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Analysis of Pressure Wave Velocity in a Steel Pipeline with Inserted Fiber Optic Cable

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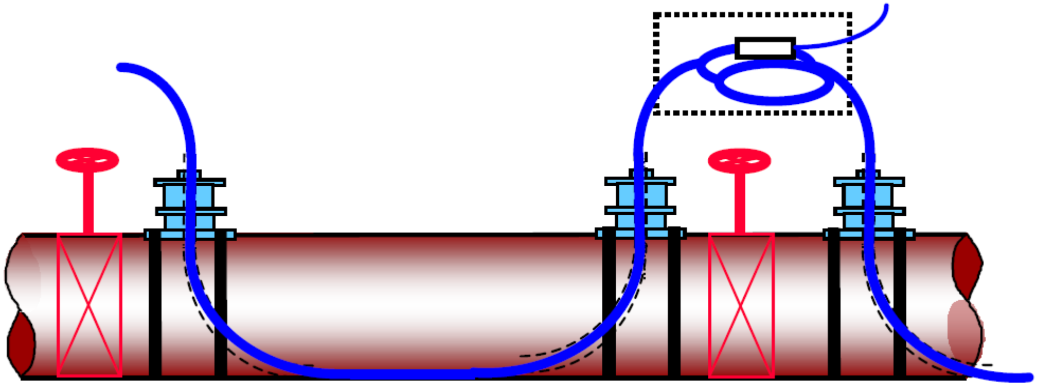
Introduction

Jeyapalan (2007)



Water hammer phenomenon in a pipeline

Pressure increase during water hammer phenomenon is described with Joukovsky's equation:

$$\Delta p = \rho c \Delta v$$

where:
Δp – pressure increase [Pa],
ρ – density of water [kg/m^3],
p – pressure wave velocity [m/s],
Δv – change in flow velocity [m/s].

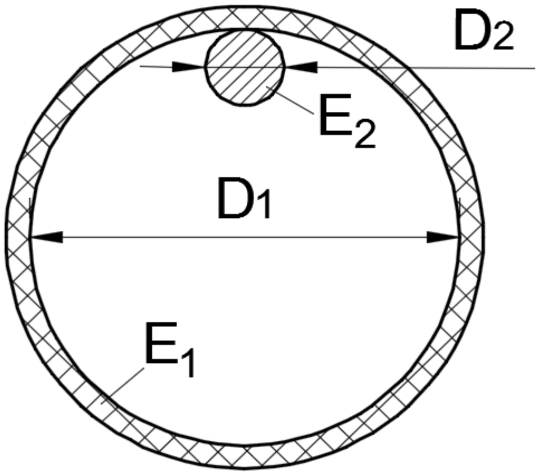
Water hammer phenomenon in a pipeline

The pressure wave velocity is described with Korteweg-Joukovsky equation:

$$c = \frac{\sqrt{\frac{K}{\rho}}}{\sqrt{1 + \frac{KD}{Ee}}}$$

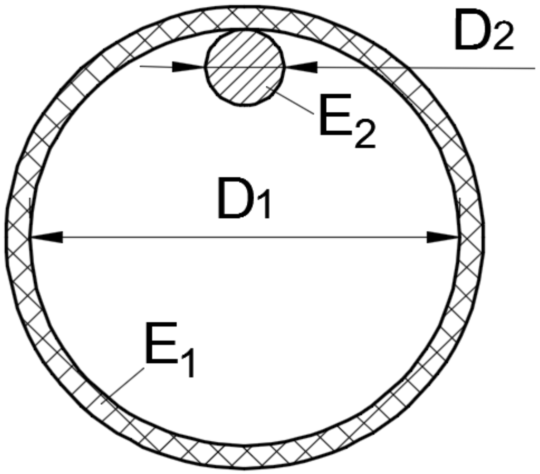
where:
K– water compressibility[Pa],
D – inner pipe diameter [m],
E – Young's modulus of the pipe [Pa],
e – pipe wall thickness[m].

Scheme of placement of fiber optic cable in the pipe



D_1 – inner diameter of the pipe [m],
D_2 – outer diameter of the fiber optic cable [m],
E_1 – Young's modulus of the pipe [Pa],
E_2 – Young's modulus of the fiber optic cable [Pa].

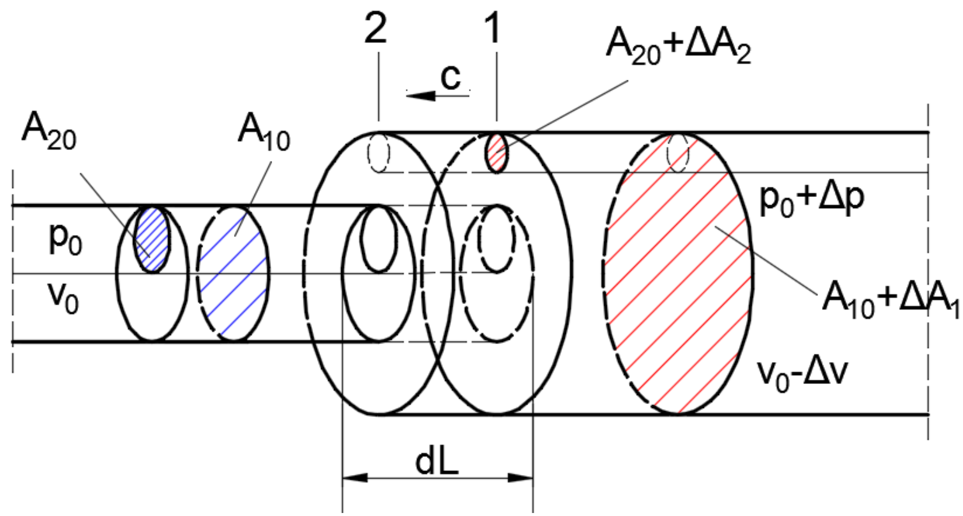
Scheme of placement of fiber optic cable in the pipe



$$A = A_1 - A_2$$

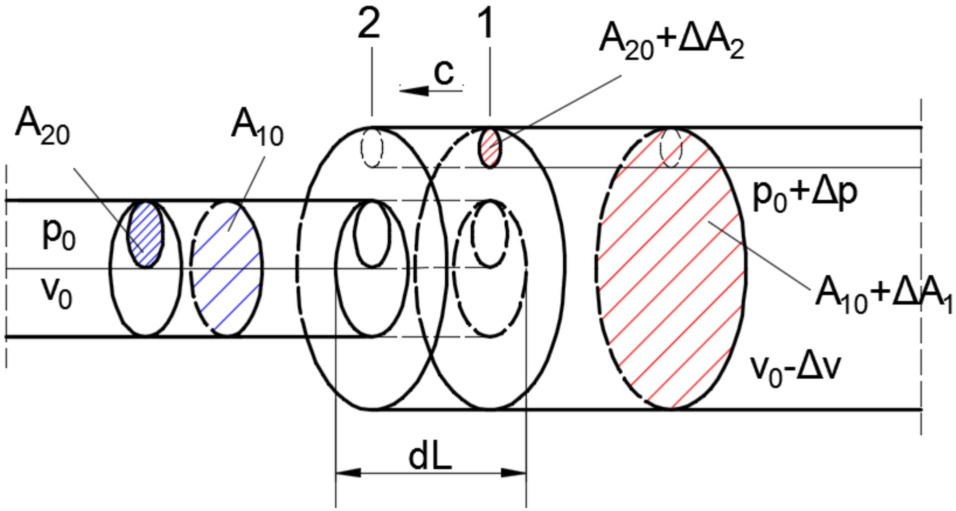
A – cross sectional area of liquid stream in the pipeline [m²],
A_1 – cross sectional of pipe diameter D_1 [m²],
A_2 – cross sectional of fiber optic cable diameter D_1 [m²].

Water hammer phenomenon in the pipeline with inserted fiber optic cable



A_{10} – cross sectional of pipeline diameter D_1 before the inititation of the phenomenon [m^2],
A_{20} – cross sectional of fiber optic cable diameter D_1 before the initiation of the phenomenon [m^2].

Water hammer phenomenon in the pipeline with inserted fiber optic cable



$$A_1 = A_{10} + \Delta A_1$$
$$A_2 = A_{20} + \Delta A_2$$

$$\Delta A_2 < 0$$

$$[\rho(A_1 - A_2) - \rho_0(A_{10} - A_{20})]dL = [\rho_0(A_{10} - A_{20})v_0 - \rho(A_1 - A_2)v]dt$$
$$[\rho_0(\Delta A_1 - \Delta A_2) + \Delta\rho(A_{20} - A_{10})]dL = [\rho_0 \Delta v(A_{10} - A_{20})]dt$$
$$dL = cdt$$
$$c = \frac{\rho_0 \Delta v(A_{10} - A_{20})}{\rho_0(\Delta A_1 - \Delta A_2) + \Delta\rho(A_{10} - A_{20})}$$

Elastic deformation

$$\Delta\sigma = E\frac{\Delta l}{l} = E\frac{\Delta D}{D}$$

Steel pipeline

$$\frac{\Delta D_1}{D_1} = \frac{\Delta p D_1}{2E_1 e_1}$$

$$\frac{\Delta A_1}{A_{10}} = 2\frac{\Delta D_1}{D_1}$$

$$\frac{\Delta A_1}{A_{10}} = \frac{\Delta p D_1}{E_1 e_1}$$

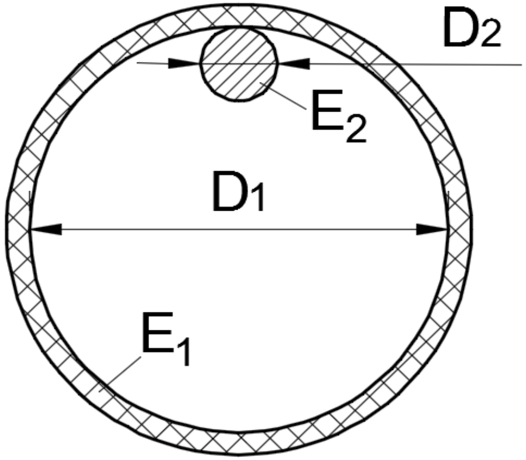
Fiber optic cable

$$\Delta l = \frac{1}{2}\pi D_2 \frac{\Delta p}{E_2}$$

$$\Delta A_2 < 0$$

$$\frac{\Delta A_2}{A_{20}} = -\frac{\Delta p}{E_2}$$

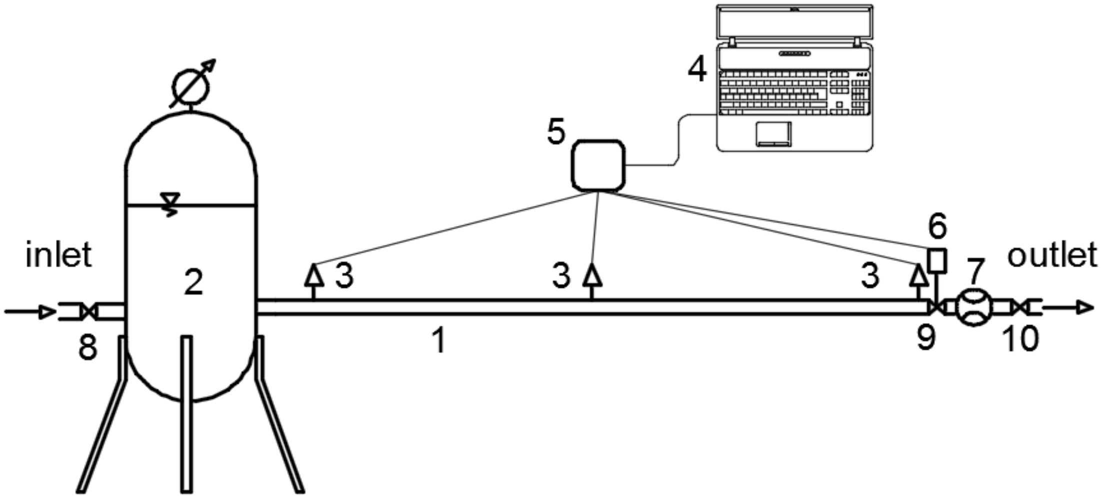
Water hammer phenomenon in the pipeline with inserted fiber optic cable



Pressure wave velocity can be expressed as follows:

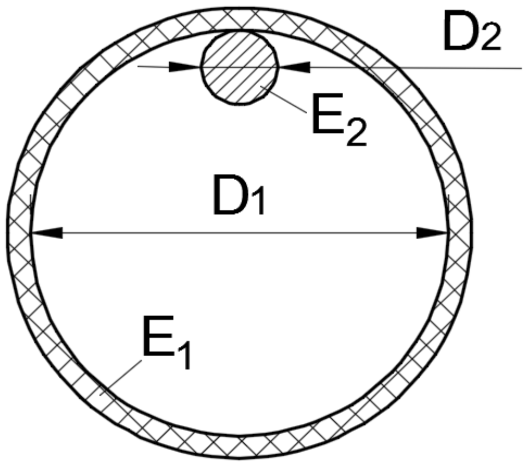
$$c = \frac{\sqrt{\frac{K}{\rho}}}{\sqrt{1 + \frac{A_1}{A}\frac{KD_1}{Ee_1} + \frac{A_2}{A}\frac{K}{E_2}}}$$

Experimental tests



1. The steel pipe
2. Pressure tank
3. Pressure sensors
4. Recorder of pressure samples – laptop
5. Analog-to-digital card
6. Valve closure time meter
7. Inductive flowmeter
8. Valve
9. Valve of initiating the water hammer phenomenon
10. Water flow regulation valve

Experimental tests

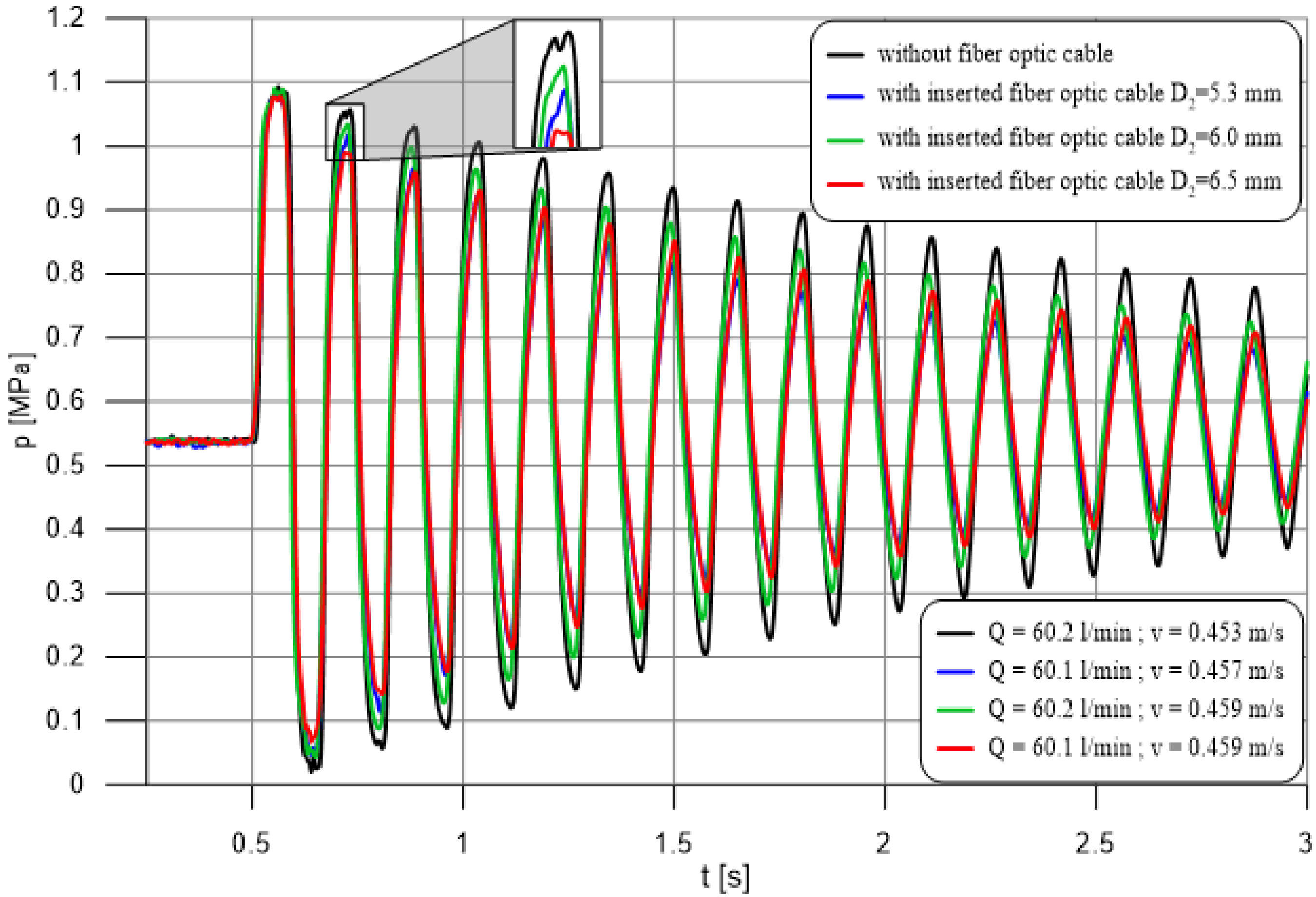


$$D_1 = 0.0531\ m$$
$$e_1 = 0.0035\ m$$

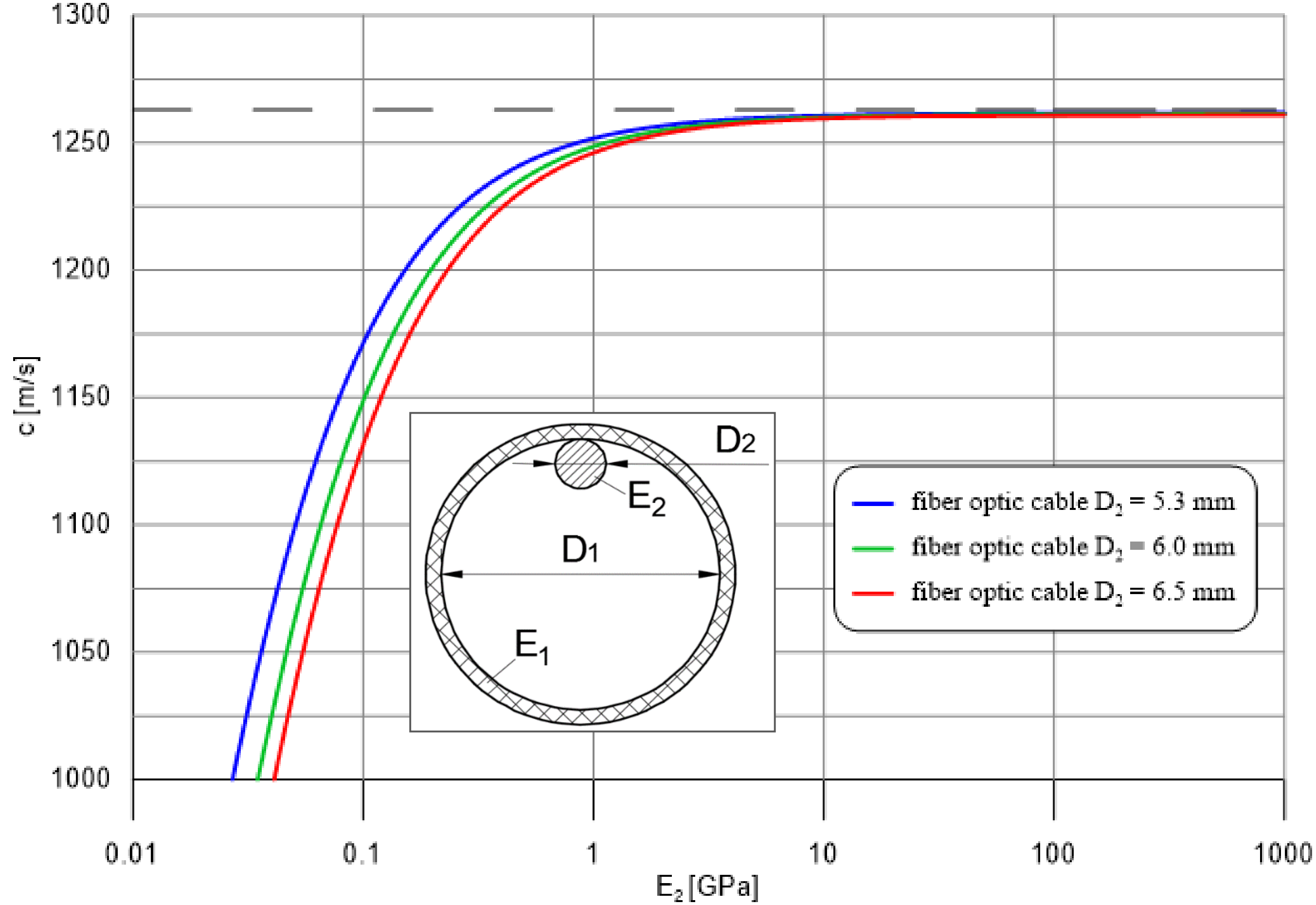
$$D_2 = 5.3\ mm$$
$$D_2 = 6.0\ mm$$
$$D_2 = 6.5\ mm$$

$$Q = 60\ l/min$$

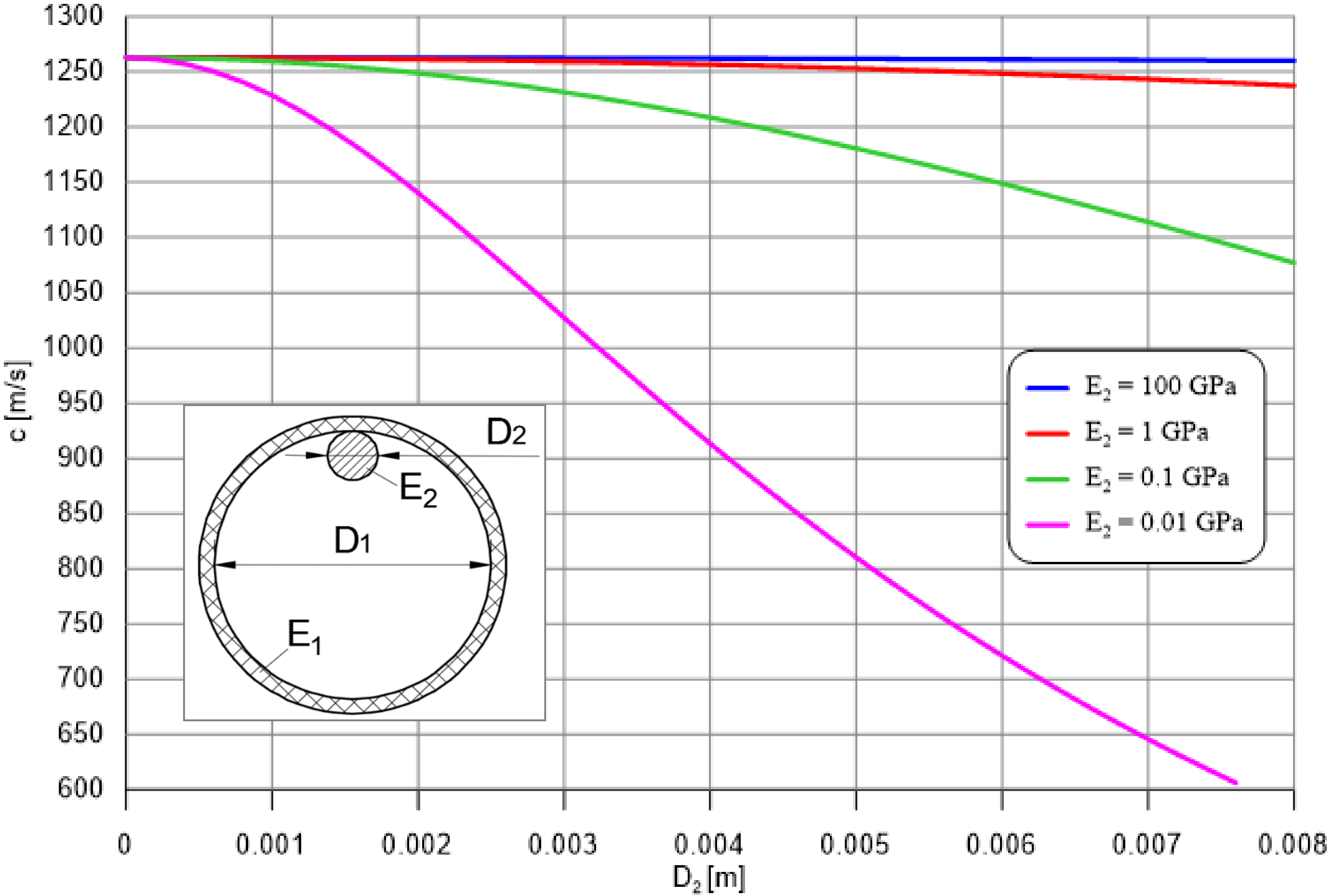
Results of the experiment



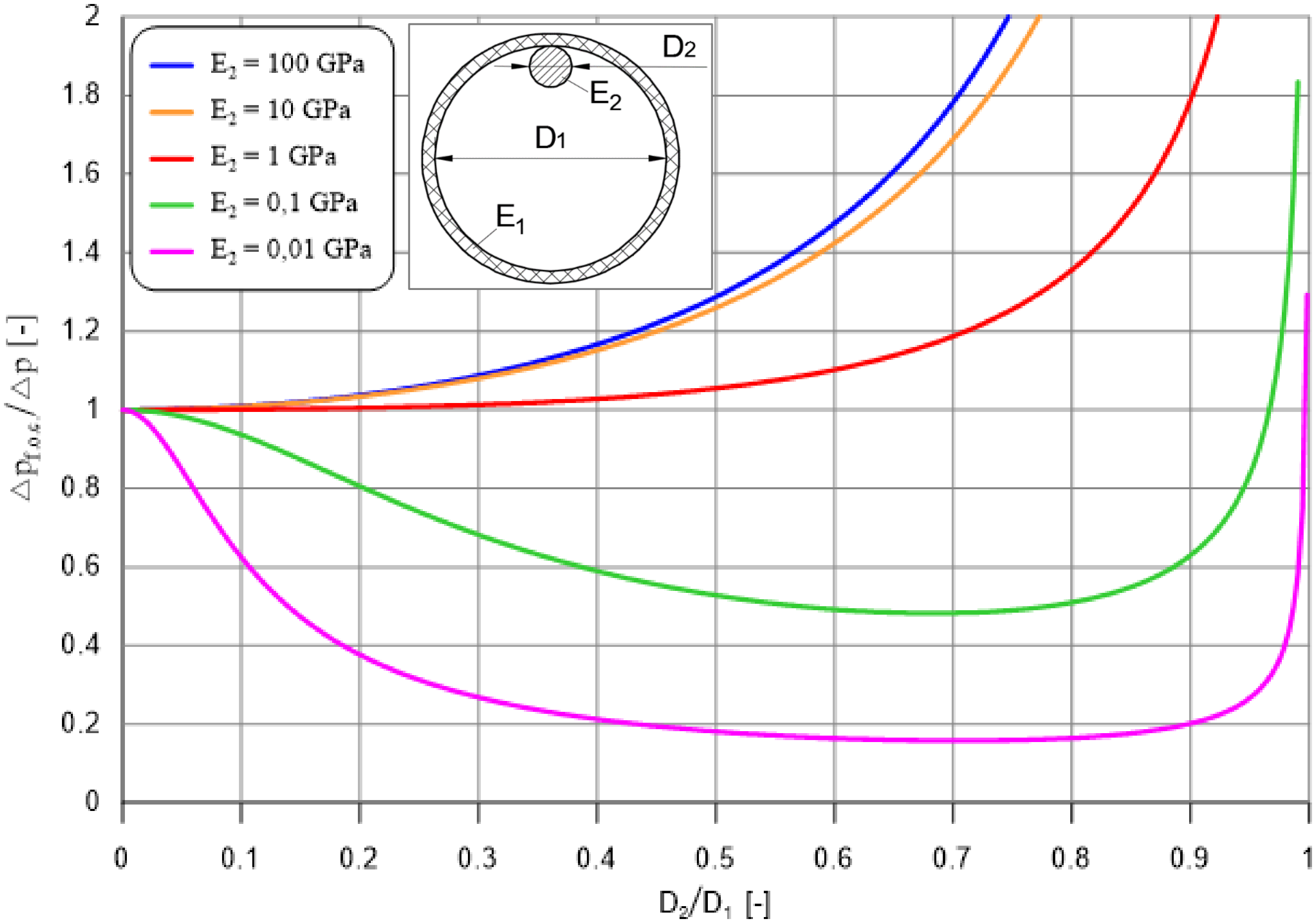
Theoretical analysis of pressure wave velocity



Theoretical analysis of pressure wave velocity



Theoretical analysis of pressure wave velocity



Conclusion and summary

- Pressure wave velocity in the pipeline with inserted fiber optic cable depends on the diameter and elasticity of the cable
- The greater the diameter and the lower cable's Young's modulus, the greater effect it has on reducing pressure wave velocity
- If the fiber optic cable has a similar elasticity compared to the pipe's material, its effect on the pressure wave velocity is practically negligible
- Inserting a cable with a low Young's modulus can have a significant impact on reducing the increase of pressure caused by water hammer