Russian Academy of Sciences
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Mathematical modelling of sand-gravel bed evolution in one dimension

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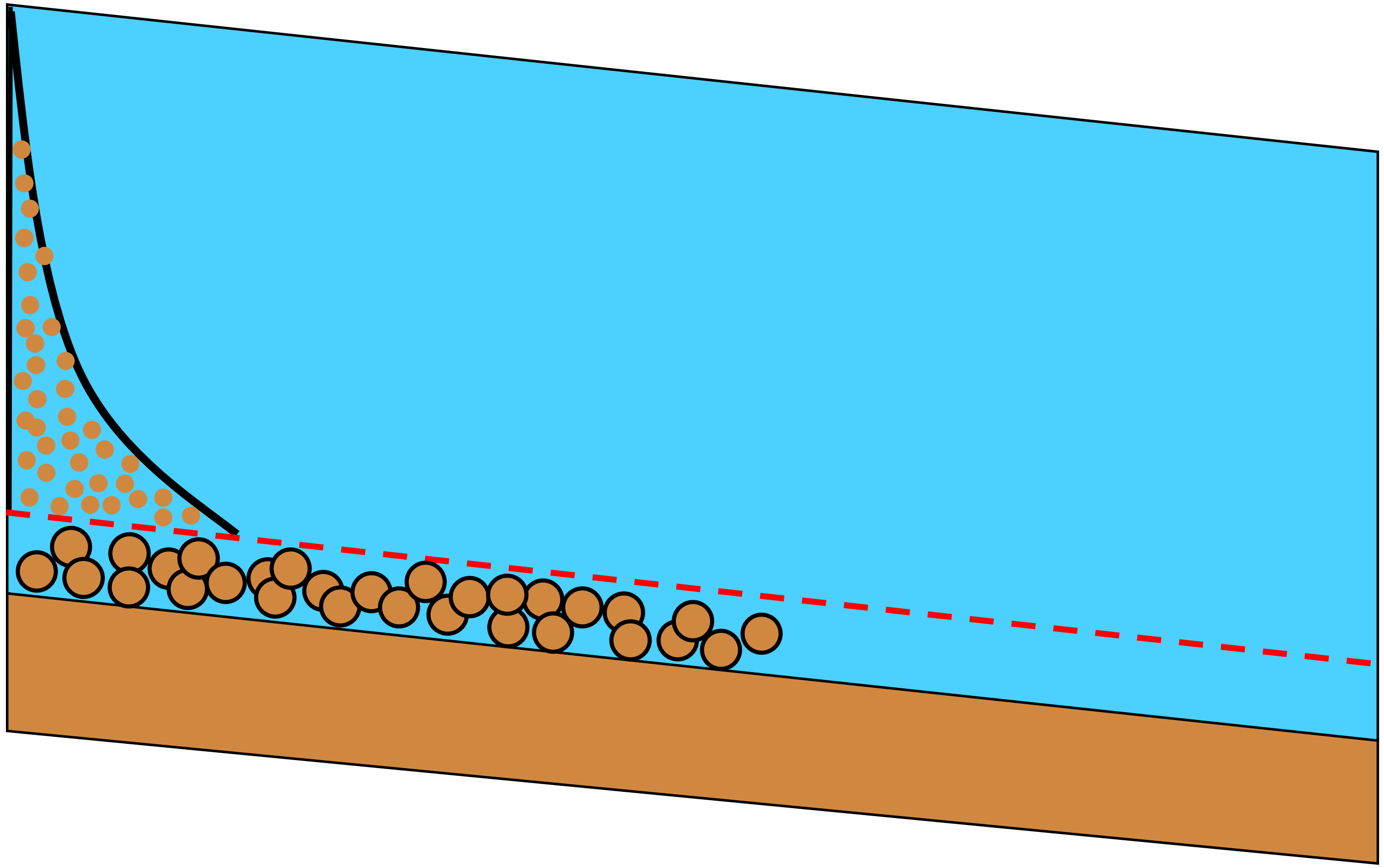
Presentation outline

1. Introduction
2. The purpose of the research
3. Mathematical formulation
4. The model verification
5. Conclusion

Introduction

Sediment

- Suspended sediment →
- Bed-load sediment →



Bed-load sediment mostly forms local-scale bed shape

Introduction

Bed-load transport models

[Shamov, 1959]

$$G = k\left(\frac{v_m}{v_s}\right)^3 (v_m - v_s)\left(\frac{d_{50}}{H}\right)^{1/4}$$

[Bagnold, 1966]

$$G = \frac{e_b}{\tan \alpha_*}\psi_G$$

Phenomenological parameters

[van Rijn, 1986]

$$G = 0.1 H d_{50} u'_{*,c}\frac{\tau^{3/2}}{D_*^{0.3}}$$

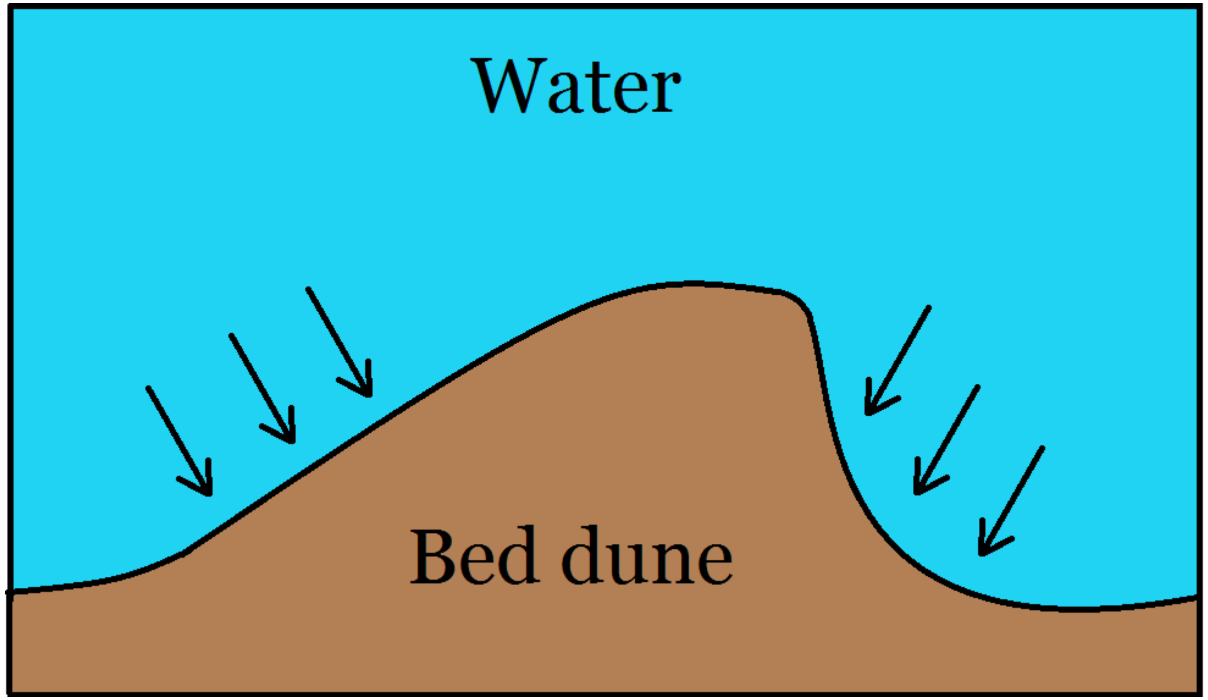
[Wu, Wang and Jia, 2000]

$$\frac{G_k}{p_k\sqrt{\gamma_s(\gamma - 1) g d_k^3}} = 0.0053\left[\left(\frac{n'}{n}\right)^{3/2}\frac{\tau_b}{\tau_{ck}} - 1\right]^{2.2}$$

The purpose of the research

To model accurately **local-scale** bed evolution, e.g.:

- degradation behind the dam;
- reservoir siltation;
- dunes development.

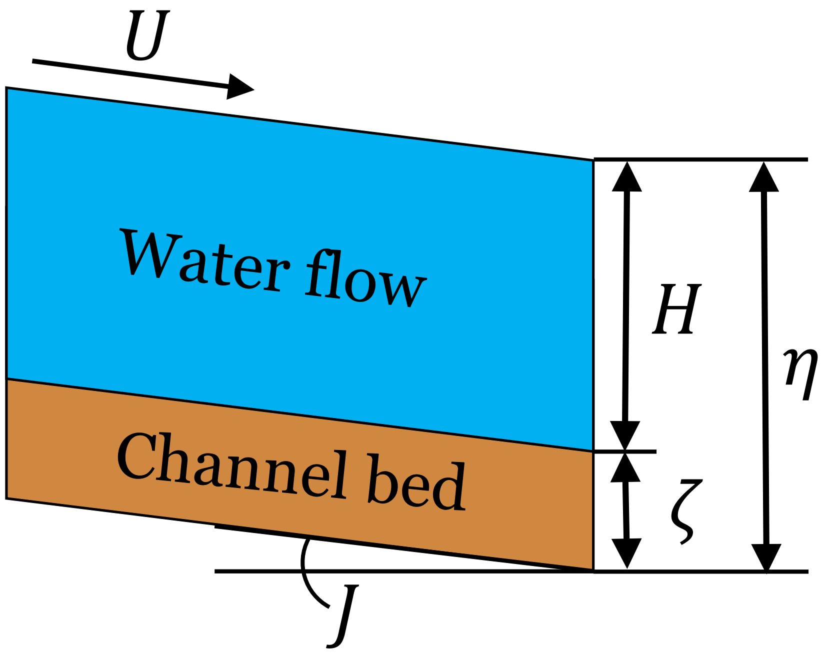


Analytical bed-load transport model

Mathematical formulation

The stationary hydrodynamic equations in shallow water approach [Grishanin, K.V., 1974]

(1) $$\frac{\partial}{\partial x}\left(\frac{U^2}{2g}+\eta\right)+\frac{\tau}{gH\rho_w}=0, \qquad Q=UH,$$



The Chezy-Manning formula for bed shear stress [Grishanin, K.V., 1974]

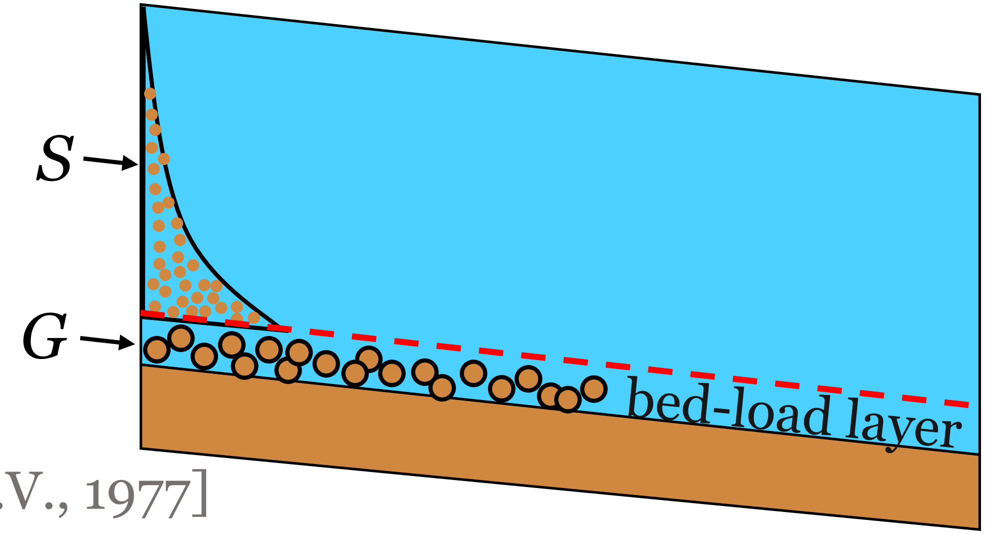
(2) $$\tau=\rho_w g\frac{U^2}{C^2}, \qquad C=\frac{H^{1/6}}{n_s}, \qquad n_s=\frac{H^{2/3}\sqrt{J}}{U}.$$

Where Q - the water rate, τ - the bed shear stress, n_s - the Manning roughness coefficient.

Mathematical formulation

The Exner equation [Exner, F.M., 1925]

$$(3)\quad (1-\varepsilon)\rho_s \frac{\partial \zeta}{\partial t} + \frac{\partial G}{\partial x} = -\alpha \frac{W}{H}(S_* - S),$$



The suspended transport equations [Karaushev, A.V., 1977]

$$(4)\quad \frac{\partial S}{\partial t} + \frac{\partial SU}{\partial x} = \alpha \frac{W}{H}(S_* - S), \qquad S_* = \begin{cases} \beta \dfrac{U^3}{WH}, & W < u_*, \\ 0, & W \geq u_*; \end{cases} \qquad u_* = \sqrt{\frac{\tau}{\rho_w}}.$$

Phenomenological parameters

Where ρ_s - the water density, ε - the bed material porosity, W - the particle fall velocity, $\alpha \in (0;1)$ - the adaptation coefficient, $\beta = 0.2$, S_* - the suspended transport capacity

Mathematical formulation

The analytical bed-load rate formula [Petrov, P.G., 1991]

Granulometrical and physical-mechanical parameters of the bed material

Local bed slope

Bed-shear stress

(5) $$G = G_0 \tau^{3/2} \left[(1-\chi) - \left(1 - \frac{\chi}{2}\right) \frac{1}{\tan\varphi \cos\gamma} \frac{\partial \zeta}{\partial x} \right],$$

(6) $$G_0 = \frac{4}{3} \frac{\rho_s m}{\kappa \sqrt{\rho_w} (\rho_s - \rho_w) g \tan\varphi \cos\gamma}, \qquad m = \begin{cases} 1, \chi < 1, \\ 0, \chi \geq 1; \end{cases}$$

(7) $$\chi = \sqrt{\frac{\tau_*}{\tau}}, \qquad \tau_* = \frac{3}{8} \frac{\kappa^2 d (\rho_s - \rho_w) g \tan\varphi \cos\gamma}{c_x}.$$

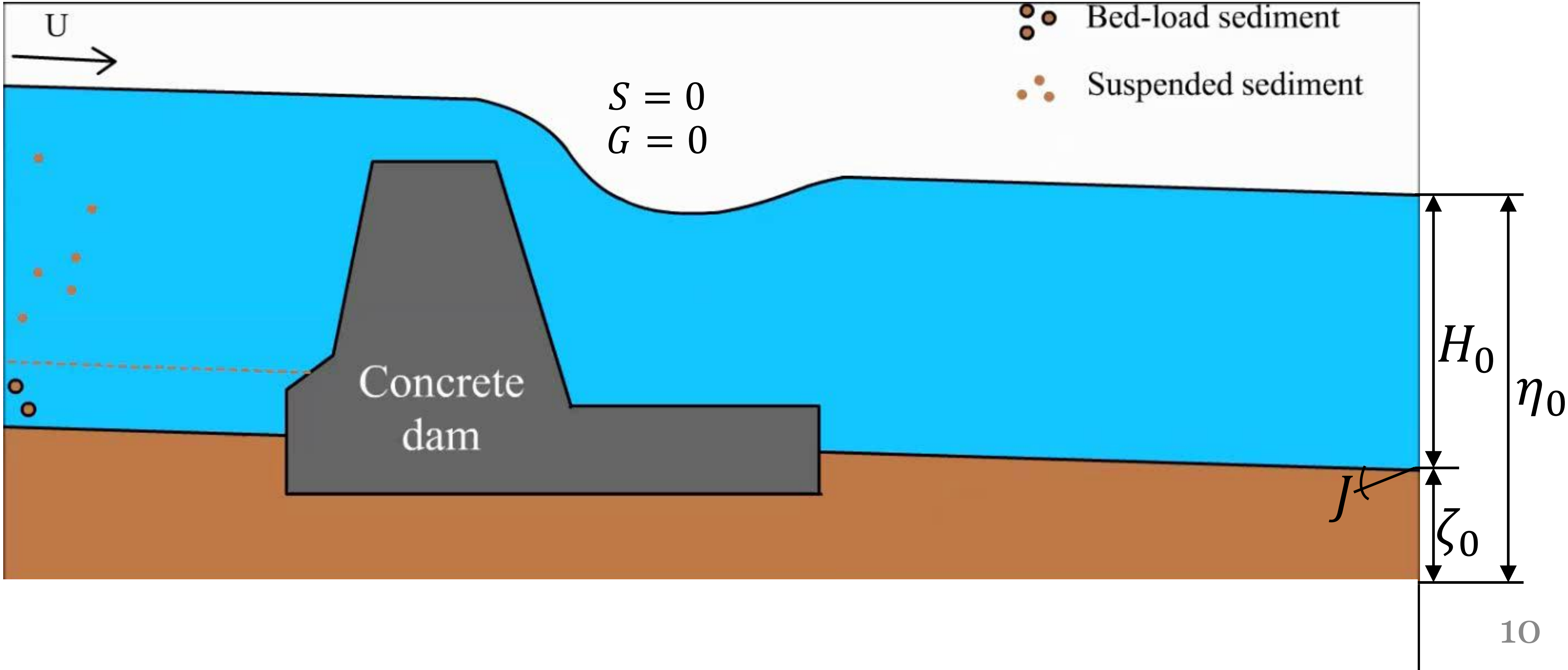
Where τ_* - the critical bed-shear stress, φ - the internal friction angle of particles, c_x - the frontal particle drag coefficient, κ - the Karmans constant.

The model verification

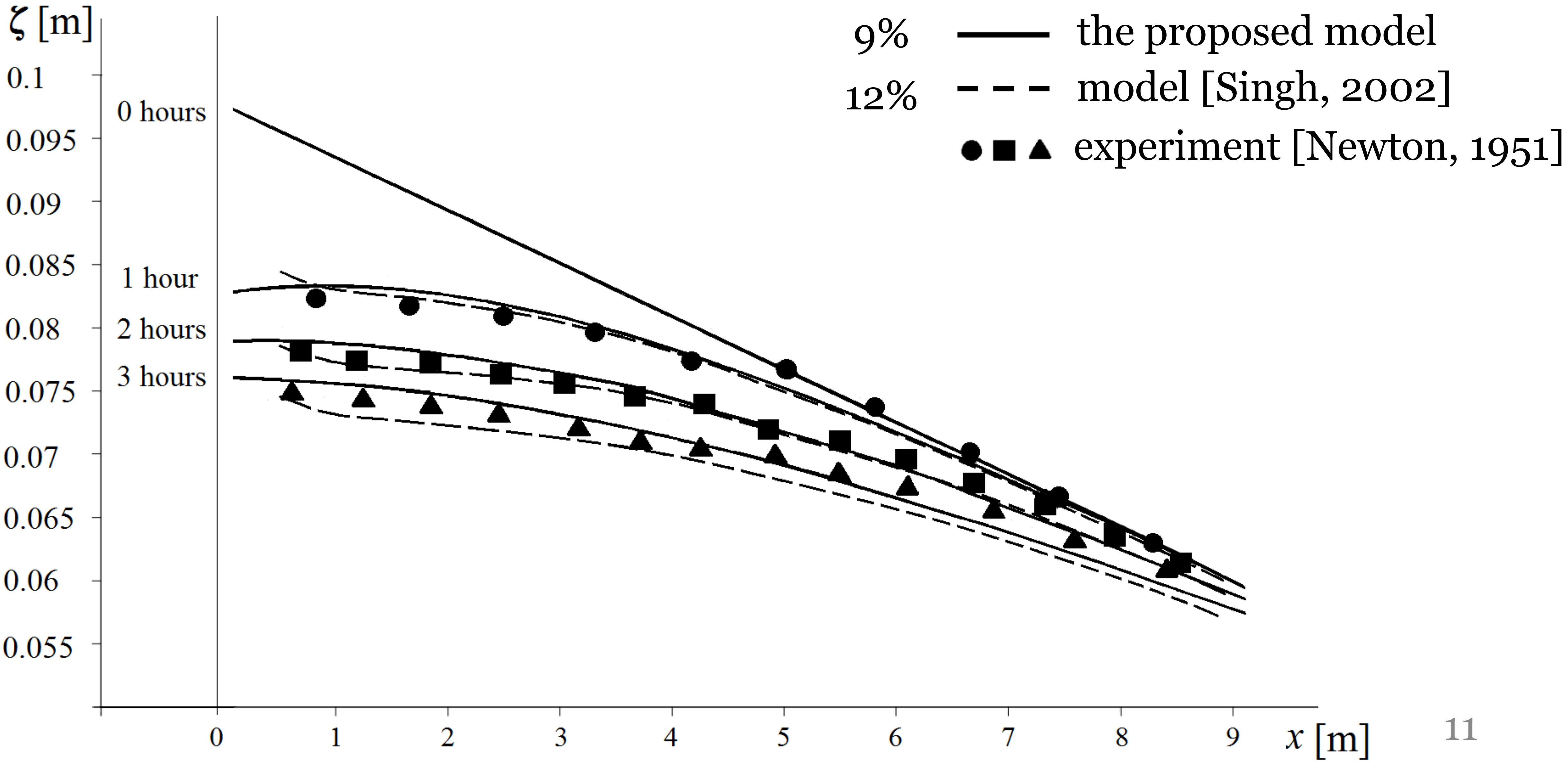
In order to estimate the ability of the model to describe local-scale bed evolution the next problems are solved:

- local bed degradation behind the dam,
- local bed aggregation,
- trench evolution.

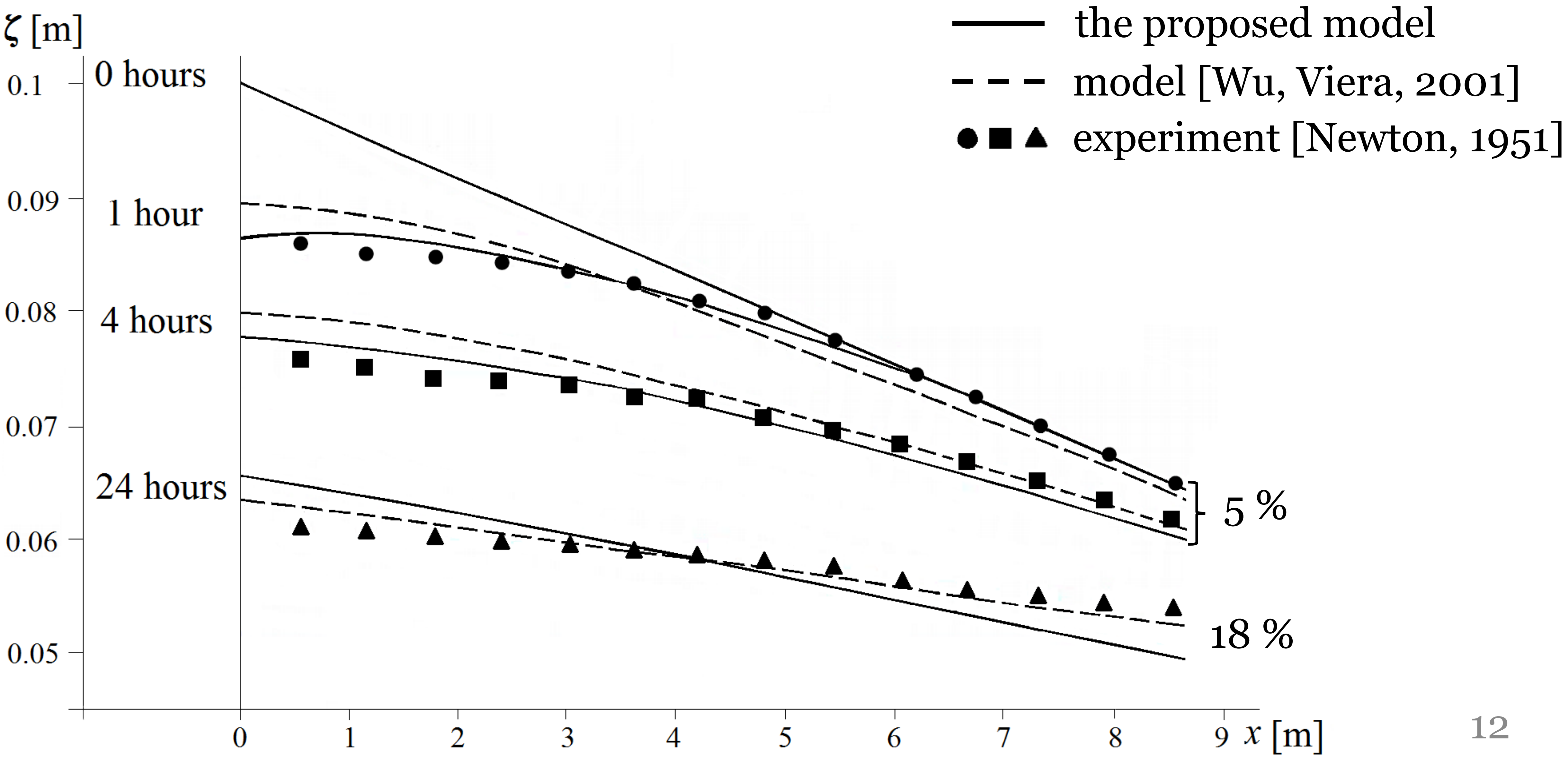
Bed degradation



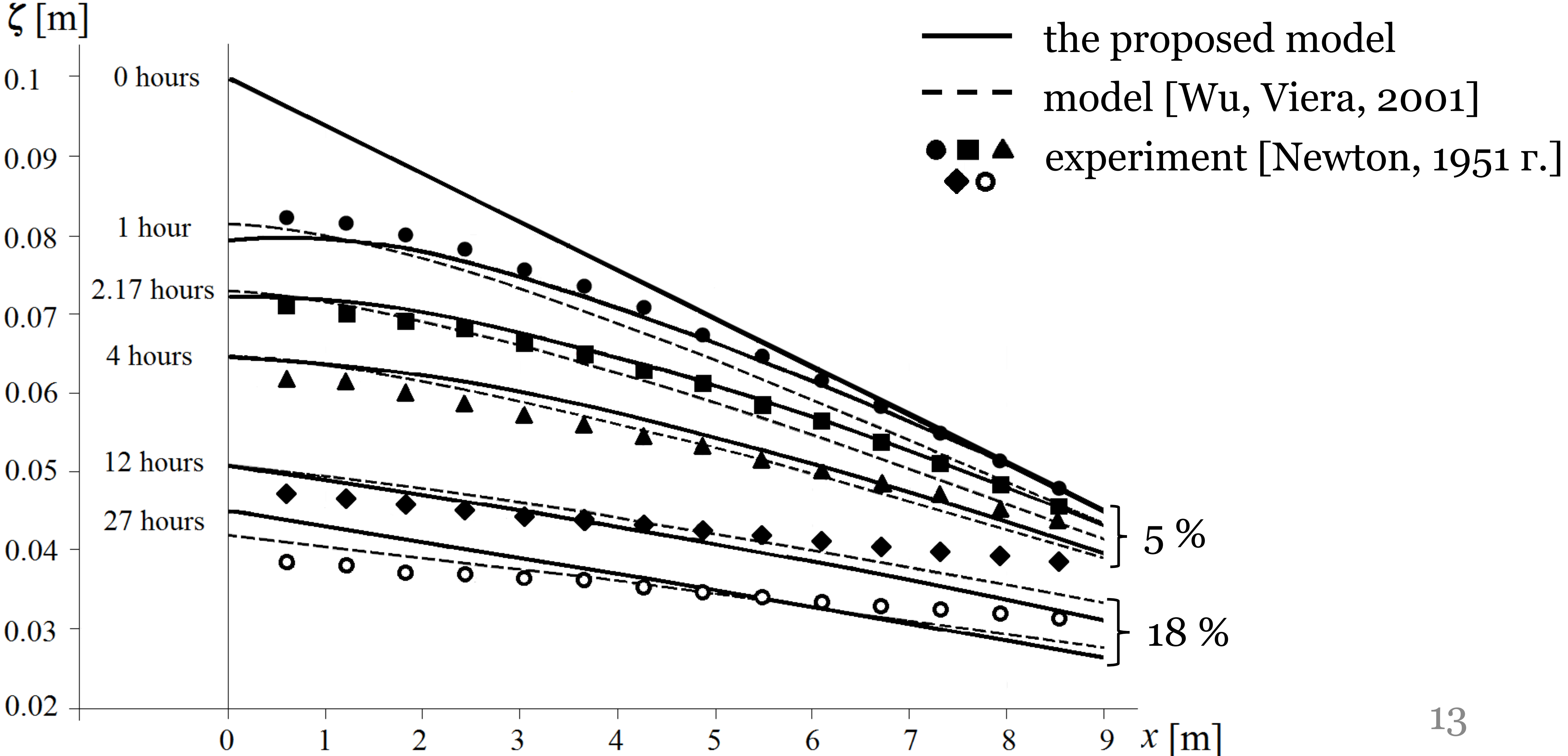
Bed degradation



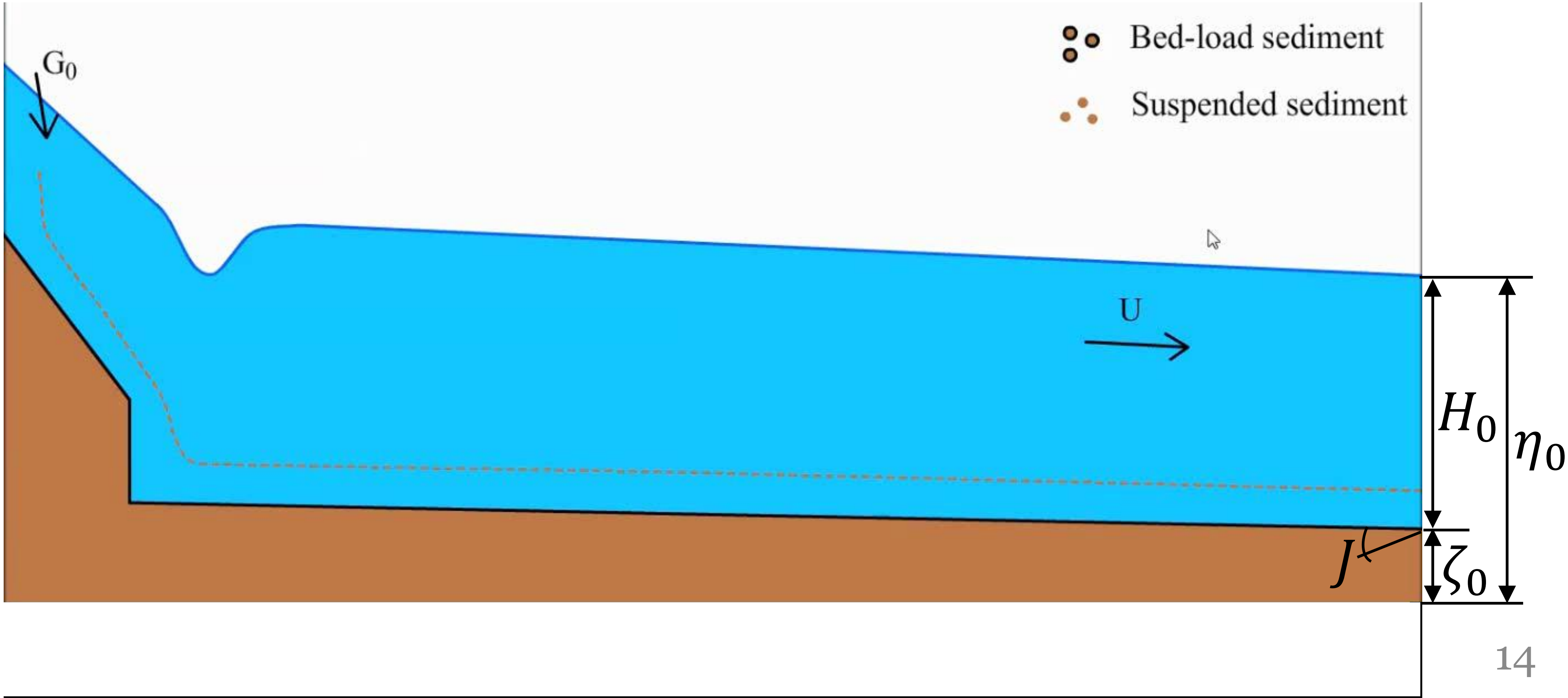
Bed degradation



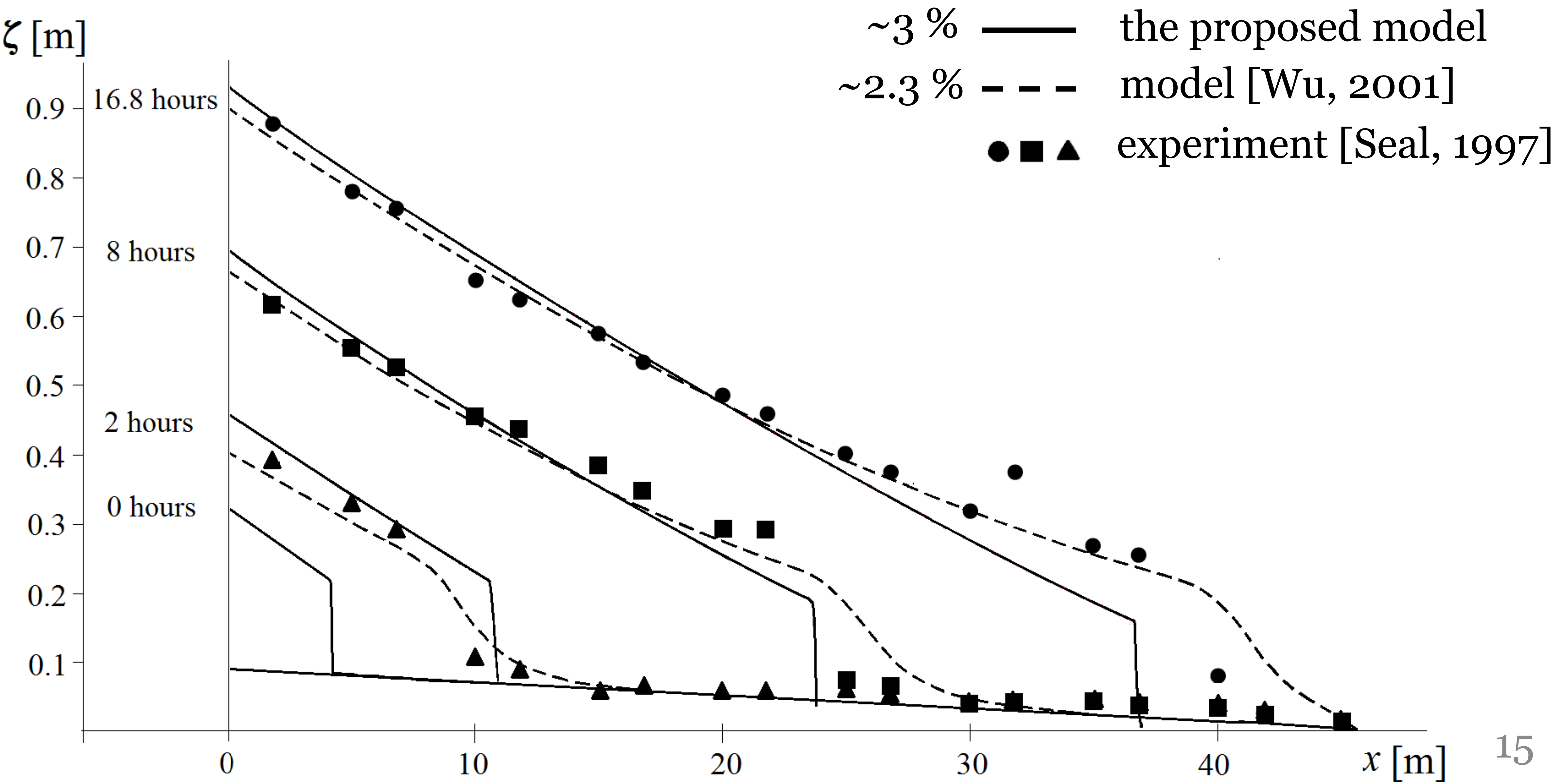
Bed degradation



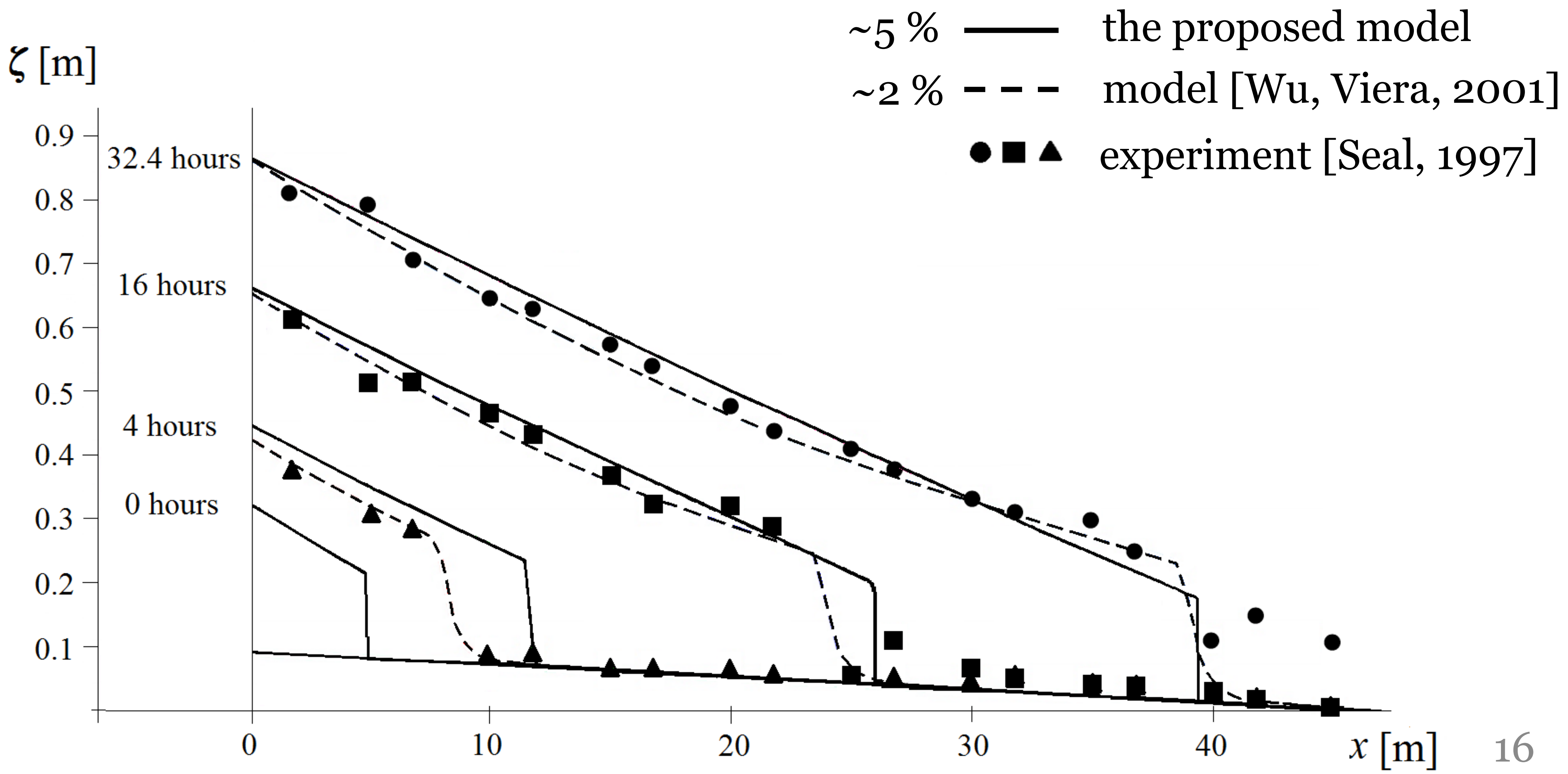
Bed aggregation



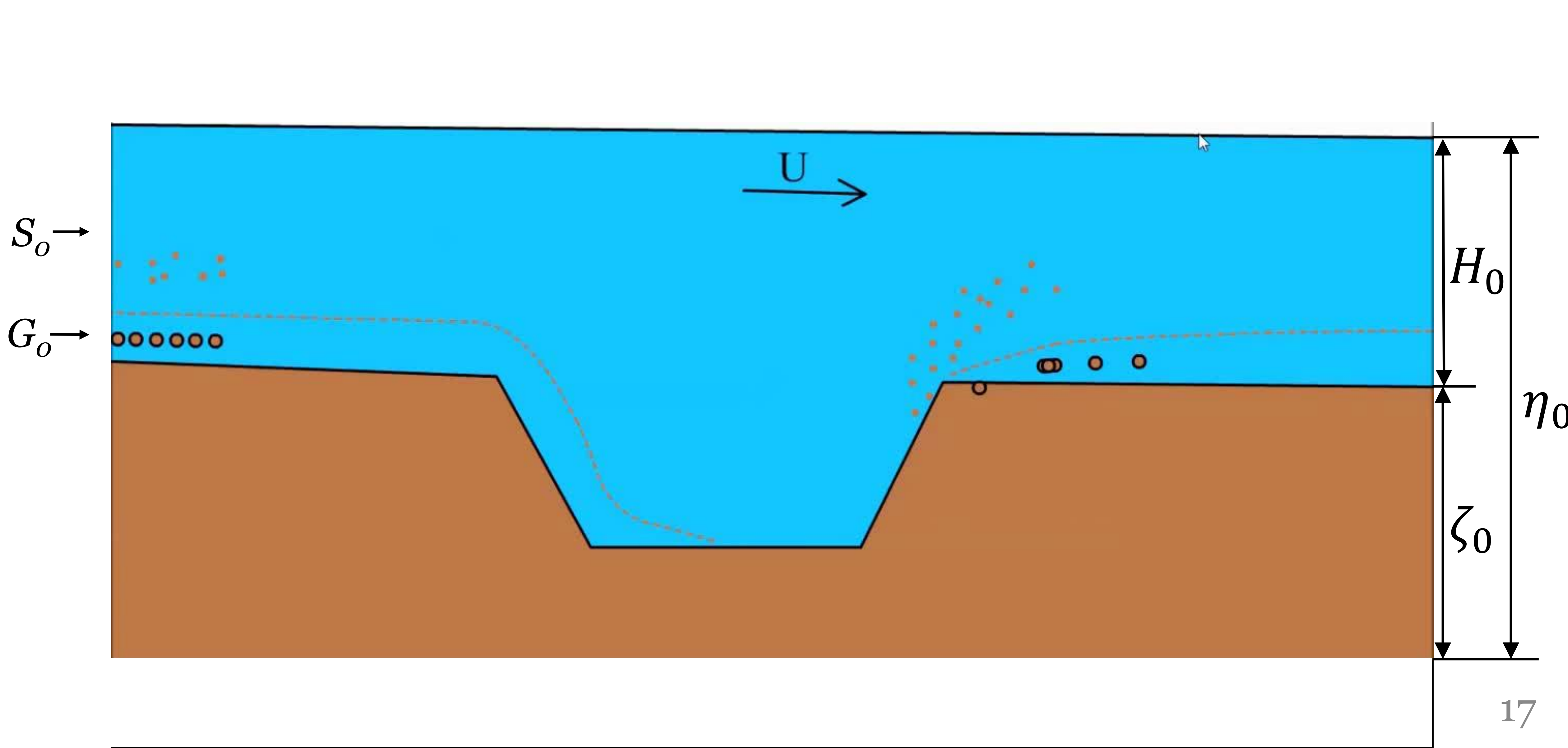
Bed aggregation



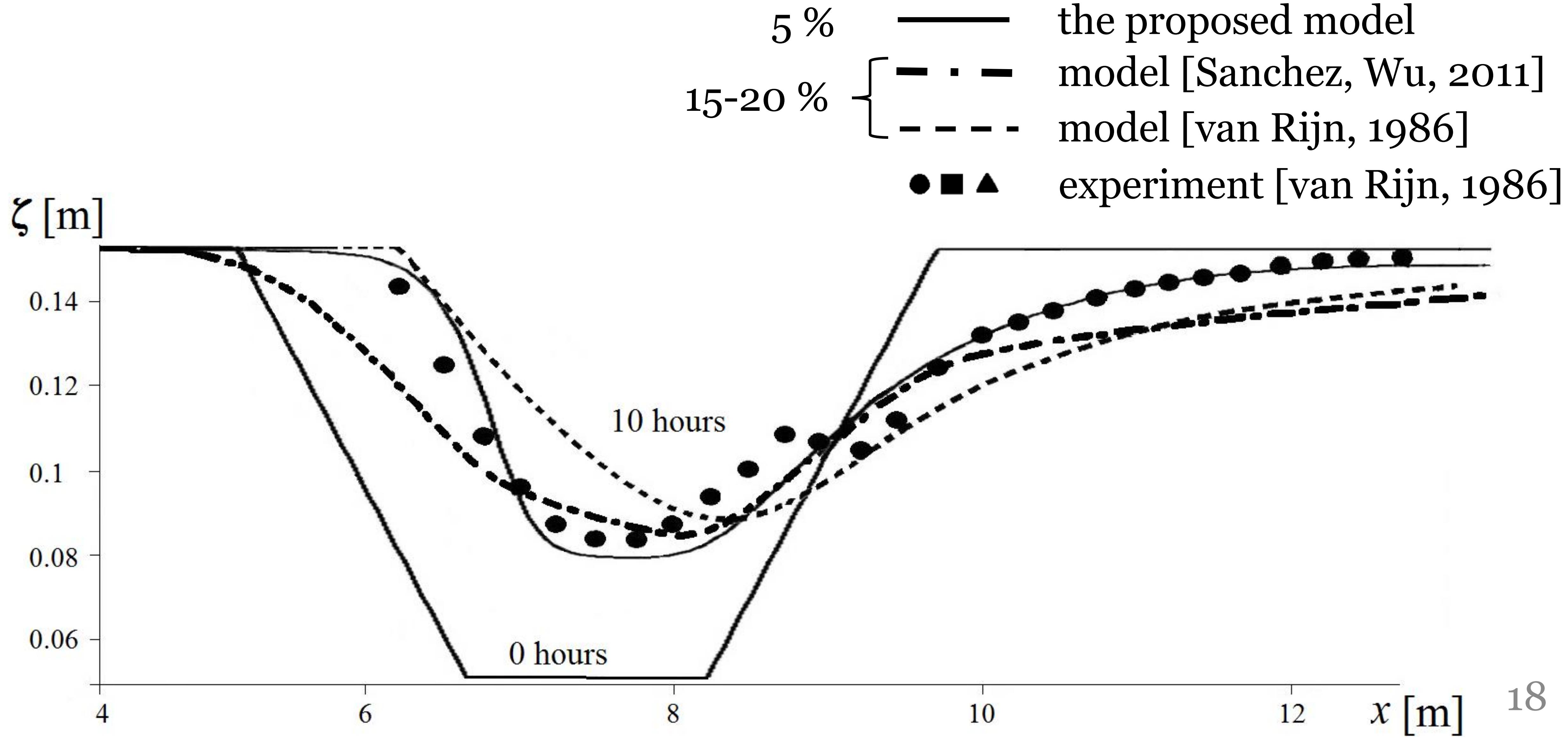
Bed aggregation



Trench evolution



Trench evolution



Conclusion

- The proposed model included the analytical bed-load rate formula is verified on three practice problems.
- The proposed model is adequately describes local-bed evolution.
- It has almost the same accuracy as models [Sanchez, A., Wu, W., 2011; Singh, V., 2002; van Rijn, L. C. 1986; Wu, W, 2001; Wu, W., Vieira, D.A., 2002].

<u>Next research</u>

- By using the analytical bed-load rate formula with 2D hydrodynamic and 2D suspended sediment models to research the bed microforms evolution and estimate their influence on the sediment rate and hydraulic resistance.

Thank you!

Photo from http://www.gazprom.ru/press/news/2013/april/article161128/