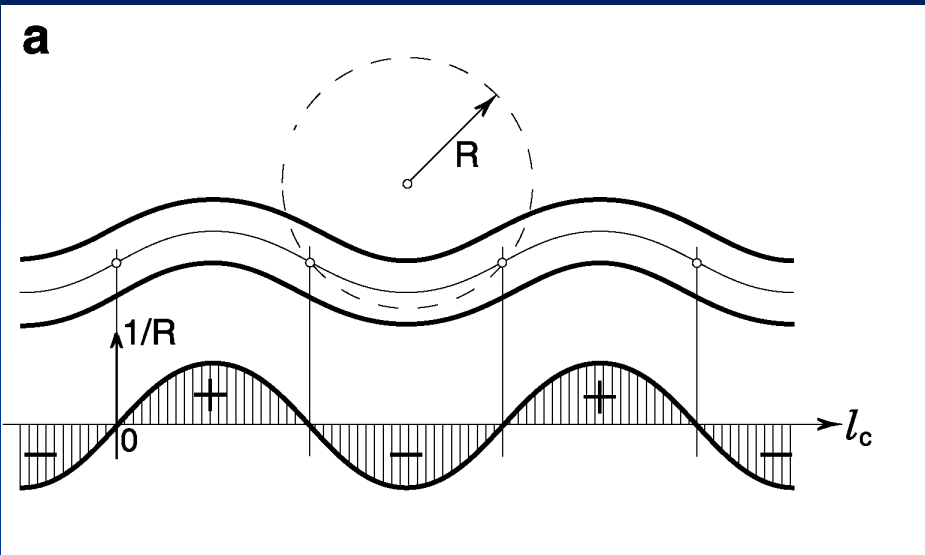
Sine-generated curve:

$$\theta = \theta_0 \cos\left(2\pi \frac{l_c}{L}\right)$$

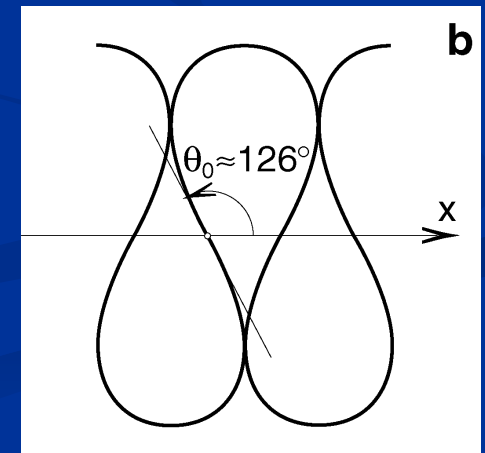
[Due to Leopold and Langbein 1966  
(following work by  
Von Schelling 1951, 1964)]

Sinuosity:

$$\sigma = \frac{L}{\Lambda_M} = \frac{1}{J_0(\theta_0)}$$

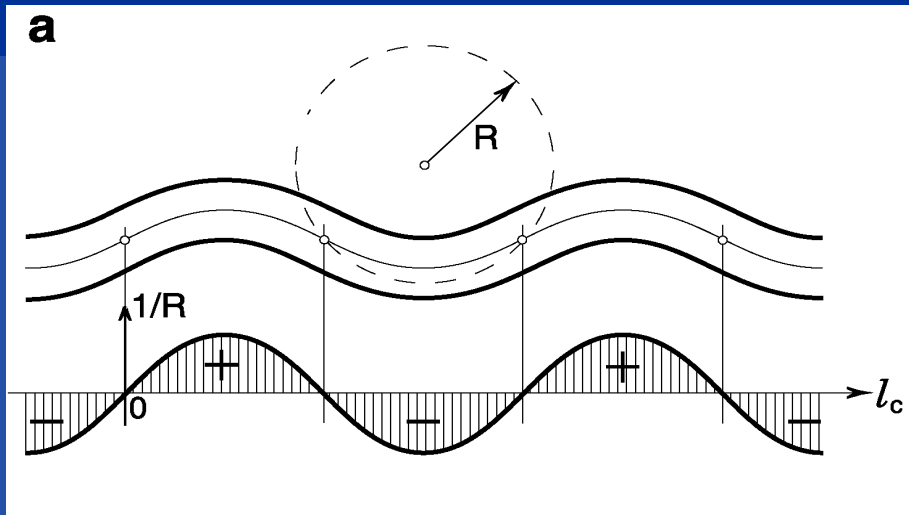


- Continuous variation of curvature along the channel centreline;
- Variation of  $B/R_a$  with deflection angle as shown in Fig.;
- Loops begin to touch when  $\theta_0 \approx 126^\circ$



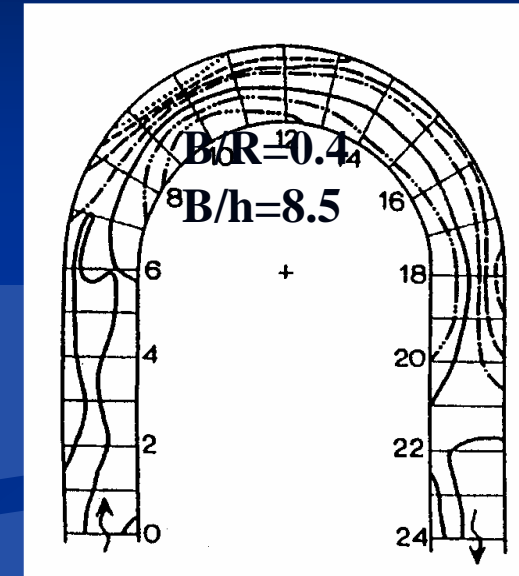
# Comparison between circular channels and sine-generated channels

## Sine-generated channels



- Continuous variation of curvature along the channel centreline;

## Circular bends



(Sutmuller & Glerum 1980)

+

$B/R=0$

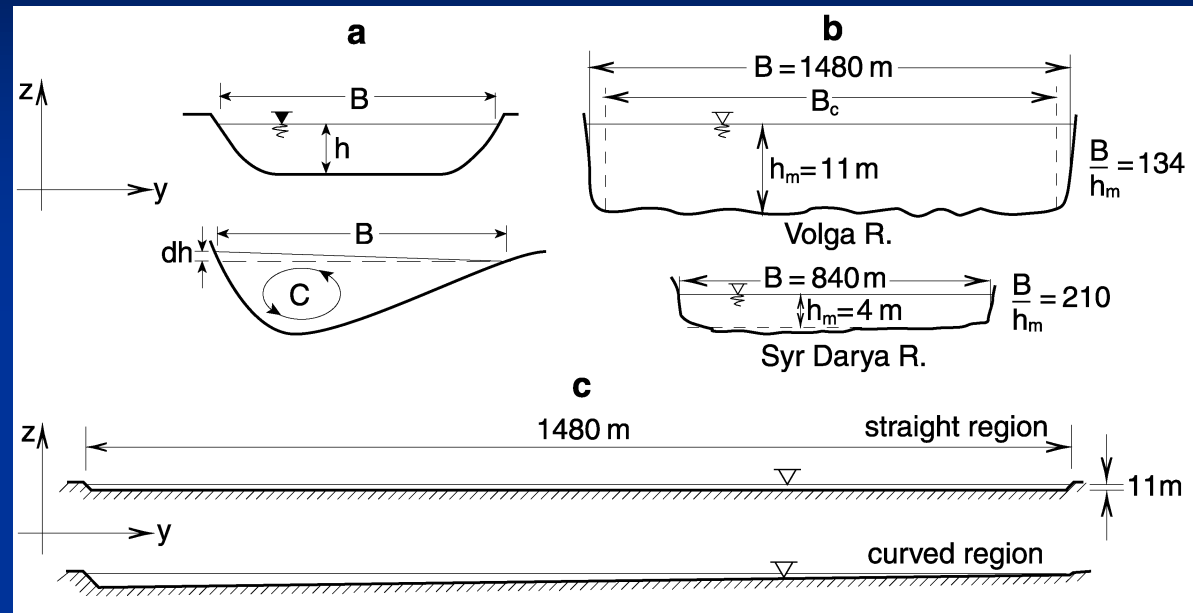
$B/R=0$





# Geometric characteristics of meandering streams: Width-to-depth ratio

- Rivers are very wide objects:

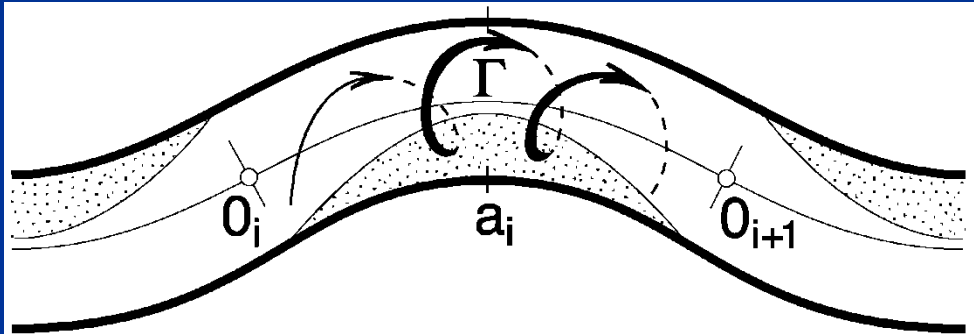


(from Yalin and da Silva 2001, Fluvial Processes Monograph, IAHR)

Their width-to-depth ratio  $B/h$  usually  $>10$  and often  $\gg 10$ , while most research so far focused on  $B/h <10$  !

# Cross-circulation $\Gamma$

Radial velocity due to  $\Gamma$  (Chang 1988, Yalin 1992, Jia and Wang 1999, Chen and Duan 2006):

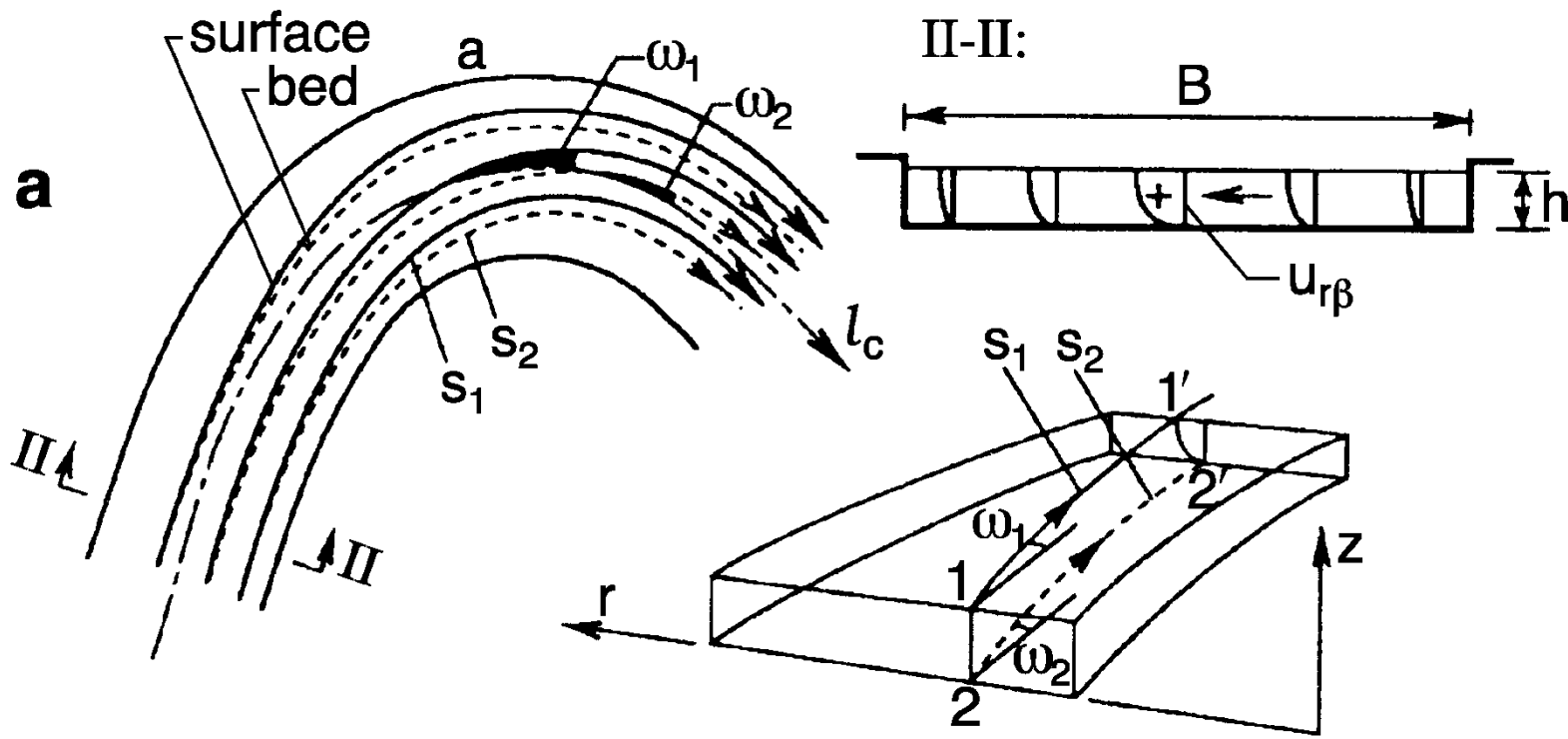


$$\frac{u_{r\Gamma}}{\bar{u}} \sim f\left(\frac{B}{R}\right)\left(\frac{h}{B}\right)$$

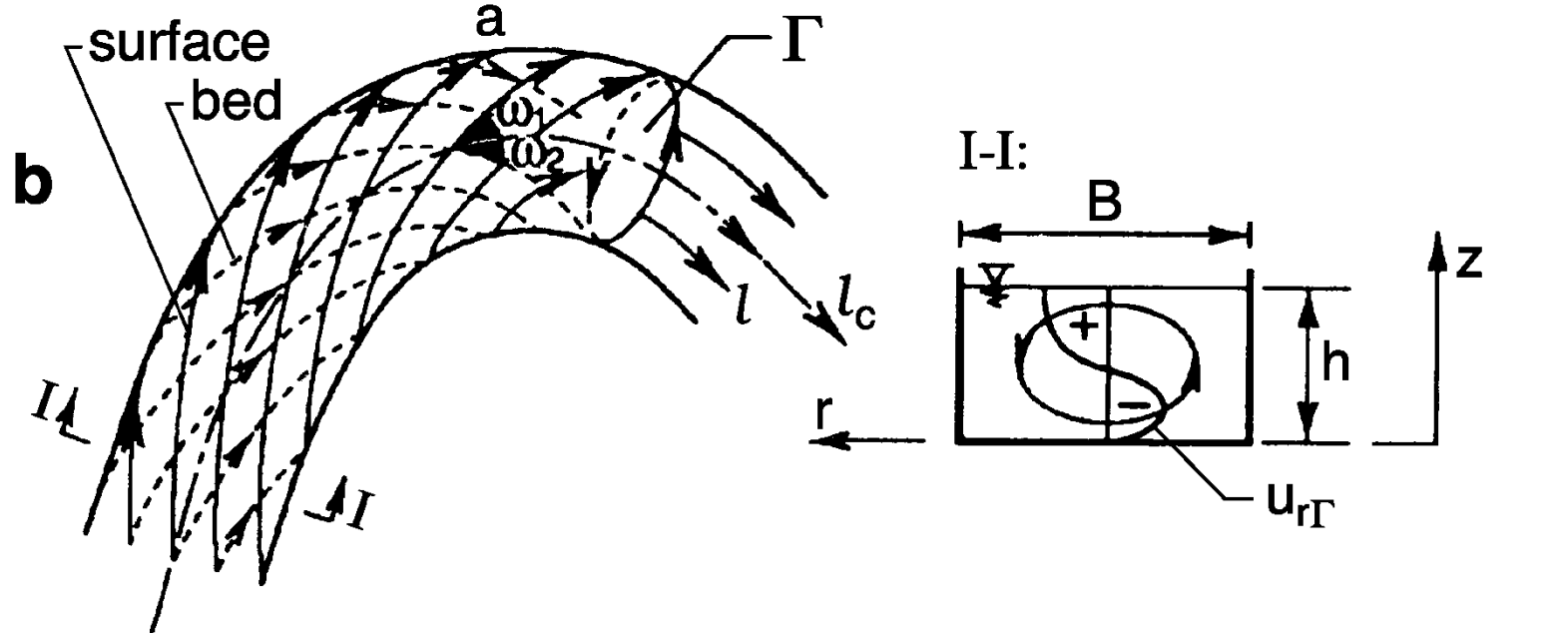
$$\left(\frac{u_{r\Gamma}}{\bar{u}}\right)_a \sim f[\theta_0 J_0(\theta_0)] \frac{h}{B}$$

- For any given  $\theta_0$ , the importance of  $\Gamma$  progressively decreases with the increment of  $B/h$ ;
- The structure of meandering flows is highly dependent on  $B/h$ : what is true for small  $B/h$ , does not necessarily apply to large  $B/h$

Although substantial amount of research on meandering has been carried out in the past, often this does not reflect the conditions in real rivers



**Lateral Oscillation:**  
 due to  $d(1/R)/dlc$

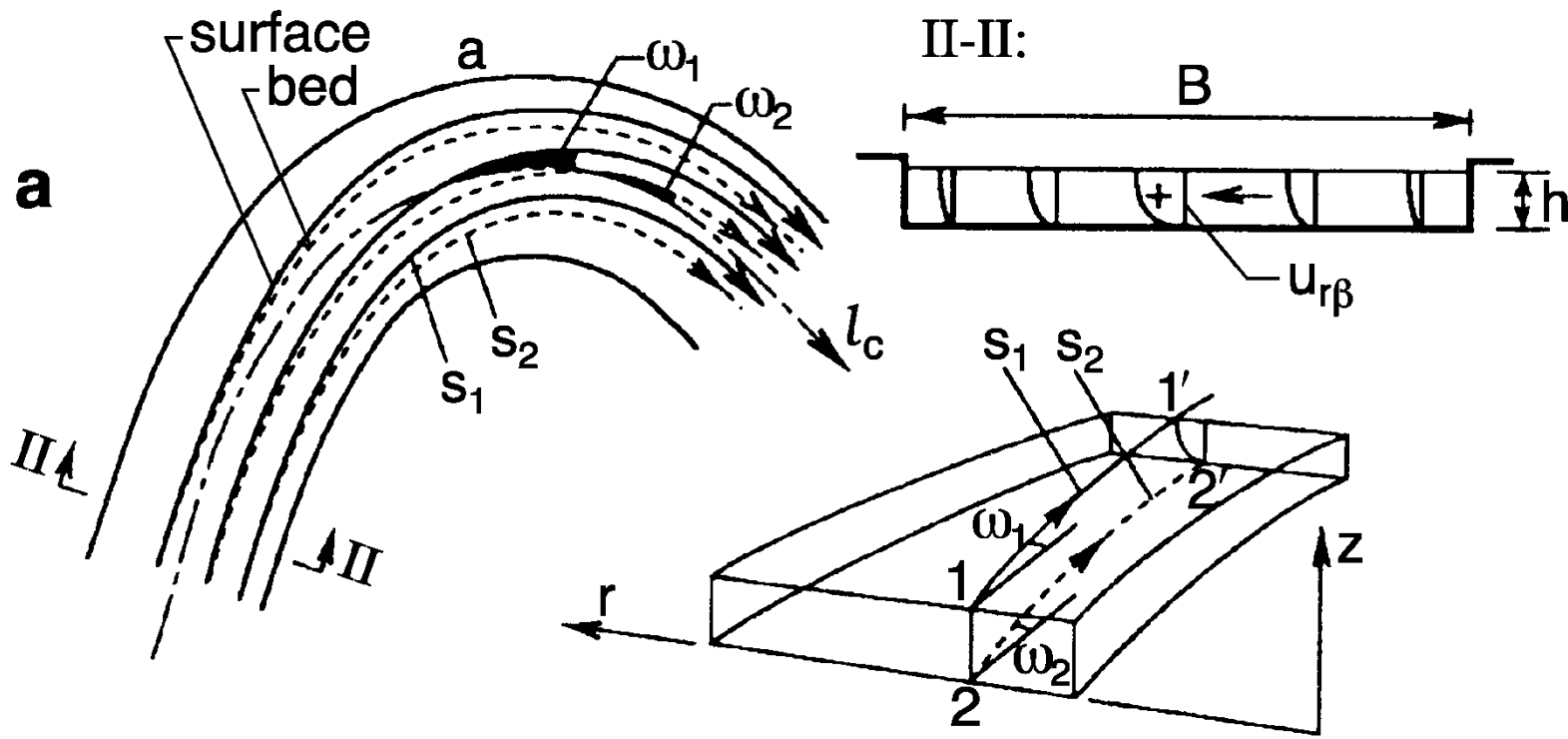


**Cross-Circulation:**  
 due to  $1/R$

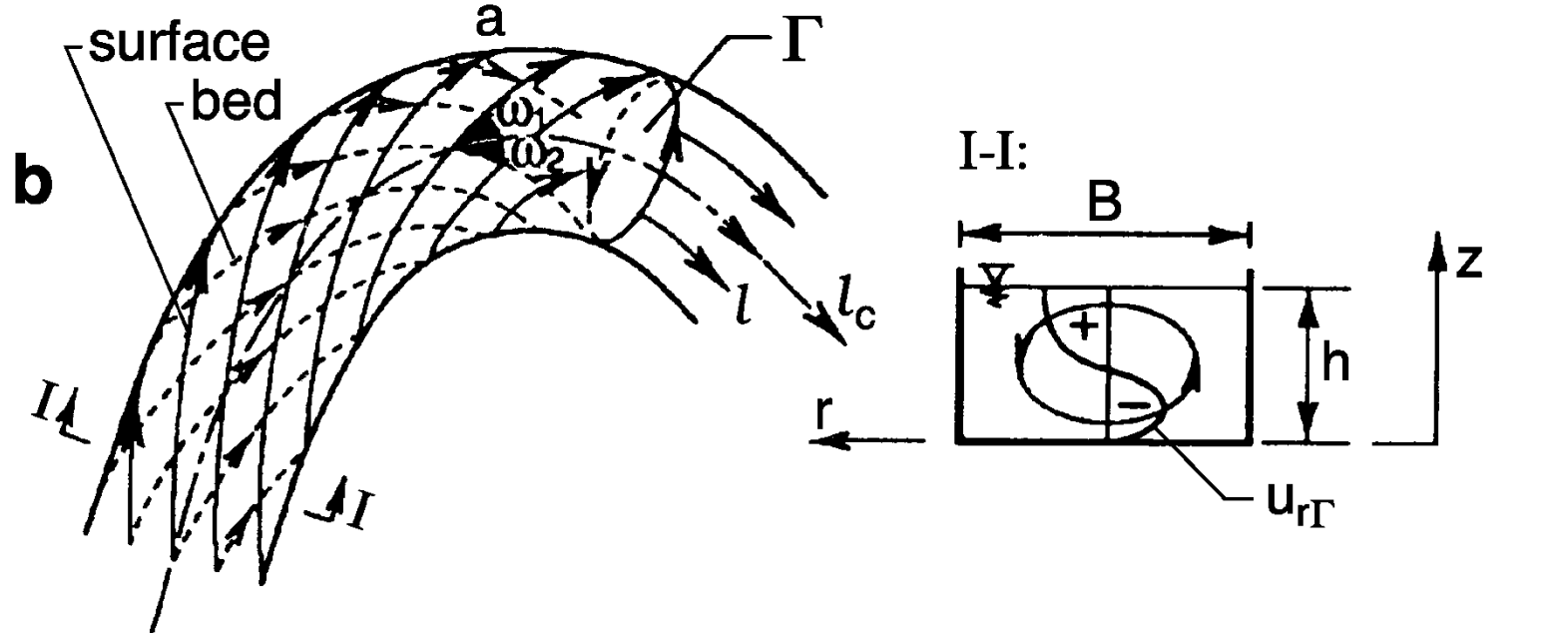
# Meandering flow kinematics

(after Engelund 1974, Smith and McLean 1984, Nelson and Smith 1989, Yalin 1992, among many others)

- Circular flow (fully developed bend flow) = uniform flow upon which the cross-circulation  $\Gamma$  is superimposed (cross-circulation is the only mechanism that can produce erosion-deposition)
- Sine-generated stream = Laterally oscillating flow (the “convective” base) upon which the cross-circulation  $\Gamma$  is superimposed

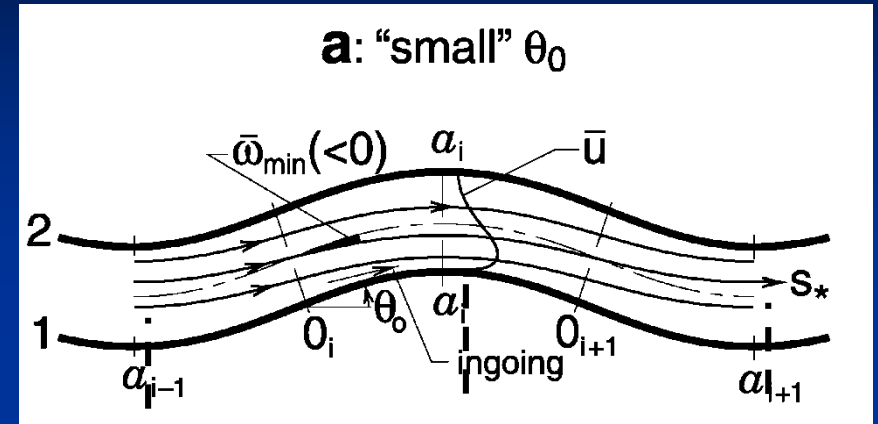
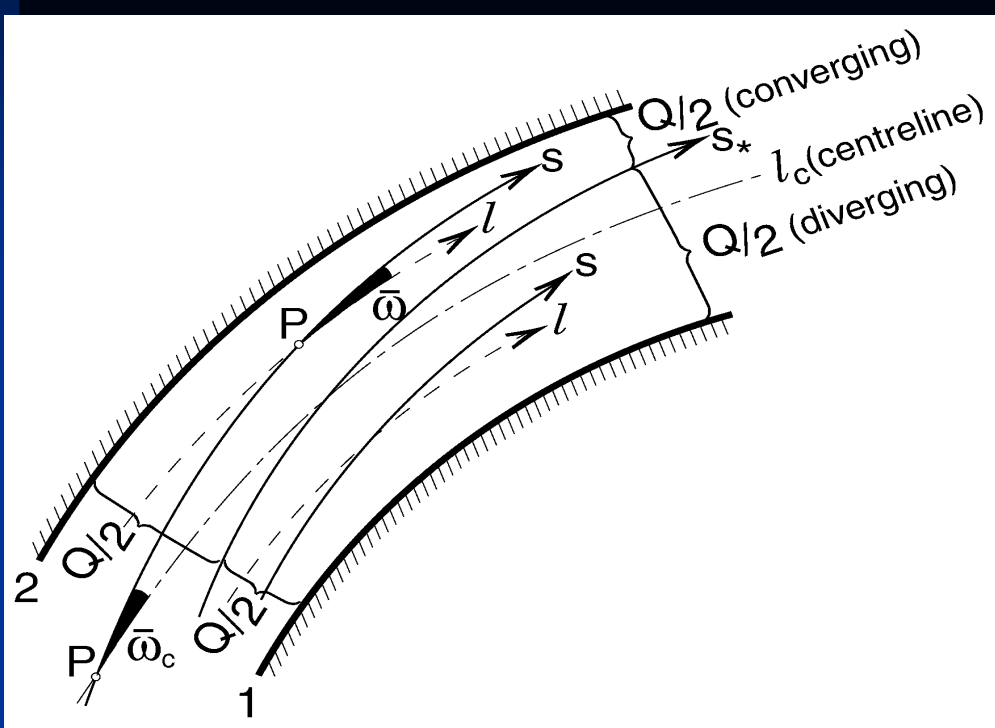


**Lateral Oscillation:**  
 due to  $d(1/R)/dlc$



**Cross-Circulation:**  
 due to  $1/R$

# The convective base (vertically-averaged flow)



**DIV**

**CONV**

**CONV**

**DIV**

- Flow is formed by laterally adjacent convergence-divergence zones
- In the case of sine-generated streams the convergence-divergence zones have the length  $L/2$  and periodically alternate along  $l_c$

$\bar{\omega}$  = deviation angle  
 $\bar{\omega} = 0$  at the beginning and end of each [CD]

# Vertically-averaged initial flow in sine-generated meandering streams

da Silva et al. (2006), ASCE JHE:

(Whiting and Dietrich 1993, da Silva 1995, Termini 1996; da Silva et al. 2006)

$$B = 0.40m$$

$$\Lambda_M = 2\pi B$$

$$D = 2.2mm$$

$$B/h = 13$$

$$\theta_0 = 30^\circ$$

$$50^\circ$$

$$70^\circ$$

$$90^\circ$$

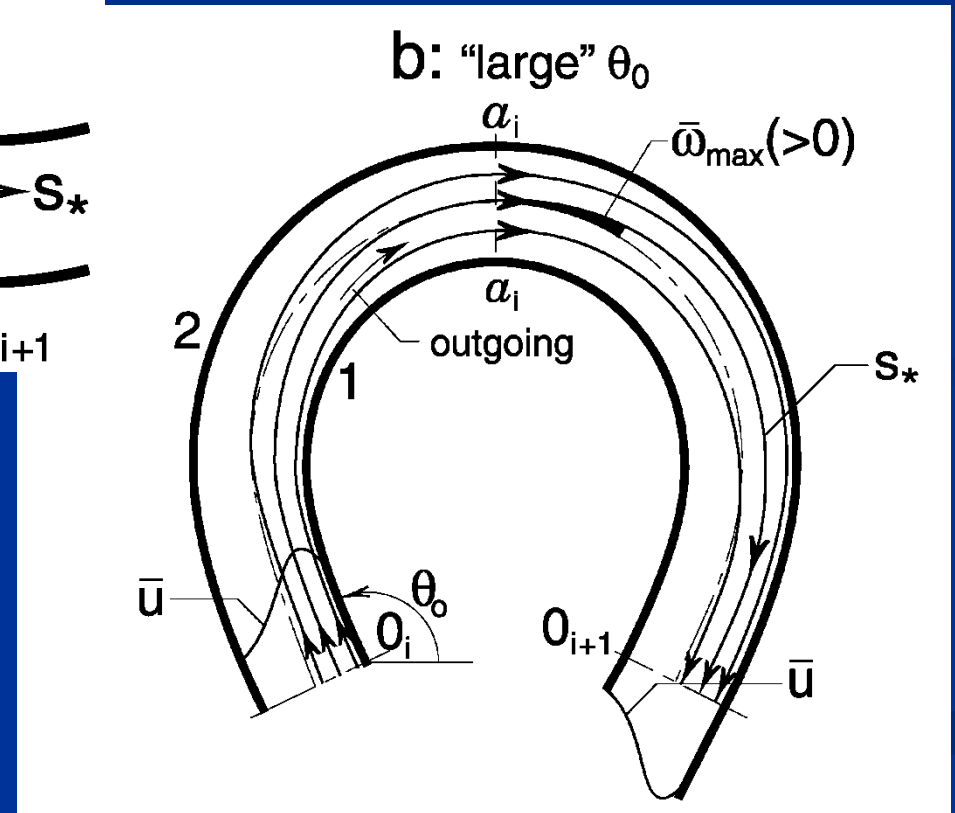
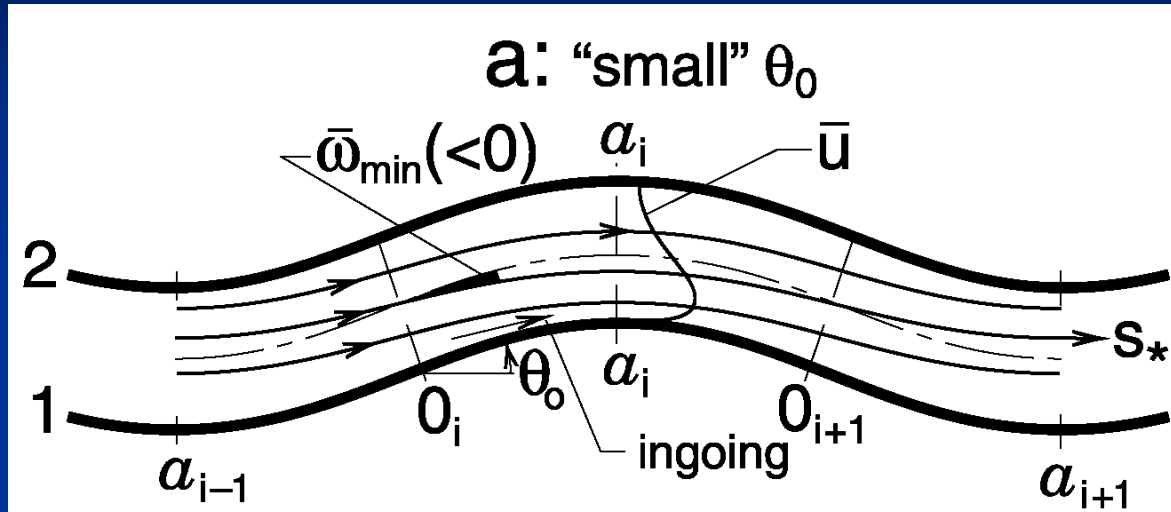
$$110^\circ$$





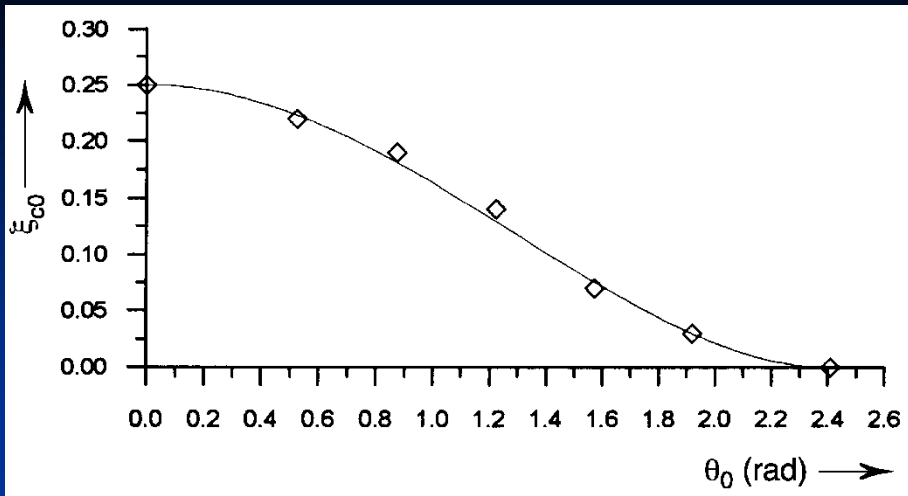
# Vertically-averaged initial flow in sine-generated meandering streams

(after da Silva et al. 2006)

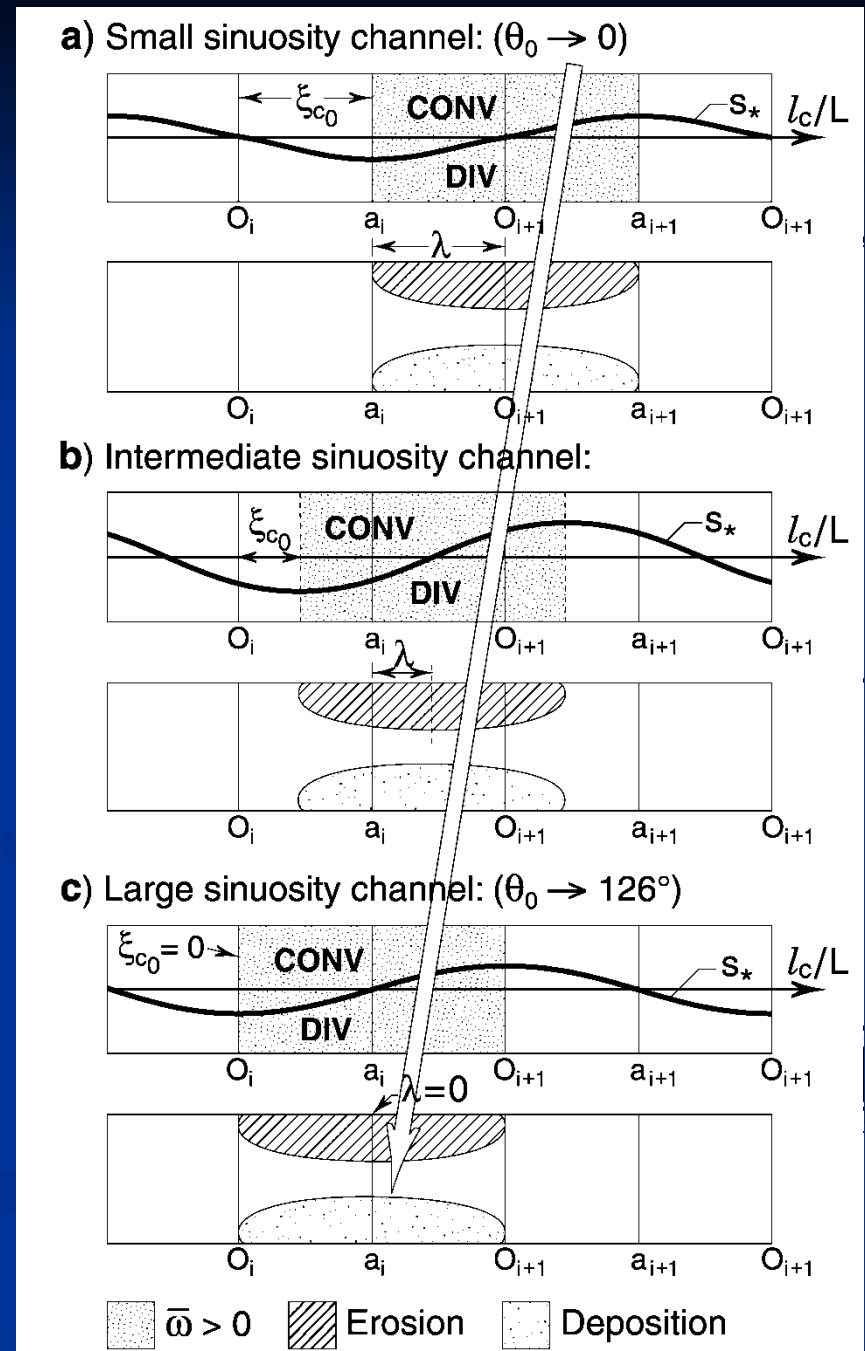
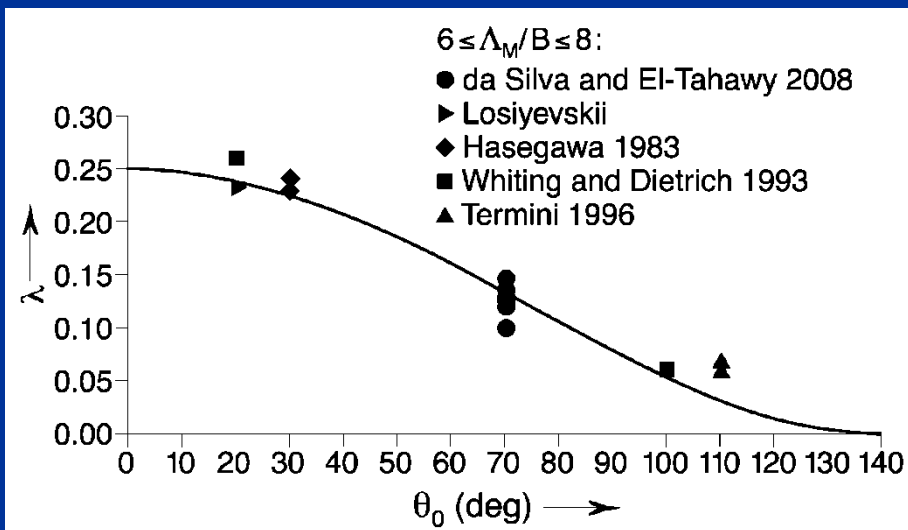


Location in flow plan of [CD]'s:

- is not of a standard type
- strongly dependent on  $\theta_0$
- weakly dependent on  $B/h_{av}$



• Location of CONV/DIV does not depend on B/h

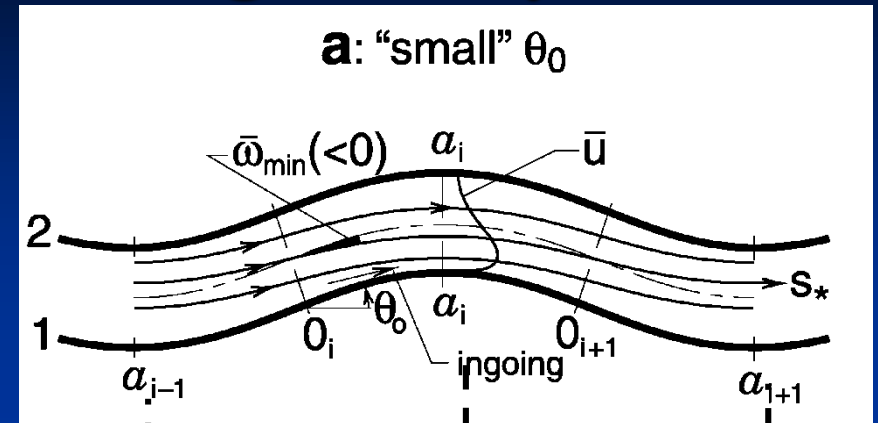


# Bed deformation and nature of bed geometry

- In “wide” meandering streams, the cross-circulation plays only a secondary role in determining the bed geometry (Matthes 1941, Hooke 1974, Yalin 1992, etc.)

- The bed deformation is mainly due to the convective behaviour of flow, and thus of the sediment transport rate (Nelson and Smith 1989, Struiksma 1985, etc., etc.)

$$(1 - p) \frac{\partial z_b}{\partial t} = -\nabla \mathbf{q}_s$$

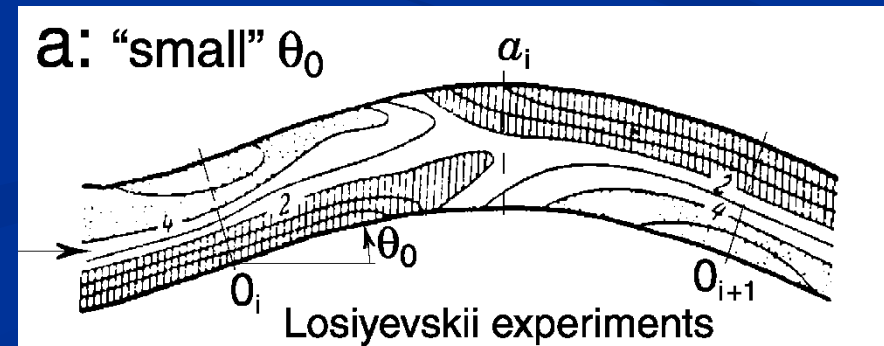


**DIV =**  
**Dep.**

**CONV =**  
**Eros.**

**CONV =**  
**Eros.**

**DIV =**  
**Dep.**



## Question:

- Large  $B/h_{av}$ :  $\Gamma \rightarrow 0$ , and its role becomes negligible
- As  $B/h_{av}$  decreases,  $\Gamma$  becomes more prominent

## Question:

For what values of  $B/h_{av}$  does  $\Gamma$  become relevant?

“It appears that the longstanding debate on the relevance of cross-circulation is thus somewhat out of focus: it is not whether but when is the cross-circulation relevant (or prominent)”

M.S. Yalin 1992  
River Mechanics

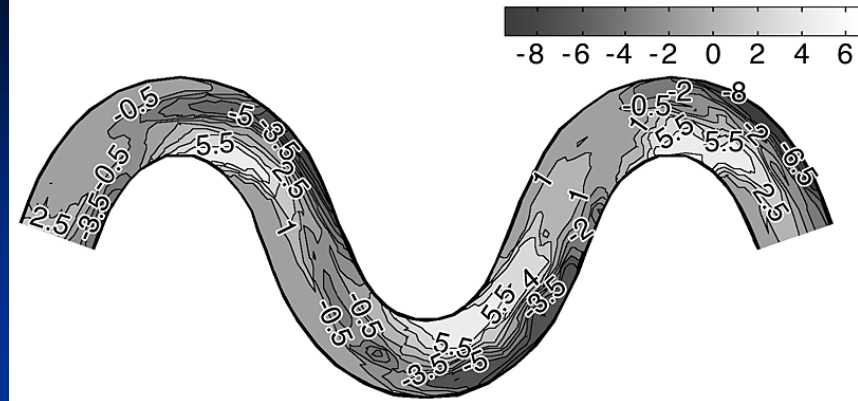
# Experimental runs



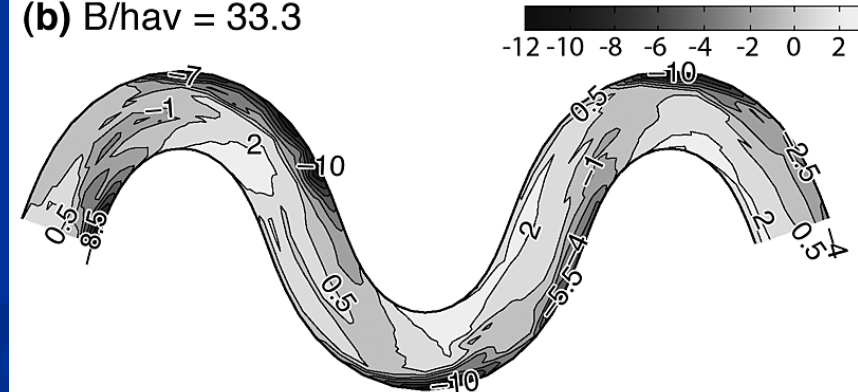
$\theta_0 = 70^\circ$ ;  $B = 0.80\text{m}$ ; 8 runs

$10.7 \leq B/h_{av} \leq 66.7$      $(c_f)_{av} = 15.0$

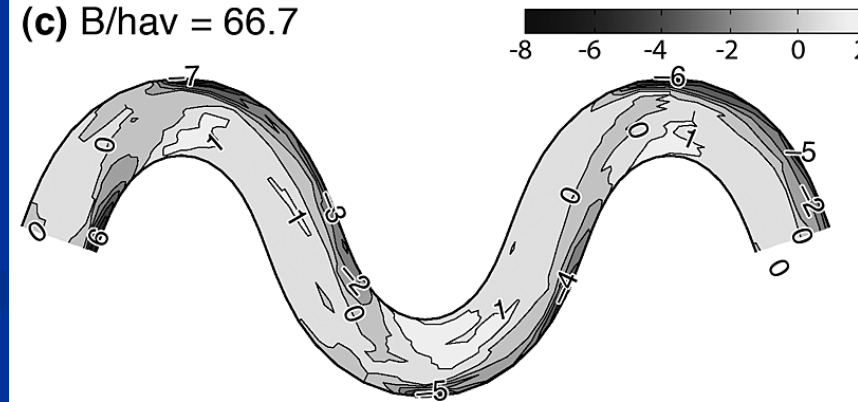
(a)  $B/h_{av} = 11.0$  (Run 2 by da Silva and El-Tahawy 2008)

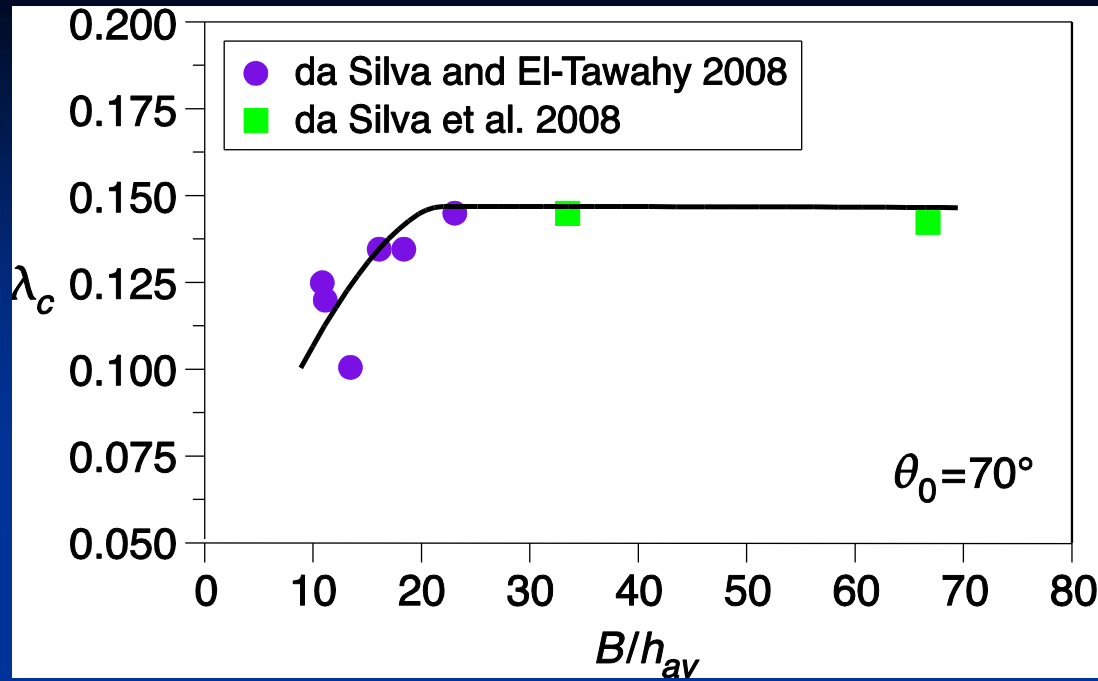


(b)  $B/h_{av} = 33.3$



(c)  $B/h_{av} = 66.7$

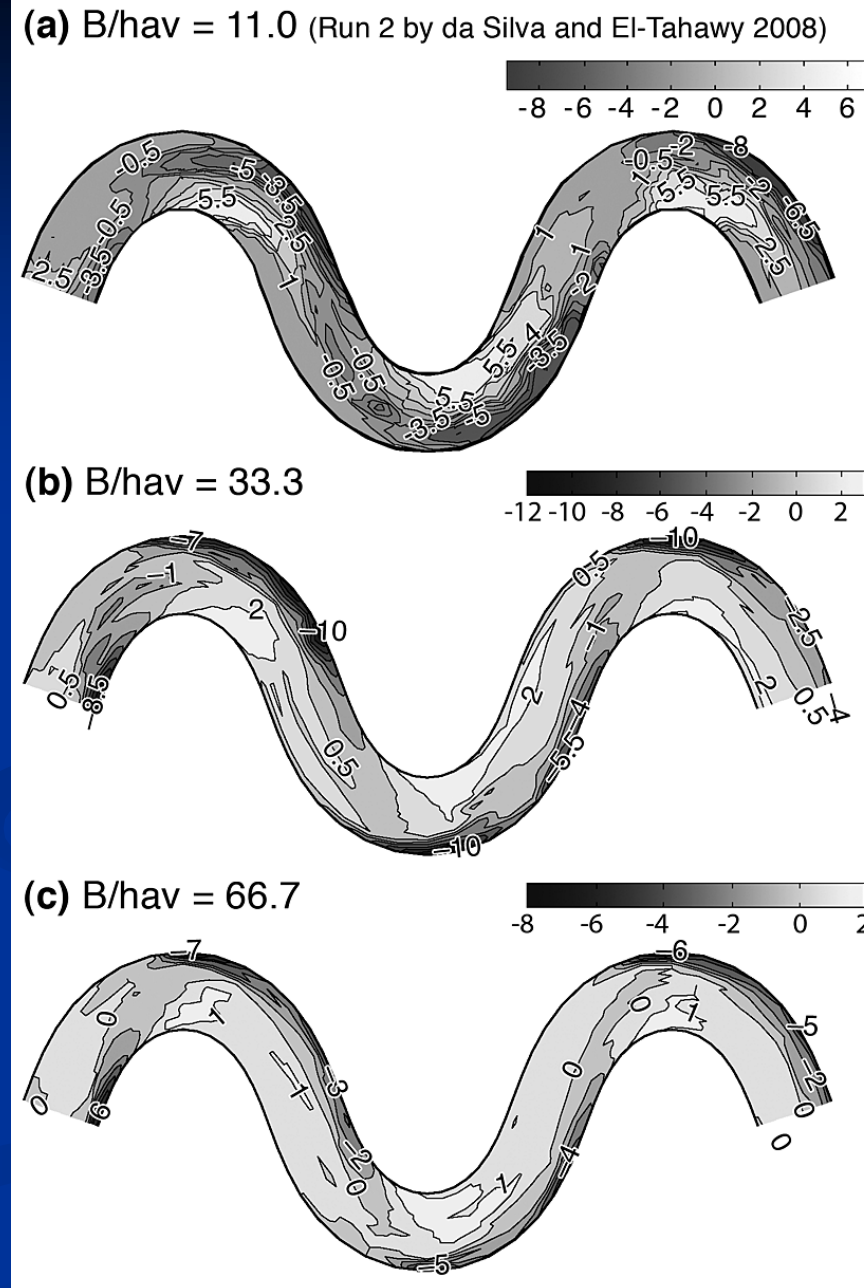


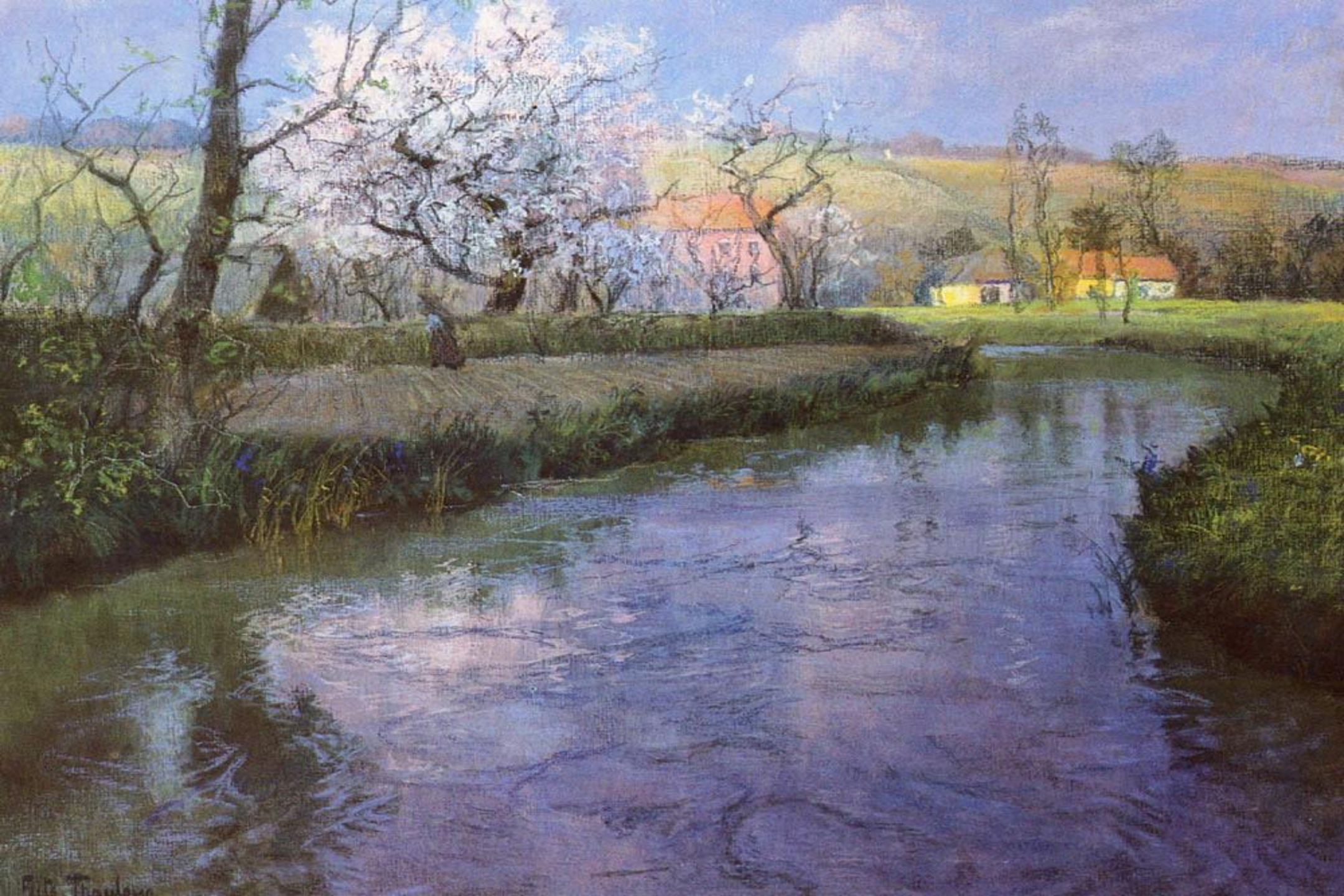


$\theta_0 = 70\text{deg}$ ;  $B = 0.80\text{m}$ ; 8 runs

$10.7 \leq B/h_{av} \leq 66.7$      $(c_f)_{av} = 15.0$

**Conclusion 1: Critical  $B/h_{av}$  is approximately equal to 20 if  $\theta_0 = 70\text{deg}$**





Ed. Toulon