GDAŃSK UNIVERSITY OF TECHNOLOGY

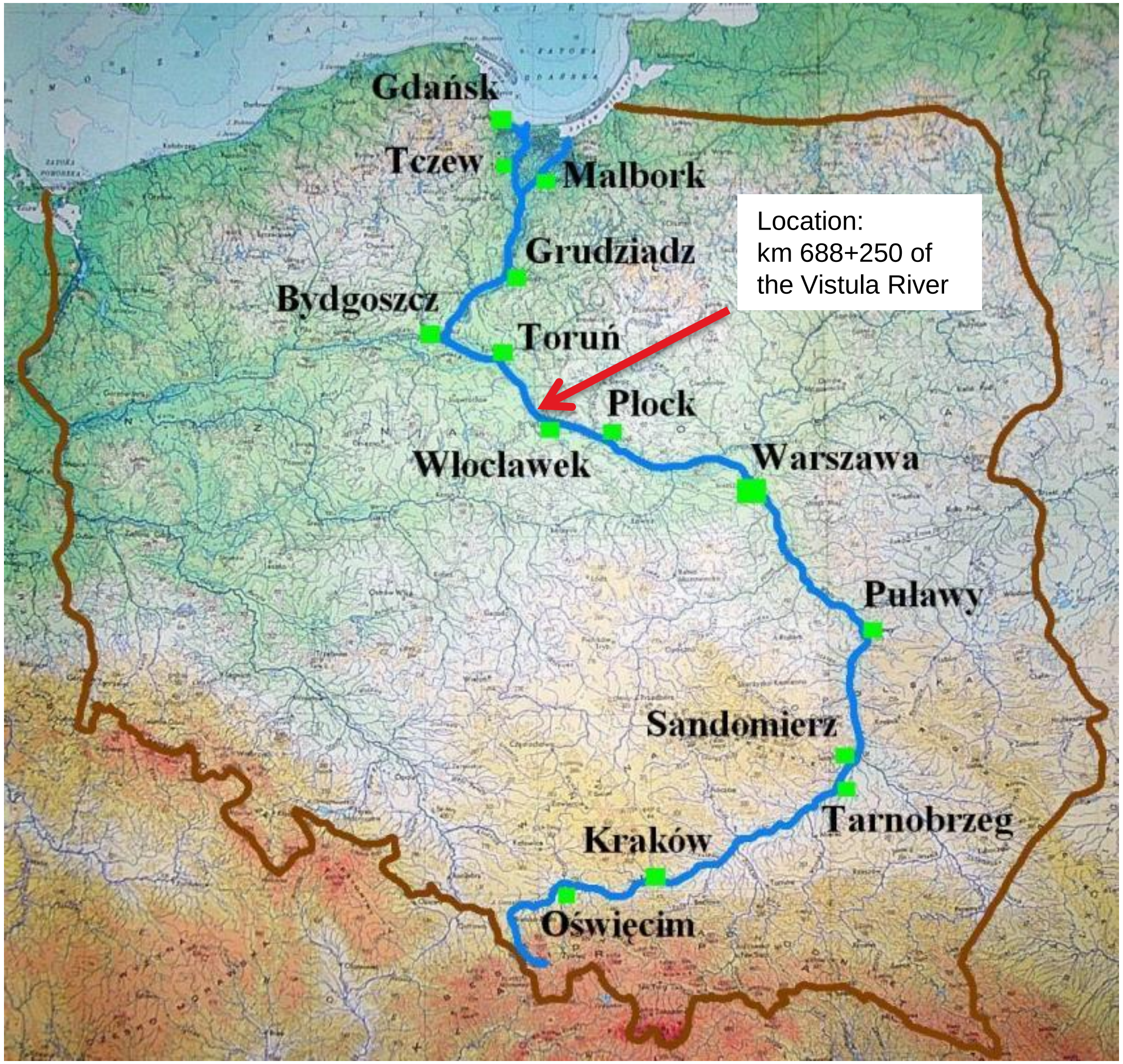
Mathematical modeling of the impact range of sewage discharge on the Vistula water quality in the region of Włocławek

FACULTY OF CIVIL AND ENVIRONMENTAL ENGINEERING

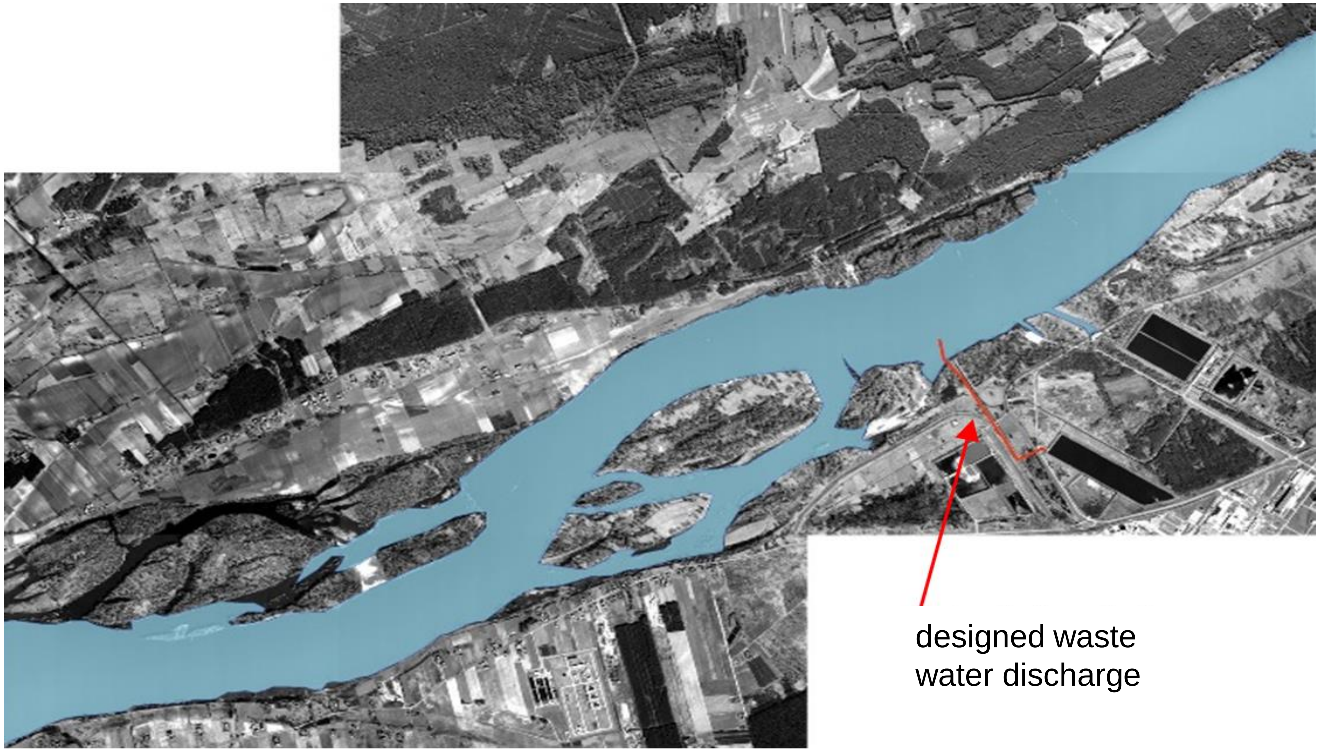
Piotr Zima

May 2017

Location of designed waste water discharge



Discharge parameters



designed waste water discharge

- the pipeline having a diameter of 800 mm
- distance from the shore 150 m

Model development

- Construction of a comprehensive model describing the process of pollution transport requires linking together into one functional unit at least two important elements:

  - the model of water flow in the river
  - and the model of pollution transport

- Pollution concept applies in this case to every factor which is introduced into the river in excess, meaning both: an increase in water temperature and substances dissolved in water

Description of the velocity field in the river

- In this work a simplified model was used:

$$\Delta\Delta\psi = \Delta\left(\frac{1}{h}\Delta\psi - \frac{1}{h^2}\nabla\psi\nabla h\right) = 0$$

- where: $\psi(x,y)$ - stream function
  $h(x,y)$ - depth in the river

- Solution of this problem in a two-dimensional space (x,y) is a stream function

$$\frac{\partial\psi}{\partial y} = h \cdot u_x, \quad -\frac{\partial\psi}{\partial x} = h \cdot u_y$$

- Finally, using this relation we can calculate the components of the velocity vector in the whole domain

- Transport of a non-degradable solute in a two-dimensional space can be written by the following equation:

- $$\frac{\partial hc}{\partial t}+\frac{\partial\left(hu_x c\right)}{\partial x}+\frac{\partial\left(hu_y c\right)}{\partial y}=\frac{1}{h}\frac{\partial}{\partial x}\left(hD_{xx}\frac{\partial c}{\partial x}+hD_{xy}\frac{\partial c}{\partial y}\right)+\frac{1}{h}\frac{\partial}{\partial y}\left(hD_{yx}\frac{\partial c}{\partial x}+hD_{yy}\frac{\partial c}{\partial y}\right)$$

- Temperature changes are described by the equation of unsteady heat transfer (with no external sources):

- $$\frac{\partial hT}{\partial t}+\frac{\partial\left(hu_x T\right)}{\partial x}+\frac{\partial\left(hu_y T\right)}{\partial y}=\frac{1}{h}\frac{\partial}{\partial x}\left(hD_{xx}\frac{\partial T}{\partial x}+hD_{xy}\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial y}\left(hD_{yx}\frac{\partial T}{\partial x}+hD_{yy}\frac{\partial T}{\partial y}\right)$$

- The coordinates of the dispersion tensor D are defined as follows:

$$D_{xx} = D_L n_x^2 + D_T n_y^2$$
$$D_{xy} = D_{yx} = (D_L - D_T) n_x n_y$$
$$D_{yy} = D_L n_y^2 + D_T n_x^2$$

- where **n** = [n_x,n_y] is the directional velocity vector:

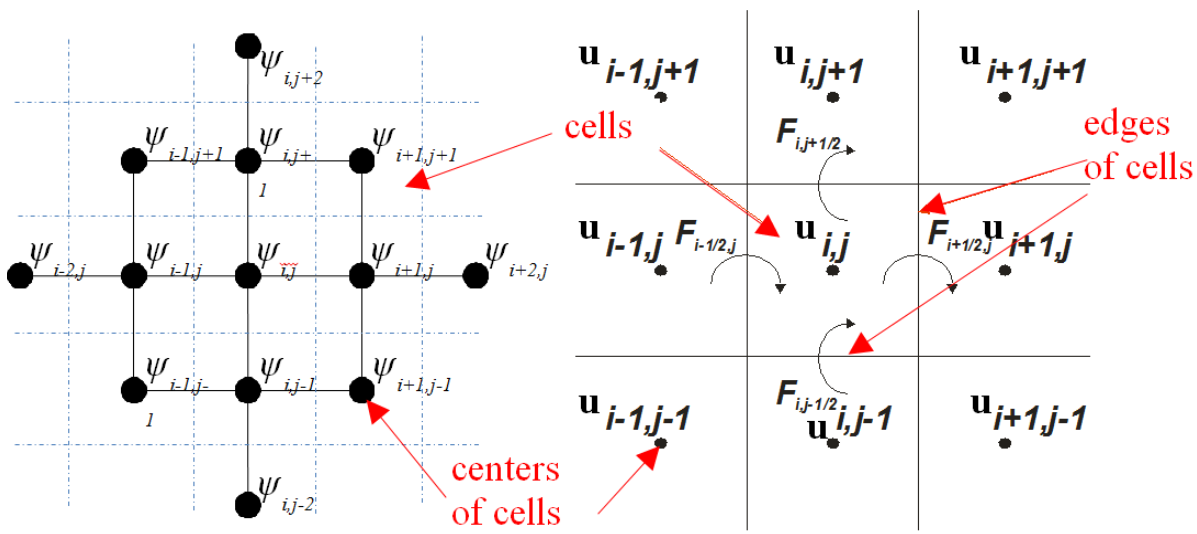
$$n_x = \frac{u_x}{|\mathbf{u}|}, n_y = \frac{u_y}{|\mathbf{u}|}$$

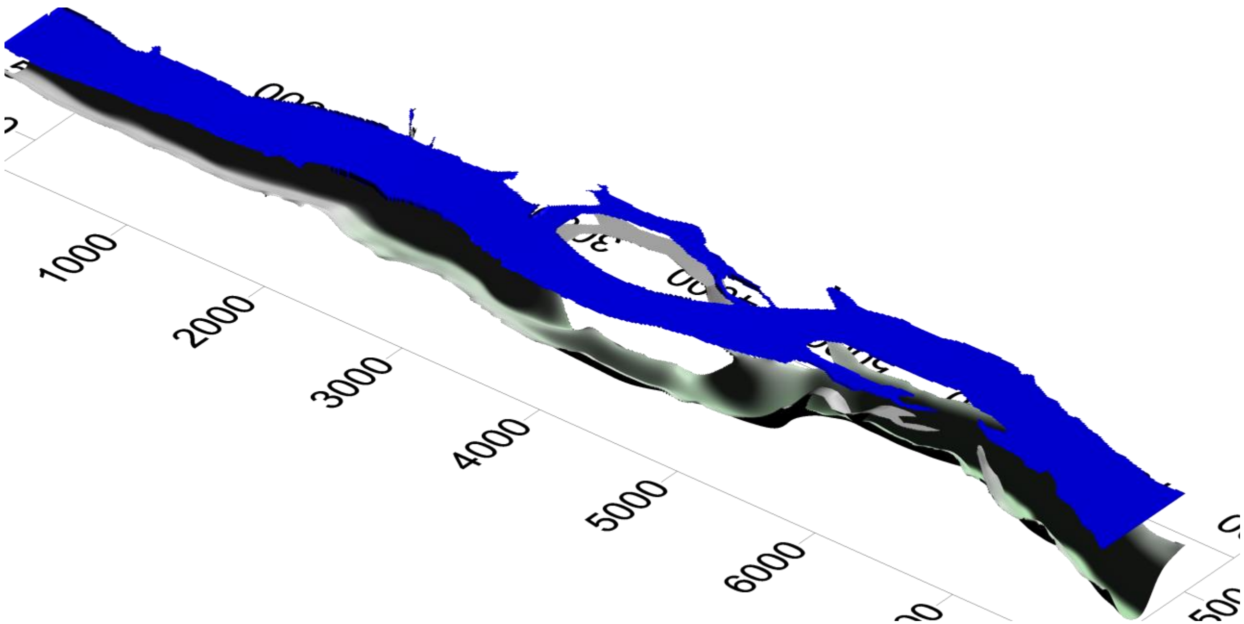
- Longitudinal D_L and transverse D_T coordinates of the dispersion tensor are described by the Elder formulas (v* dynamic velocity):

$$D_L = \alpha \cdot h \cdot v^* \quad 30 < \alpha < 3000$$
$$D_T = \beta \cdot h \cdot v^* \quad 0.15 < \beta < 0.30$$

- In this study, the finite difference method (FDM) for the biharmonic equation and the finite volume method (FVM) for the unsteady transport equations were used

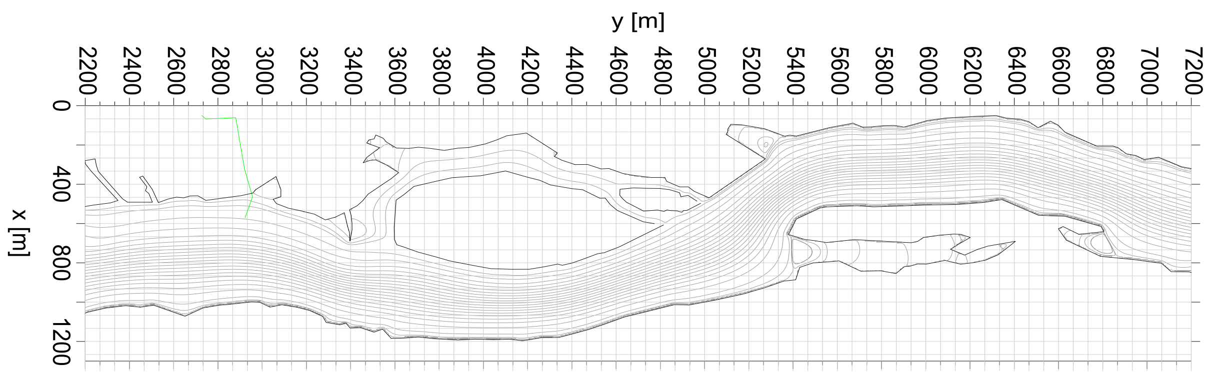
- In order to apply these methods, we divided the domain into square elements (cells) with $x=y=\Delta$. In FDM a 13-points scheme was used (left side). In FVM the the flux vector components F_x and F_y on the edges were calculated (right side)

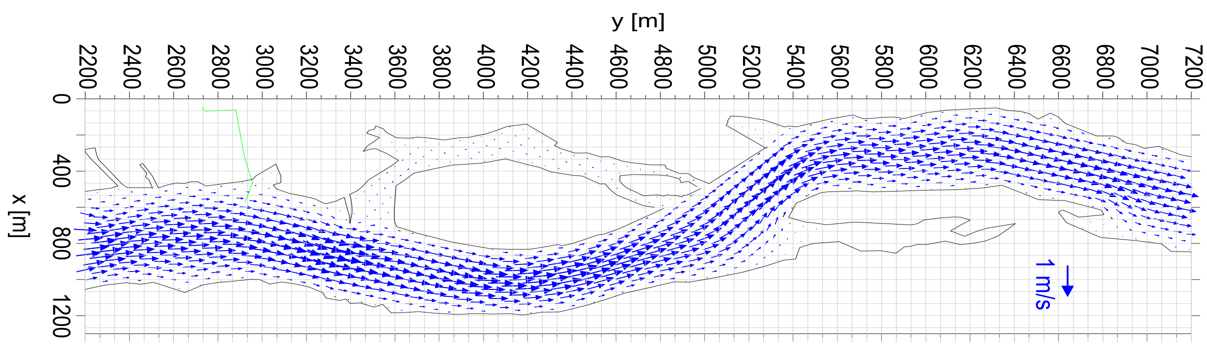


- Execution of numerical simulations of pollutant transport requires defining the solution area. It is a mathematical interpretation (model) of the actual conditions in which the flow takes place in the river
- This model is a result of the bathymetry of the river and the water level in identified flow conditions. The result of the calculations of the area for the analyzed case (from km 685+300 to km 693+300 of the Vistula River) is shown below
- 

Numerical simulations
Velocity distribution

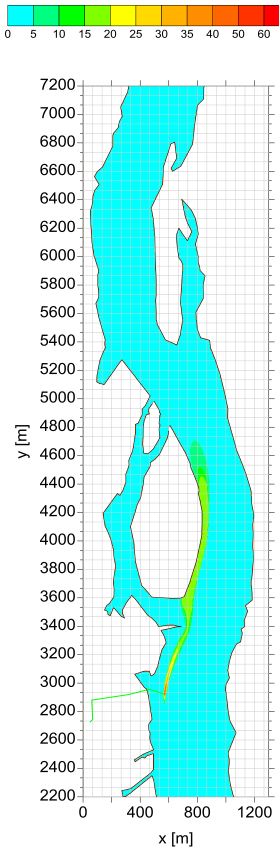
- The boundary conditions were defined on the basis of IMWM data for the MLQ flow. Discharge of wastewater was assumed as the maximum flow rate Q_{max}=0.686 m^3/s with maximum velocity v_{max} = 1.36 m/s
- The calculations performed yielded an image of the velocity distribution in the relevant section of the Vistula River Δ= 5 m

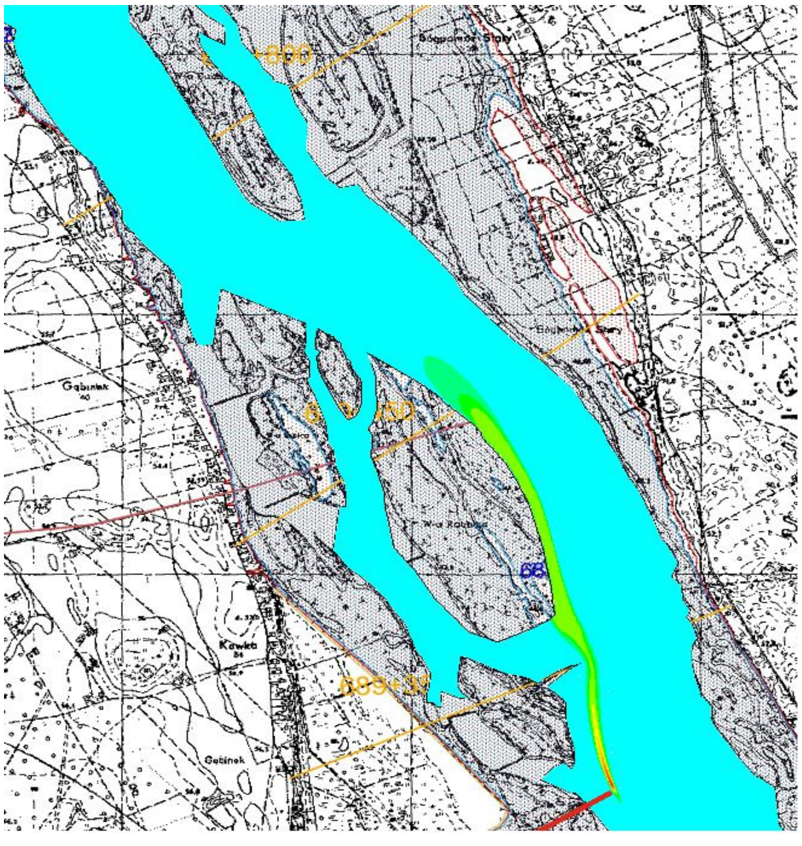




Numerical simulations
Solution of the advection-dispersion equation

- In order to solve the transport equation, the same area as in case of the velocity distribution was adopted. Mixing with river water was considered. The distribution of solute concentration and temperature was calculated (exampel below)





(distribution of concentration of the non-degradable dissolved matter (tracer) in the area of the dump

GDAŃSK UNIVERSITY OF TECHNOLOGY

Summary

In order to obtain the results of numerical simulations of pollutant transport in the section of the Vistula River (from km 685+300 to km 693+300) a mathematical model had to created

First, velocity distribution was calculated under steady flow conditions for MLQ = 293 m^3/s (flow rate and water level)

Calculations of pollution transport were carried out in unsteady conditions for pollution in the form of a non-degradable substance (tracer) and temperature higher than in the river

Discharge of sewage into the river was adopted in accordance with the project at approx. 150 m from the shore

Generally, the estimated range of pollutants was about 2 km from the dump (the area where the indicator was higher by at least 10% than the concentration or temperature in the river)

GDAŃSK UNIVERSITY OF TECHNOLOGY

HISTORY IS WISDOM
FUTURE IS CHALLENGE